International Journal of Scientific Research in Computer Science, Engineering and Information Technology
 | Volume 2 | Issue 4 | ISSN : 2456-3307

A Study on Transfinite Graph with Least Power Electric Circuit

S. Nathiya, Dr. S. Subramanian

Department of Mathematics, PRIST University, Thanjavur, Tamil Nadu, India

ABSTRACT

Graph theory is helpful in various practical problems solving in circuit or network analysis and data structure. It leads to graph practically not possible to analyse without the aid of computer. In electrical engineering the word is used for edge, node for vertex and loop for circuit. An electrical network is the set of electronic components i.e. resistors, inductors and capacitors etc. Electric network analysis and synthesis are the study of network topology. Electric network problem can be represented by drawing graphs. In this paper, we present a circuit network in the concept of graph theory application and how to apply graph theory to model the circuit network. We study the vulnerability of electrical networks through structural analysis from a graph theory point of view. We measure and compare several important structural properties of different electrical networks, including a real power grid and several synthetic grids, as well as other infrastructural networks. The properties we consider include the minimum dominating set size, the degree distribution and the shortest path distribution. We also study the network vulnerability under attacks in terms of maximum component size, number of components and flow vulnerability. Our results suggest that all grids are more vulnerable to targeted attacks than to random attacks. We also observe that the electrical networks have low tree width, which explains some of the vulnerability. We prove that with a small tree width, a few important structural properties can be computed more efficiently.
Keywords : Electrical Infrastructure, Vulnerability, Graph Theory.

I. INTRODUCTION

The North American power grid is a giant network of more than 15 000 generators in 10 000 power plants, and hundreds of thousands of miles of transmission lines. Analysts estimate it to be worth over $800 billion. In 2000, the transmission and distribution infrastructure was valued at $358 billion. Geographically, the power grid forms a network of over 1 million kilometres of high voltage lines that are continuously regulated by sophisticated flow control equipment (Albert 2004; Amin, 2003; 2002; EPRI, 1999). A connected graph without closed path i.e. tree was implemented by Kirchhoff in 1847 and he employed graph theoretical concept in the calculation of currents in network or circuits and was improved upon J.C.Maxwell in 1892.Ever since, graph theory has been applied in electrical network analysis .An electrical network is a collection of components and device interconnected electrically .The network components are idealized of physical device and system, in order to for them to represent several properties, they must obey the Kirchhoff's law of currents and voltage. A graph representation of electrical network in terms of line segments or arc called edges or branches and points called vertices or terminals. The robustness of the electrical networks can be studied in many different ways. For instance, one can develop a scenario in which a critical node or a transmission line fails and use a model of the grid or the grid itself to emulate the reaction of that failure. This method can directly measure the impact of the event but it is often difficult to build a realistic model of the grid or use the real grid. Another possible method is to study the structural properties of the grid and associate them with the robustness of the grid. For example, the degree distribution of the grid provides topological information on the grid. If the degree distribution has a heavy tail, one can expect that even a random attack on the grid can easily shatter it into disconnected components. On the other hand, a thin tail distribution may indicate that the grid is vulnerable to only targeted attacks. The focus of our work is to use the structural measures of a grid to determine its robustness. While this approach, admittedly, only studies a static approximation of the power grid, it has the potential of giving fast worst case estimates of various kinds of failures. For instance, the number of nodes needed to shatter the graph is an upper bound on the number of nodes that can fail before the whole grid fails, because the shattering model does not take cascading failures into account. For instance, knowing whether the grid is one large connected component is useful in determining the feasibility of transferring power between any two nodes on the network. If a natural or man-made disaster removes a few nodes of the grid it would be important to know the size of the maximum connected component of the remaining network. The size of the maximum connected component can be computed more efficiently for networks that have small 'tree-width'. We also study 'tree-width', an important structural property of the power grid, which helps explain some of the source of vulnerability of the grid.

