International Journal of Scientific Research in Computer Science, Engineering and Information Technology
 | Volume 2 | Issue 3 | ISSN : 2456-3307

Technique for Solving Some Type of Improper Integral

Chii-Huei Yu

Department of Information Technology, Nan Jeon University of Science and Technology, Tainan City, Taiwan

ABSTRACT

This paper uses the mathematical software Maple as a problem-solving tool to study some type of improper integral. We use an important method in advanced calculus: differentiation with respect to a parameter, to find the infinite series form of this type of improper integral. The research method used in this paper is to get the answers through manual calculation, and then use Maple to verify the answers. This research way not only allows us to find the calculation errors, but also help us to amend the original thinking direction, because we can verify the correctness of our theory from the consistency of manual and Maple calculations.
Keywords: Improper Integral, Differentiation with Respect to a Parameter, Infinite Series Form, Maple

I. INTRODUCTION

This article describes how to use Maple to do mathematical research. The superiority of Maple lies in its simple instructions and ease of use, which enable beginners to learn the operating techniques in a short period. In addition, through the numerical and symbolic computations performed by Maple, the logic of thinking can be converted into a series of instructions. The computation results of Maple can be used to modify previous thinking directions, thereby forming direct and constructive feedback that can aid in improving understanding of problems and cultivating research interests. Inquiring through an online support system provided by Maple or browsing the Maple website (www.maplesoft.com) can facilitate further understanding of Maple and might provide unexpected insights. For the instructions and operations of Maple, we can refer to [1-5].

In advanced calculus and engineering mathematics courses, the study of the improper integral problem is an important issue. Yu [6-25], Yu and Chen [26], Yu and Sheu [27] used complex power series method, integration term by term theorem, differentiation with respect to a parameter, Parseval's theorem, area mean value theorem, and generalized Cauchy integral formula to solve some types of integrals. This paper considers the following some type of improper integral :

$$\int_0^1 \frac{x^a - \sum_{k=1}^{m} \frac{1}{(m-k)!} a^{m-k} (\ln x)^{m-k}}{(\ln x)^m} dx, \qquad (1)$$

where a is a real number, and m is a positive integer such that $-1 < a \le 1$ or $a = -1$ and $m \ge 2$. The infinite series form of this type of improper integral can be determined by an important method in advanced calculus: differentiation with respect to a parameter, that is the major result of this article: Theorem K. Moreover, some examples are used to do calculation practically, and the research method adopted in this paper is to obtain the answers through manual calculation, and then use Maple to verify the answers. Such a way of research not only allows us to find the calculation errors, but also help us to amend the original thinking direction, because we can verify the correctness of our theory from the consistency of manual and Maple calculations.

II. METHODS AND MATERIAL

First, we introduce the notations and theorems used in this paper.

2.1. Notations:

The p-times derivative of function $f(x)$ is denoted as $f^{(p)}(x)$, where p is a positive integer.

2.2. Theorems:

2.2.1. Differentiation with respect to a parameter ([28, p405]):

Suppose that I_1, I_2 are real intervals, and assume that two variables function $f(x,y)$ and its first order partial derivative with respect to y, $f_y(x,y)$ are all defined on $I_1 \times I_2$. If the following two conditions are satisfied: (i) For all $y \in I_2$, Lebesgue integrals $\int_{I_1} f(x,y)dx$ and $\int_{I_1} f_y(x,y)dx$ are all exist, (ii) $f_y(x,y)$ is a continuous function defined on $I_1 \times I_2$ such that $\int_{I_1} f_y(x,y)dx$ is uniformly convergent on I_2. Then $F(y) = \int_{I_1} f(x,y)dx$ is differentiable on I_2, and its derivative $\frac{d}{dy}F(y) = \int_{I_1} f_y(x,y)dx$.

2.2.2. Abel limit theorem ([29, p245])：

Assume that $r > 0$ and c_n are all real numbers for all non-negative integers n. Let $F(a) = \sum_{n=0}^{\infty} c_n a^n$, where $-r < a < r$. If $\sum_{n=0}^{\infty} c_n r^n$ exists, then $\lim_{a \to r^-} F(a)$ exists and $\lim_{a \to r^-} F(a) = \sum_{n=0}^{\infty} c_n r^n$.

The following theorem is the main result in this paper, we obtain the infinite series form of the improper integral (1).

