

Overview Applications of Graph Theory in Real Field

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ABSTRACT

The field of Mathematics plays a vital role in the various types of field. Because of the gradual research was done in graph theory, Graph Theory has become very large subject in Mathematics, which is used, structural models. The structural arrangements of various objects or technologies lead to new invention and modifications in the existing environment for enhancement in those fields. The origin of Graph Theory started with the problem of Königsberg Bridge in 1735. In the modern world, planning effective routes is necessary for business and industry, with applications as varied as product distribution. In this paper, we demonstrate various graph with their definition, basic concept and finally their importance and applications in the real field. This paper gives an overview of the applications of Graph Theory in the different types of fields. In our researches, we have identified different types of graphs that are used in most important real field applications and then tried to give their clear idea from the Graph Theory.

Keywords : Bipartite Graph, Connected Graph, Social Media Networks, Graph Coloring, Median Graph.

I. INTRODUCTION

Graph theory is a branch of the discrete mathematics. Graphs are important because the graph is a way of expressing information in pictorial form. A graph shows information that equivalent to many words. Graph theoretical ideas are extremely utilized by computer science applications. Particularly in research areas of computer science such data mining, image segmentation, clustering, the image capturing, Social Media networking etc. Graph theory is the study of graphs, which are mathematical formation used to model pairwise relations between objects. A "graph" in this thesis is made up of "vertices" or "nodes" and lines called edges that connect them. A graph can be undirected, meaning that there is no distinction between the two vertices connected with each edge, or its edges can be directed from one vertex to another; see graph (mathematics) for more than clear definitions and for other changes in the types of graph that are usually considered. A graph is drawn up of vertices V (nodes) and edges E (lines) that connect them. A graph is an ordered pair $G = (V, E)$ comprising a set of vertices V with a set of edges E .

Graph theory is originated with the problem of Königsberg Bridge, in 1735. This problem conduct to the concept of Eulerian Graph. Euler studied the problem of Königsberg Bridge and established a structure to resolve the problem called Eulerian graph. In 1840, A.F Mobius presented the idea of a complete graph and bipartite graph and Kuratowski proved that they are planar by means of recreational problems. Gustav Kirchhoff enacted the concept of a tree, (a connected graph without cycles) in the year 1845, and he enlisted graph theoretical concepts in the calculation of currents in electrical networks or circuits. In the year 1852, Thomas Guthrie founded the famous four-color problem. Then in the year 1856, Thomas. P. Kirkman and William R. Hamilton measured cycles on polyhedra and contrived the concept called Hamiltonian graph by studying trips that visited certain sites exactly once. In 1913, H. Dudeney entitled a puzzle problem. Even though the four-color problem was invented, Kenneth Appel and Wolfgang Haken solved it only after a century. This is considered as the origin of Graph Theory. Caley deliberate particular analytical forms from differential calculus cogitation the trees. This had much significance in theoretical chemistry. This lead to the excogitation of enumerative Graph Theory.

Anyhow, the term “Graph” was innovated by Sylvester in 1878 where he drew an analogy between “Quantic Invariants” and co-variants of algebra and molecular diagrams. In the year 1941, Ramsey worked on colorations, which lead to the identification of another branch of Graph Theory called extremely Graph Theory. In the year 1969, Heinrich has solved the Four Colour problem by using a computer. The study of asymptotic graph connectivity gave arise to random graph theory.

Many problems that are considered hard to determine or implement can easily solve use of graph theory. There are various types of graphs as a part of graph theory. Each type of graph is associated with a special property. The most application makes use of one of this graph in order to find the solution to the problems. Because of the representation power of graphs and flexibility, any problem may be represented as graphs and easily solved. A problem that is solved by graph theory includes Resource allocation, distance minimization, network formation, optimal path identification, data mining, circuit minimization, image capturing, image processing.