II. METHODS AND MATERIAL

Basic Definition of Graph Theory

Graphs are amenable for pictorial representation of a system using two basic components vertex and edges. A vertex is represented by a dot and an edge is represented by line segment connecting the dots associated with the edge. If the edges of a graph direct one vertex to the other vertex, then the graph is called as a directed graph. Otherwise graph is called an undirected graph. [2] Formally, a graph G= (V, E) contains a finite set V= (v_1, v_2,…..,v_n) of elements called vertices and a finite set (e_1 ,e_2,…..,e_m) of elements called edges. In an undirected graph G= (V, E), the edges are unordered pairs, and each edge e_1 in E is associated with two vertices v_1 and v2, and it is written as either e_1= (v_1,v_2) or e_1= (v_2, v_1).But, In a directed graph, each edge e1 in E is associated with an ordered pairs of vertices (v1, v2) and it is denoted the directed edge e_1 from v_1 to v_2. Two vertices v_1 and v_2 of a graph are adjacent, if there is an edge, v_1v_2 connecting them, then vertices are them considered incident to the edge v_1v_2 . [5]

2.1. Definition

The capacity of a power grid is the maximum flow that can be sent from the generator nodes to the consumer nodes (load serving nodes), subject to the transmission line capacity constraints, generator capacity constraints, and the substation capacity constraints.

2.2. Definition

The flow vulnerability of a power grid, subject to node or link deletion, is the percentage decrease in the grid capacity.

3. Analysis of Electrical Circuit

Ohm's law states that for an edge 'e', the current flowing across that edgeI_e is given by$I_e = \frac{pc}{rc} = pc.ce$ We see that this means that i{uv} = -i{vu} and the negative current as positive currents flowing the different way. The weight of an edge as the conductance of that edge, which denote Ce for a given edge e .The resistance of an edge re is defined as$r_e = \frac{1}{ce}$. Both the resistance and conductance are independent of edge such as r(vu) = r(uv) and c(uv) = c(vu).

4. Kirchhoff's Circuit Law

Kirchhoff's voltage law states that for a closed loop SV=0 or SV rise is equal to SV drops. The total resistance of 'n' resistors in series is RT= R1+R2+R3+………+Rn and the total power are PT= P1+P2+P3+…….Pn In series, So that the same current flows through all the components but a different potential voltage can exist across every one. In parallel, so that the same potential difference exists across every components but each component may carry a different current. Representation of circuit and its graph: A graph model is used to represented circuit network inn graph by tracing the nodes of the circuit and edges contain in circuit.