Theorem K：*Suppose that* a *is a real number, and* m *is a positive integer such that* $-1 < a \le 1$ *or* $a = -1$ *and* $m \ge 2$*, then*

$$\int_0^1 \frac{x^a - \sum_{k=1}^{m} \frac{1}{(m-k)!} a^{m-k} (\ln x)^{m-k}}{(\ln x)^m} dx$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{1}{(n+1)(n+2)\cdots(n+m)} a^{n+m}. \quad (2)$$

Proof Since $\int_0^1 x^a dx = \frac{1}{a+1}$ is uniformly convergent, it follows from differentiation with respect to a parameter that

$$\frac{d}{da}\left(\int_0^1 \frac{x^a - 1}{\ln x} dx\right) = \int_0^1 x^a dx. \quad (3)$$

Since $\int_0^1 \frac{x^a - 1}{\ln x} dx$ is also uniformly convergent, then again by differentiation with respect to a parameter, we obtain

$$\frac{d}{da}\left(\int_0^1 \frac{x^a - a\ln x - 1}{(\ln x)^2} dx\right) = \int_0^1 \frac{x^a - 1}{\ln x} dx. \quad (4)$$

And so on, we get

$$F(a) = \int_0^1 \frac{x^a - \sum_{k=1}^{m} \frac{1}{(m-k)!} a^{m-k} (\ln x)^{m-k}}{(\ln x)^m} dx$$

satisfies the following ordinary differential equation:

$$\begin{cases} F^{(m)}(a) = \dfrac{1}{a+1} \\ F(0) = F'(0) = F''(0) = \cdots = F^{(m-1)}(0) = 0 \end{cases}. \quad (5)$$

Case 1. If $-1 < a < 1$. Since $\frac{1}{a+1} = \sum_{n=0}^{\infty} (-1)^n a^n$, it follows that

$$F(a) = \sum_{n=0}^{\infty} (-1)^n \frac{1}{(n+1)(n+2)\cdots(n+m)} a^{n+m}.$$

Case 2. If $a = 1$. Since the infinite series

$$\sum_{n=0}^{\infty} (-1)^n \frac{1}{(n+1)(n+2)\cdots(n+m)} \quad (6)$$

exists, then by Abel limit theorem, the desire result holds.

Case 3. If $a = -1$ and $m \ge 2$. Since

$$\sum_{n=0}^{\infty} \frac{1}{(n+1)(n+2)\cdots(n+m)} \quad (7)$$

exists, it also follows from Abel limit theorem that the desire result holds. q.e.d.

III. EXAMPLES

In the following, for the improper integral problem discussed in this paper, a few examples are proposed to do calculation practically, and we use Maple to calculate the approximations of these improper integrals and their infinite series forms to verify our answers.

Example 3.1. By Theorem K, we have

$$\int_0^1 \frac{x^{1/3}-\frac{1}{3}\ln x-1}{(\ln x)^2}dx=\sum_{n=0}^{\infty}\frac{(-1)^n}{(n+1)(n+2)}\left(\frac{1}{3}\right)^{n+2}. \tag{8}$$

Next, we employ Maple to verify the correctness of Eq. (8):

>evalf(int((x^(1/3)-ln(x)/3-1)/(ln(x))^2,x=0..1),14);

0.0502427632691

>evalf(sum((-1)^n*(1/3)^(n+2)/((n+1)*(n+2)),n=0..infinity),14);

0.05024276326901

Example 3.2. Using Theorem K yields

$$\int_0^1 \frac{x^{-1/4}-\frac{1}{32}(\ln x)^2+\frac{1}{4}\ln x-1}{(\ln x)^3}dx$$

$$=\sum_{n=0}^{\infty}\frac{(-1)^n}{(n+1)(n+2)(n+3)}\left(\frac{-1}{4}\right)^{n+3}$$

$$=-\sum_{n=0}^{\infty}\frac{1}{(n+1)(n+2)(n+3)}\left(\frac{1}{4}\right)^{n+3}. \tag{9}$$

We also use Maple to verify the correctness of Eq. (9).

>evalf(int((x^(-1/4)-(ln(x))^2/32+ln(x)/4-1)/(ln(x))^3, x=0..1),14);

-0.00278558287707

>evalf(-sum((1/4)^(n+3)/((n+1)*(n+2)*(n+3)),n=0..infinity),14);

-0.002785582877063

Example 3.3. By Theorem K, we have

$$\int_0^1 \frac{x^{1/\sqrt{2}}-\frac{1}{96}(\ln x)^4-\frac{\sqrt{2}}{24}(\ln x)^3-\frac{1}{4}(\ln x)^2-\frac{\sqrt{2}}{2}\ln x-1}{(\ln x)^5}dx$$

$$=\sum_{n=0}^{\infty}\frac{(-1)^n}{(n+1)(n+2)(n+3)(n+4)(n+5)}\left(\frac{1}{\sqrt{2}}\right)^{n+5}. \tag{10}$$

Below we use Maple to verify the answer:

>evalf(int((x^(1/sqrt(2))-(ln(x))^4/96-(ln(x))^3*sqrt(2)/24-(ln(x))^2/4-ln(x)*sqrt(2)/2-1)/(ln(x))^5,x=0..1),14);

0.0013274751954515

>evalf(sum((-1)^n*(1/sqrt(2))^(n+5)/((n+1)*(n+2)*(n+3)*(n+4)*(n+5)),n=0..infinity),15);

0.0013274751954588

IV. CONCLUSION

From above, we see that theorem K is the main theoretical basis for solving the improper integral problem discussed in this article. On the other hand, we know that Maple plays an important role in problem-solving. Moreover, we can also use Maple to design some improper integral problems, and try to find the solutions of these problems. So, Maple can give us the inspiration to solve the problems.