II. Related Work

By virtue of the gradual research was done in graph theory, graph theory has become relatively vast subject in mathematics. Graph theory includes different types of graphs, each having basic graph properties and some additional properties. These properties separate a graph from there type of graphs. These properties arrange vertex and edges of a graph is some specific structure. There are different operations that can be performed over various types of graph. Therefore, graph theory can be considered the large and complicated subject. On the other side, graphs are used in more applications as a powerful tool to solve large and complicated problems. The problems that can be solved by graphs cover many fields such as Chemistry, Biology, Computer Science and Operational Research. Hence, graphs theory is useful in many applications and these applications are widely used in real the field. Almost each field today makes use of graph theory, such as search computer networks. Hence, to absolutely implement these applications and to operate them, it is essential to have the clear idea of graph theory. Often material is not able to cover all the corners of graph theory. Materials that successfully give every small detail of graph theory fail to give brief details about

where those concepts are used in real life applications. Materials covering the application of graph theory often fail to describe the basics of the graphs and their characteristics. The authors of this paper make an attempt to give basics fundamentals of graph theory along with the proper knowledge of where these fundamentals are used i.e. their application. This paper contains definitions of different types of graphs by which helps to provide the proper understanding of graph theory. After that major application of these graph theory are given in various subjects. The major fields that extensively usage graphs are Biochemistry, Genomics, Electrical Engineering - communication networks and coding theory, Computer Science algorithms and computations, Operation Research - scheduling. Various application of graph theory in the real field has been identified and represented along with what type of graphs are used in that application. The Authors tries to give a basic conceptual understanding of all such type of graphs.

III. Basic Concept of Graph Theory

Earlier we may realize the application of graphs we need to know some definitions that are part of graph theory. The Author of this paper has identified some definitions and has delineated it is so easy to realize manner.

3.1 Graph: A graph-generally denoted $G(V, E)$ or $G=(V, E)$ – consists of the set of vertices V unitedly with a set of edges E . The number of vertices in a graph is normally denoted n while the number of edges is normally denoted m [1].

Example: The graph given in the figure-1 has vertex set $V=\{1,2,3,4,5,6\}$ and edge set $=\{(1,2),(1,5),(2,3),(2,5),(3,4),(4,5),(6,6)\}$.

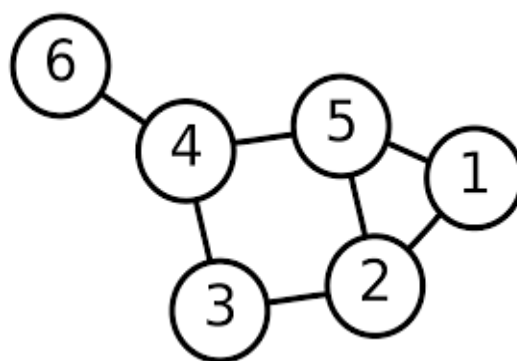


Figure 1- Simple Graph

3.2 Vertex: The vertex is the point at which two rays (edges) of an angle or two edges of a polygon meet.

3.3 Edge: An edge is a line at which vertices are connected in the graph. Edges are denoted by $E = (U, V)$ it is a pair of two vertices.

3.4. Undirected graph: An undirected graph is a graph in which edges have no orientation. The edge (p,q) is identical to the edge (q,p) , i.e., they are not ordered pairs, but sets $\{p,q\}$ (or 2- multi-sets) of vertices. The maximum number of edges in an undirected graph without a loop is $n(n - 1)/2$.

3.5. Directed graph: A directed graph in which each edge is represented by an ordered pair of two vertices, e.g. (V_i, V_j) denotes an edge from V_i to V_j (from the first vertex to the second vertex).

3.6. Connected graph: A graph $G = (V, E)$ is expressed to be connected graph if there exists a path between every pair of vertices in graph G .

3.7. Loop: Edges drawn from a vertex to itself is called a loop.

3.8. Parallel edges: In a graph $G = (V, E)$ if a pair of vertices is allowed to join by more than one edges, those edges are called parallel edges and the resulting graph is called multi-graph.

3.9. Simple graph: A graph $G = (V, E)$ has no loops and no multiple edges (parallel edges) is called simple graph.

3.10. Adjacent vertices: In a graph $G = (V, E)$ two vertices are said to be adjacent (neighbor) if there exists an edge between the two vertices.

3.11. Adjacency matrix: Every graph has associated with it an adjacency matrix, which is a binary $n \times n$ matrix A in which $a_{ij} = 1$ and $a_{ji} = 1$ if vertex v_i is adjacent to vertex v_j and $a_{ij} = 0$ and $a_{ji} = 0$ otherwise. The natural graphical representation of an adjacency matrix is a table, such as shown below.

