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New Contractive Mapping Based Invariant Points in Topological Space

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ABSTRACT

This paper proposes the new contractive mapping based invariant points in topological space by the study of topological semigroup over the fixed point. This mapping is defined in metric space into itself under the new condition involved the inequality sign of less than or equal to. The continuous and equi-continuous properties are studied over the convex subset. The fixed-point properties in convex space is generalised in this paper. The reversibility of the topological semigroup is applied to propose the new invariant points.

Keywords: Invariant Point, Topological Semigroup, Fixed Point.

I. INTRODUCTION

The study of surface is topology. Topology is the study of special characteristic of the set. The points of the set and its neighbourhood properties referred a topological space. In this paper, we review the topological space over the topological semigroup properties. By this, we will find some new invariant points in topological space.

Levine [10] presented an application of the semi-open sets and semicontinuity in topological space in 1963. In 1966, Velicko [18] introduced H-closed topological space. The convex space in reviewed in this paper.

Crossley et al [2] found some new properties on semi-topological space in 1972. In 1982, Masshour et al [11] studied precontinuous mappings. Next year, Gould et al [9] discussed the continuity properties of the functions. In 1985, Reilly et al [6] put the new concept of continuity in topological space.

In 1999, Gould [8] generalised the continuity for variation. The generalised topology is established in 2002 by Csazar [4]. Next year, he [5] gave the new definition of the connected set. He [3] overviewed the open sets in 2005. He [7] modified the previous results in 2008.

Next year, Min [13] proposed the ordered continuity. Shen [17] generalised the connectedness in 2009. In 2010, He [14] developed generalised topological space.
Bai et al [1] discovered the new irresolute function in 2011.

This paper defines a new contractive mapping in the metric space with some new characteristics.

II. METHODS AND MATERIAL

Topological Space : A topology on a set X is a collection τ of subsets of X, that,

a. Contains ϕ, X.
b. Closed under the formation of finite intersections, and,

c. Closed under the formation of arbitrary unions. Hence the ordered pair (X, τ) is called a topological space.

III. RESULTS AND DISCUSSION

Proposed Results : Let S be a topological semigroup and I be a closed ideal of S. First, we propose the new mapping of contraction, which is given in below:

3.1. **Definition (New Contractive Mapping) :** A mapping T of a metric space E into itself is said to be contractive if the following conditions holds:

1. $d(Tx, Ty) < d(x, y).$
2. $x \leq y.$
3. $x - \varepsilon, y + \varepsilon \in E.$
4. $\frac{x}{y} < \varepsilon.$
5. $\frac{x\varepsilon}{y} \leq \frac{y}{x\varepsilon}.$

The next definition is based on $\varepsilon -$ contractive *mapping,* which is presented below:

3.2. **Definition** ***(New $\varepsilon -$ contractive mapping) :*** A mapping T is said to be $\varepsilon -$contractive mapping if it holds the followings:

1. $0 < d(\frac{x}{y}) < \varepsilon.$
2. $d(Tx, Ty) < d(\frac{x}{y}).$

The following theorem lies with the invariant points in the topological space.

3.1. Theorem: I / S is an invariant point in topological space.

Proof: Since, S / I is a Hausdorff $k_w -$space.

Let a map be π defined by,

$\pi : S \to S / I.$

Suppose,

$I \neq S.$

Then,

A map of product topology is defined by,

$S \times S \to S.$

This map is continuous (jointly).

The Rees Congruence is defined on the ideal by,

$$\rho_I = \Delta_S \cup (I \times I).$$

Where,

$\Delta_S = \{(s, s) : s \in S\}.$

Hence,

It holds the compact subsets. Thus an invariant point is s in topological space.

This completes the proof. Next is lemma on isomorphism of the invariant points in topological space.

3.2. **Lemma:** $I = J.$

Proof: Let I and J be the two ideals of the topological semigroup S.

By Topological Rees Congruence,

I and J are the closed ideal.

Thus,

$$S / I = J / I.$$

But,

$$\rho_I : I \times I,$$
$$\rho_J : J \times J.$$

Since,

ρ_I and ρ_J, both is the closed map, hence,

$$I = J.$$

IV. CONCLUSION

The new invariant point defines the Euclidean distance in rational fixed point condition. The fixed point considered as the contractive point under the semi topological space. An invariant point on the Hausdorff space is studied in this paper. This point shows that the compact space over the compact subsets is closed. The union of the countable collection of the compact subset is the resultant of this point. Another new result came as the semigroup which called topological semigroup, if the multiplication be jointly continuous.

V. REFERENCES

[1]. S. Z. Bai and Y.-P. Zuo, On g-α-irresolute functions, Acta Math. Hungar. 130, no. 4 (2011), 382-389.

[2]. S. G. Crossley and S. K. Hildebrand, Semi-topological properties, Fund. Math. 74 (1972), 233-254.

[3]. A. Cs'asz'ar, Generalized open sets in generalized topologies, Acta Math. Hungar. 106, no. 1-2 (2005), 53-66.

[4]. A. Cs'asz'ar, Generalized topology, generalized continuity, Acta Math. Hungar. 96 (2002), 351-357.

[5]. A. Cs'asz'ar, γ-connected sets, Acta Math. Hungar. 101 (2003), 273-279.

[6]. A. Cs'asz'ar, Normal generalized topologies, Acta Math. Hungar. 115, no. 4 (2007), 309-313.

[7]. A. Cs'asz'ar, δ- and θ-modifications of generalized topologies, Acta Math. Hungar. 120 (2008), 275-279.

[8]. D. B. Gauld, S. Greenwood and I. L. Reilly, On variations of continuity, Topology Atlas, Invited Contributions 4 (1) (1999), 1-54.

[9]. D. B. Gauld, M. Mr sevi'c, I. L. Reilly and M. K. Vamanamurthy, Continuity properties of functions, Colloquia Math. Soc. Janos Bolyai, 41 (1983), 311-322.

[10]. N. Levine, Semi-open sets and semicontinuity in topological spaces, Amer. Math. Monthly 70 (1963), 36-41.

[11]. A. Mashhour, M. Abd. El-Monsef and S. El-Deeb, On precontinuous and weak precontinuous mappings, Proc. Math. Phys. Soc. Egypt 53 (1982), 47-53.

[12]. W. K. Min, Almost continuity on generalized topological spaces, Acta Math. Hungar. 125, no. 1-2 (2009), 121-125.

[13]. W. K. Min, A note on θ(g, g')-continuity in generalized topological spaces, Acta Math. Hungar., 125 (4) (2009), 387-393.

[14]. W. K. Min, (δ, δ')-continuity on generalized topological spaces, Acta Math. Hungar. 129, no. 4 (2010), 350-356.

[15]. M. Mr. sevi'c, I. L. Reilly and M. K. Vamanamurthy, On semi-regularization topologies, J. Austral. Math. Soc. (Series A) 38 (1985), 40-54.

[16]. I. L. Reilly and M. K. Vamanamurthy, On α-continuity in topological spaces, Acta Math. Hungar. 45 (1985), 27-32.

[17]. R. X. Shen, A note on generalized connectedness, Acta Math. Hungar. 122, no. 3 (2009), 231-235.

[18]. N. V. Veliˇcko, H-closed topological spaces, Mat. Sbornik 70 (112) (1966), 98-112.