V. REFERENCES

[1]. Prof. M. L. Abell and J. P. Braselton, Maple by Example, 3rd ed., Elsevier Academic Press, 2005.

[2]. R. M. Corless, Essential Maple, Springer Verlag, New York, 1994.

[3]. F. Garvan, The Maple Book, Chapman & Hall/CRC, 2001.

[4]. A. Heck, Introduction to Maple, 3rd ed., Springer-Verlag, New York, 2003.

[5]. D. Richards, Advanced Mathematical Methods with Maple, Cambridge University Press, 2002.

[6]. C. H. Yu, “Application of Parseval’s theorem on evaluating some definite integrals,” Turkish Journal of Analysis and Number Theory, Vol. 2, No. 1, pp. 1-5, 2014.

[7]. C. H. Yu, “Evaluation of two types of integrals using Maple, ” Universal Journal of Applied Science, Vol. 2, No. 2, pp. 39-46, 2014.

[8]. C. H. Yu, “Studying three types of integrals with Maple, ” American Journal of Computing Research Repository, Vol. 2, No. 1, pp. 19-21, 2014.

[9]. C. H. Yu, “The application of Parseval’s theorem to integral problems,” Applied Mathematics and Physics, Vol. 2, No. 1, pp. 4-9, 2014.

[10]. C. H. Yu, “A study of some integral problems using Maple, ” Mathematics and Statistics, Vol. 2, No. 1, pp. 1-5, 2014.

[11]. C. H. Yu, “Solving some definite integrals by using Maple, ” World Journal of Computer Application and Technology, Vol. 2, No. 3, pp. 61-65, 2014.

[12]. C. H. Yu, “Using Maple to study two types of integrals,” International Journal of Research in Computer Applications and Robotics, Vol. 1, Issue. 4, pp. 14-22, 2013.

[13]. C. H. Yu, “Solving some integrals with Maple,” International Journal of Research in Aeronautical and Mechanical Engineering, Vol. 1, Issue. 3, pp. 29-35, 2013.

[14]. C. H. Yu, “A study on integral problems by using Maple, ” International Journal of Advanced Research in Computer Science and Software Engineering, Vol. 3, Issue. 7, pp. 41-46, 2013.

[15]. C. H. Yu, “Evaluating some integrals with Maple,” International Journal of Computer Science and Mobile Computing, Vol. 2, Issue. 7, pp. 66-71, 2013.

[16]. C. H. Yu, “Application of Maple on evaluation of definite integrals, ” Applied Mechanics and Materials, Vols. 479-480 (2014), pp. 823-827, 2013.

[17]. C. H. Yu, “Application of Maple on the integral problems,” Applied Mechanics and Materials, Vols. 479-480 (2014), pp. 849-854, 2013.

[18]. C. H. Yu, “Using Maple to study the integrals of trigonometric functions,” Proceedings of the 6th IEEE/International Conference on Advanced Infocomm Technology, Taiwan, No. 00294, 2013.

[19]. C. H. Yu, “A study of the integrals of trigonometric functions with Maple,” Proceedings of the Institute of Industrial Engineers Asian Conference 2013, Taiwan, Springer, Vol. 1, pp. 603-610, 2013.

[20]. C. H. Yu, “Application of Maple on the integral problem of some type of rational functions, ” (in Chinese) Proceedings of the Annual Meeting and Academic Conference for Association of IE, Taiwan, D357-D362, 2012.

[21]. C. H. Yu, “Application of Maple on some integral problems, ” (in Chinese) Proceedings of the International Conference on Safety & Security Management and Engineering Technology 2012, Taiwan, pp. 290-294, 2012.

[22]. C. H. Yu, “Application of Maple on some type of integral problem,” (in Chinese) Proceedings of the Ubiquitous-Home Conference 2012, Taiwan, pp.206-210, 2012.

[23]. C. H. Yu, “Application of Maple on evaluating the closed forms of two types of integrals,” (in Chinese) Proceedings of the 17th Mobile Computing Workshop, Taiwan, ID16, 2012.

[24]. C. H. Yu, “Application of Maple: taking two special integral problems as examples,” (in Chinese) Proceedings of the 8th International Conference on Knowledge Community, Taiwan, pp.803-811, 2012.

[25]. C. H. Yu, “Evaluating some types of definite integrals, ” American Journal of Software Engineering, Vol. 2, Issue. 1, pp. 13-15, 2014.

[26]. C. H. Yu and B. H. Chen, “Solving some types of integrals using Maple,” Universal Journal of Computational Mathematics, Vol. 2, No. 3, pp. 39-47, 2014.

[27]. C. H. Yu and S. D. Sheu, “Using area mean value theorem to solve some double integrals,” Turkish Journal of Analysis and Number Theory, Vol. 2, No. 3, pp. 75-79, 2014.

[28]. L. Flatto, Advanced Calculus, The Williams & Wilkins Co., 1976.

[29]. T. M. Apostol, Mathematical Analysis, 2nd ed., Addison-Wesley Publishing Co., Inc., 1975.