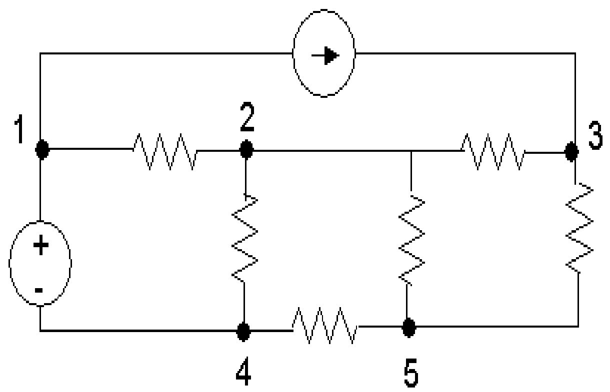

Figure 1. Here is the graph of the circuit

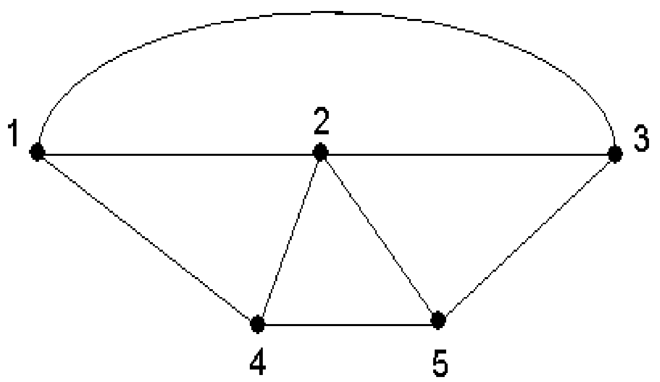

Figure 2: Network Graph

A circuit is a path which ends at the vertex it begins. An electric circuit is a closed loop formed by source, wires, load, and a switch, when switch is turned on the electrical circuit is complete and current flows from negative terminals of the power source. An electrical circuit is categories in to three type namely series, parallel and series and parallel circuit. The representation of graph in circuit network are one of the type of representation of graph in which the current flows in circuit and present the linking of connection between resistors series and parallel connection are determined in the circuit.[4] The representation is The schematic figure of the electric circuit is as follows,

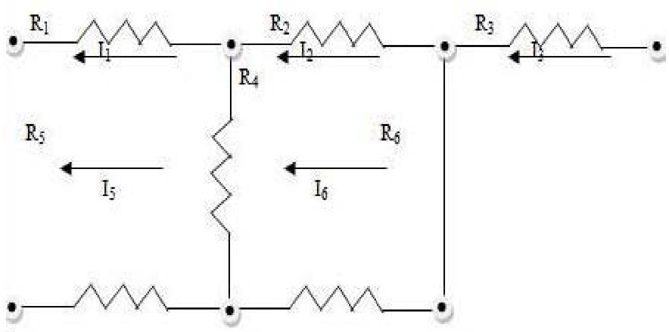

Figure 3

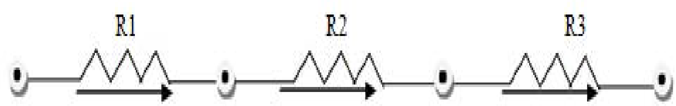

Figure 4

The electrical features of individual network components can be representing suitably in the form of primitive network matrix that describe the performance of interconnected network.

5. Tree-width of power networks

We now study a new property of power grids, called tree-width that has fundamental implications for the robustness of power grids and the efficiency of analytical methods on such grids. Intuitively, a graph has low tree-width, if it has a 'tree-like' structure. We find empirically that all the power grids studied in this paper have low tree-width. We first provide the definitions from Bodlaender (1993). A tree-decomposition of a graph G(V, E) is a pair ($\{X_i|i \in I\}$, T = (I, F)) with $\{X_i : i \in I\}$ a family of subsets of V, one for each node of T, and a tree T such that:

- $\cup i \in I\ Xi = V$
- For each edge (v, w) ∈ E, there exists an i ∈ I with v, w $\in X_i$
- For all i, j, k in I, if j is on the path from i to k in T, then $X_i \cap X_k \subseteq X_j$.

The tree-width of a tree-decomposition ($\{X_i : i \in I\}$, T = (I, F)) is defined as $\max_i \in I\ |X_i| - 1$. The tree-width of a graph G is the minimum tree-width over all possible Tree-decompositions of G. The smaller the tree-width, the closer the graph is to a tree– indeed, if the tree-width is 1, the graph is a tree. Computing the tree width of a graph is a hard problem. Computing it exactly for any given graph is known to be NP-complete (Arnborg et al., 1987). Therefore, we use the software by Gogate and Dechter (2004) for estimating the tree width of our graphs. Even with this software, we are not able to compute the tree width of all the grids exactly. Table 1 shows the tree width of different grids computed using the algorithm of Gogate and Dechter (2004). Observe that they are fairly small – for the real grid, it is no more than 10. In contrast, random graphs have very high tree width – the result by Kloks and Bodlaender (1992) shows that almost all graphs with n vertices and at least δ_n edges have tree width at least nε, where $\delta > 1$ and $\varepsilon < (\delta - 1)/(\delta + 1)$ are constants. Thus, this property conclusively shows that power grids are not like random graphs, and require very different models