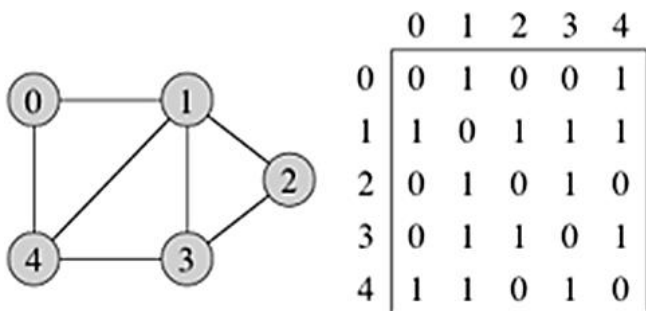


Figure 2- Example of an adjacency matrix.

3.12. The Degree of a vertex: Number of edges that are incident to the vertex is called the degree of the vertex.

3.13. Regular graph: In a graph if all vertices have same degree (incident edges) k than it is called a regular graph.

3.14. Complete graph: A simple graph $G = (V, E)$ with n mutually adjacent vertices is called a complete graph G and it is denoted by K_n or A simple graph $G = (V, E)$ in which every vertex is mutually adjacent to all other vertices is called a complete graph G .

3.15. Cycle graph: A simple graph $G = (V, E)$ with n vertices ($n \geq 3$), n edges is called a cycle graph.

3.16. Wheel graph: A wheel graph $G = (V, E)$ with n vertices ($n \geq 4$), is a simple graph which can be obtained from the cycle graph C_{n-1} by adding a new vertex (as a hub), which is adjacent to all vertices of C_{n-1} .

3.17. Cyclic and acyclic graph: A graph $G = (V, E)$ has at least one Cycle is called cyclic graph and a graph has no cycle is called Acyclic graph.

3.18. Connected graph and Disconnected Graph: A graph $G = (V, E)$ is said to be connected if there exists at least one path between every pair of vertices in G . Otherwise, G is disconnected.

3.19. Tree: A connected acyclic graph is called tree or a connected graph without any cycles is called tree.

3.20. Bipartite graph: A simple graph $G = (V, E)$ is Bipartite if we may partition its vertex set V to disjointing sets U and V such that there are no edges between U and V . We say that (U, V) is a Bipartition of G . A Bipartite graph is shown in figure 3.

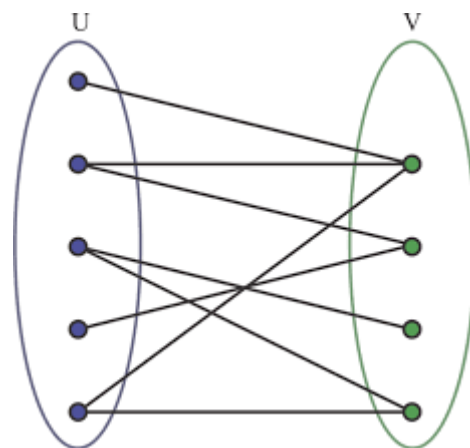


Figure 3- Bipartite graph

3.21. Complete bipartite graph: A bipartite graph $G = (V, E)$ with vertex partition U, V is known as a complete bipartite graph if every vertex in U is adjacent to every vertex in V .

3.22. Vertex coloring: An assignment of colors to the vertices of a graph G so that no two adjacent vertices of G have the same color is called vertex coloring of a graph G .

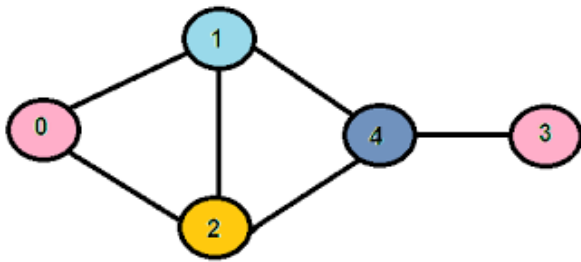


Figure 4- Vertex coloring

3.23. Chromatic number: The minimum number of colors required for the vertex coloring of a graph G , is called the chromatic number of graph G .

3.24. Line covering: Let $G = (V, E)$ be a graph. A subset C of E is called a line covering (Edge covering) of a graph G if every vertex of graph G is incident with at least one edge in C .

3.25. Vertex covering: Let $G = (V, E)$ be a graph. A subset K of V is called a vertex covering of graph G if every edge of graph G is incident with a vertex in K .

3.26. Spanning tree: Let $G = (V, E)$ be a graph. A subset M of G is called a spanning tree of graph G , if M is a tree and M contains all the vertices of graph G .

