

# On the Invariant Points in Topological Spaces

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## ABSTRACT

This paper presents the topological space over fuzzy set. There are the fuzzy points. The open set is reviewed here to develop this result. The collection of subsets of the set of the topological space is generalised in this paper. The fuzzy topological space and its operation are used in fuzzy points. The order of the subset of the topological space will be even is proved also in this paper.

**Keywords :** Topological space, Fuzzy set, Fuzzy points.

## I. INTRODUCTION

Topological space uses in various applications in modern era. The major reason behind this can be stated as its set characteristics. The characteristics of the set also. Topological spaces classifies in two sections, pure and applied. Basically applied lies with the generalised topological space. The particular axioms are responsible of the difference between these two variants of the topological spaces.

The set theory is the centre of the topological study. Various types of the set decide the topological space as its kind. The fuzzy set, vague set, rough set etc. involves in the topological space with uncertainties. This uncertainty creates other dimension of the mathematics.

Ali et al [1] proposed an idea of generalisation of topological space by applying the new operations on set theory. This result came in the year 2009. Ayguoglu et al [2] developed soft topological space in 2011. The variants of the sets are used for this result.

Cagman et al [3] establish a new decision technique based on the study matrix theory in 2010. The parametrization reduction of soft sets is applied in topological space in 2005 by Chen [5].

In 2002, Csaszar [6] generalised topological space as the generalised continuity. In 2011, Hussain et al [7]

overviewed topological space over discrete properties. Maji et al [8] proposed topological space in decision making problem in 2003. They [9] also proposed a soft set theory in the same year.

Min [10] put a new concept on soft topological space in 2011. In 1999. Molodotsov [11, 12] launched the system of computing the soft set. Munkers [13] is a noteworthy literature.

In 2005, an application of topological space came in the existence, as information theory. This is proposed by Pie et al [14].

Peyghan et al [15] presented topological space as the new applicable frame in 2013. In 2011. Shabir et al [16] shown some conjectures on topological space.

Zorlutuna et al [17] found some new results in topological space by tracing the curves in the year 2012.

This paper deals with the analysis of topological space over the fuzzy set. Some new results are obtained by generalising the topological space in fuzzy domain. We start by fuzzy set.

### Fuzzy Set

Let,

The space =  $X$ ,

The generic element of  $X = x$ ,

Thus,

$X = x$ .

A fuzzy set (class) =  $A$ ;  $A \in X$ ,

By the characterization,

A membership function =  $f_A(x)$ ,

Such that,  $x \in [0,1]$ ,

By the grade of membership;  $x \in A$ .

Hence,

The nearer the value of  $f_A(x) = 1$ ,

The higher grade of the membership:  $x \in A$ ,

As the conventional set theory term,

$$A = \{0,1\},$$

With

$$f_A(x) = 1; x \in A,$$

or,

$$f_A(x) = 0; x \notin A.$$

Example:

$X$  = The real number,

$A$  = Fuzzy set of real numbers which are much greater than 1,

Then,

$$f_A(x) \in R,$$

Its functional value might be;

$$f_A(0) = 0;$$

$$f_A(2) = 0;$$

$$f_A(7) = 0.01;$$

$$f_A(70) = 0.3;$$

$$f_A(987) = 0.89;$$

$$f_A(1800) = 1.$$

Next, the topological space is defined.

**Topological Space:** A topology on a set  $X$  is a collection  $\tau$  of subsets of  $X$ , that,

- a. Contains  $\phi, X$ .
- b. Closed under the formation of finite intersections, and,
- c. Closed under the formation of arbitrary unions.

Hence the ordered pair  $(X, \tau)$  is called a topological space. Fuzzy topology is given in next.

**Fuzzy Topology:** A fuzzy topology on a set  $X$  is a collection  $\delta$  of fuzzy sets in  $X$  satisfying:

- d.  $0,1 \in \delta$ ,

e. If  $\mu, \nu \in \delta$  then  $\mu \wedge \nu$ , and,

f.  $\mu_i \in \delta : \forall i \in I$  then  $\bigvee_{i \in I} \mu_i$ .

If  $\delta$  is a fuzzy topology on  $X$ , then the pair  $(X, \delta)$  is called a fuzzy topological space.

The proposed fuzzy points in topological space will be presented in below:

**Fuzzy Points in Topological Space:** We start this with the below theorem:

**Theorem:** Fuzzy point is an interior point.

**Proof:** Let a set be  $A$ , and  $a \in A$ . Let  $(X, \tau)$  be a topological space and an open set be such that  $U \in \tau$ .

Hence,

$$a \in U \subseteq A.$$

Let,

$$a = 0,$$

Then,

$$0 \in U \subseteq A.$$

Similarly,

$$a = 1,$$

Then,

$$1 \in U \subseteq A.$$

Therefore,  $0,1 \in U$ , which is an open set. Thus, this point is a fuzzy point.

This completes the proof.

Next we give a corollary.

**Corollary:** The order of  $\tau$  is even if  $0,1 \in X$ .

**Proof:** Let  $\tau$  be the collection of the a subset of  $X$ .

If,  $A$  be the subset of  $\tau$ .

Let,

$$A = \{0,1\} \text{ be a subset of } \tau.$$

Then,

$\sigma = (\{0\}, \{1\})$  is the collection of the subsets of  $\tau$ .

The order of  $\sigma = o(\sigma) = 2 = \text{even}$ .

This completes the proof.

## II. CONCLUSION

Topological space can be generalised by various properties of the set theory. The fuzzy set is studied in this paper. The classes of the fuzzy set lies with the collection of the subsets of the topological space. The fuzzy topology is another area which interact the proposed with the common characteristics. The operation is required to justify in fuzzy points. The homomorphism of the fuzzy set and fuzzy point is the new way of development as the application of topological space.

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