Table 1 The tree width of different grids computed using Gogate and Dechter (2004)

Grid	# Nodes	Tree width
Real	662	≤10
14	14	2
30	30	3
57	57	5
118	118	4
145	145	10
162	162	≤12
300	300	≤6

III. RESULTS AND DISCUSSION

Implications of low tree width

The low tree width of power grids has a number of implications – these are negative from an operational and robustness point of view, and are extremely positive from an algorithmic and planning point of view. For robustness of the grid, the low tree width is bad, because it leads to the vulnerability in maximum component size, as shown earlier. In fact, as we will discuss later, the vulnerability in the maximum component size can be quantified in terms of the tree width. From an algorithmic point of view, tree width has a number of positive implications. A number of very hard problems become easy, *i.e.*, solvable in polynomial time, if the tree width is low. Most of the structural measures discussed in this paper can be computed more efficiently in low tree width graphs, than in general graphs. Shortest paths in graphs can be computed much more efficiently in parallel in low tree width graphs (Cohen,1996). Optimum dominating sets can be computed in polynomial time in low tree width graphs (Telle and Proskurowski, 1997); note that the dominating set sizes reported in Table 1 are computed by running a greedy algorithm, which does not give optimal solutions in general, but only an $O(\log n)$ approximation (Vazirani, 2004). Perhaps, the most useful result for the analysis of power grids is that systems of linear equations can be solved much more efficiently if the underlying graph has low tree width (Radhakrishnan *et al.*, 1992) – since Kirchoff's laws are linear constraints, this means that DC flow can be solved more efficiently in such graphs. Finally, we study two measures of robustness – the first is the size of the largest component, and the second is the number of components. We show that the former can be quantified in terms of the tree width. We then describe a polynomial time algorithm to determine which k nodes to delete, so that either the maximum component size is minimised or the number of resulting components is maximised. Recall that in Section 3.2, we studied the effect of greedy and random node deletion on both these measures.

IV. CONCLUSION

In this research we focus on the application of graph theory to electrical network analysis and approach as an electrical network analysis. Graph theory is a very interesting topic in mathematics due to numerous applications in various fields especially in computer and electrical engineering. We use the graph theory concept and techniques that we have developed to study electrical networks. Thus, graph theory has more practical application particulars in solving electric network.

V. REFERENCES

[1]. Amin, M. (2003) 'North America's electricity infrastructure: are we ready for more perfect storms?', IEEE Security and Privacy, Vol. 1, No. 5, pp.19-25.

[2]. Arnborg, S., Corneil, D. and Proskurowski, A. (1987) 'Complexity of finding embed-dings in a k-tree', SIAM Journal of Algebraic and Discrete Methods, Vol. 8, pp.277-284.

[3]. Barrett, C.L., Eubank, S. and Marathe, M.V. (2006) 'Modeling and simulation of large biological, information and socio-technical systems: an interaction based approach', in D. Goldin, S. Smolka and P. Wegner (Eds.) Interactive Computation: The New Paradigm, Springer Verlag.

[4]. Bodlaender, H.L. (1993) 'A tourist guide through treewidth', Acta Cybernetica, Vol. 11, Nos. 1-2,pp.1-22.

[5]. Sudhakaran, Electrical circuit analysis, Tata Mc Grow-Hill Pvt ltd.

[6]. B.Bollobas, Modern Graph Theory, Springer 1998.

[7]. F.Kirchoff's,Ubar die Auflosung der Gleichungen,auf welche man bei der Untersuchung der linearen Verteilung galvanischer Strome gefuhrt wird,Ann.Phys.chem. 72(1847),497-508

[8]. Introductory Graph Theory for Electrical and Electronics Engineers,IEEE MULTIDISCIPLINARY ENGINEERING EDUCATION MAGAZINE. [5] Narasih Deo, Graph theory & its Application to computer science.