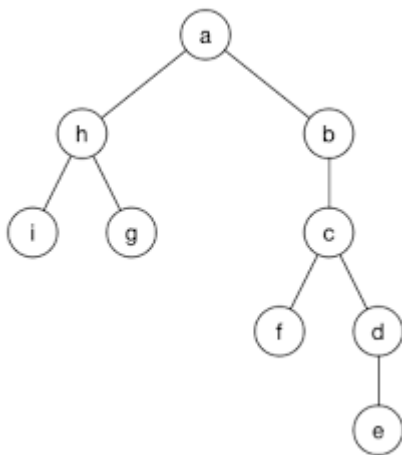


Figure 5- Spanning Tree

3.27. Cut vertex: Let $G = (V, E)$ be a connected graph. A vertex $V \in G$ is called a cut vertex of graph G , if " $G - V$ " results in a disconnected graph G .

3.28. Cut edge: Let $G = (V, E)$ be a connected graph, an edge $E \in G$ is called a cut edge of graph G , if " $G - E$ " result in a disconnected graph G .

3.29. Euler graph: A connected graph $G = (V, E)$ is said to be Euler graph (traversable), if there exists a path which includes, (which contains each edge of the

graph G exactly once) and each vertex at least once (if we can draw the graph on a plain paper without repeating any edge or letting the pen). Such a path is called Euler path.

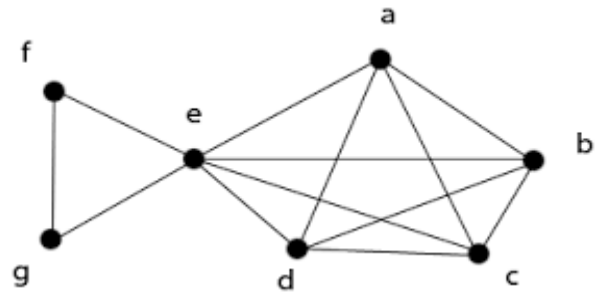


Figure 6- Euler Graph

3.30. Euler circuit: An Euler path in which a starting vertex of the path is same as ending vertex of the path is called as Euler circuit (closed path).

3.31. Hamiltonian graph: A connected graph $G = (V, E)$ is said to be the Hamiltonian graph if there exists a cycle which contains all vertices of graph G . Such a cycle is called Hamiltonian cycle.

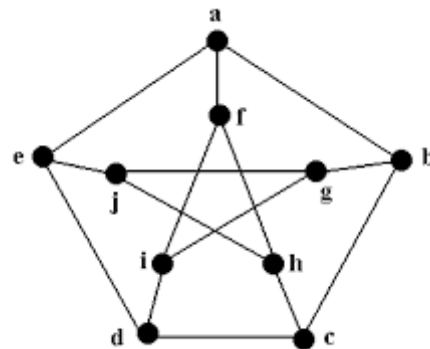


Figure 7- Hamiltonian Graph

IV. Applications of Graph Theory in Real Field

Graphs are used to model many problem of the various real fields. Graphs are extremely powerful and however flexible tool to model. Graph theory includes many methodologies by which this modelled problem can be solved. The Author of the paper have identified such problems, some of which are determined of this paper. There are many numbers of applications of graph theory; some applications are delineated as follows:

4.1. GPS or Google Maps:

GPS or Google Maps are to find out the shortest route from one destination to another. The goals are Vertices and their connections are Edges consisting distance.

The software determines the optimal route. Schools/ Colleges are also using this proficiency to gather up students from their stop to school. To each one stop is a vertex and the route is an edge. A Hamiltonian path presents the efficiency of including every vertex in the route.

4.2. Traffic Signal Lights:

To study the traffic control problem at an arbitrary point of intersection, it has to be modeled mathematically by using a simple graph for the traffic accumulation data problem. The set of edges of the rudimentary graph will represent the communication link between the set of nodes at an intersection. In the graph stand for the traffic control problem, the traffic streams, which may move at the same time at an intersection without any difference, will be joined by an edge and the streams, which cannot move together, will not be connected by an edge.

The functioning of traffic lights i.e. turning Green/Red/Yellow lights and timing between them. Here vertex coloring technique is utilised to solve contravenes of time and space by identifying the chromatic number for the number of cycles needed.

4.3. Social Networks:

We connect with friends via social media or a video gets viral, here user is a Vertex and other connected users produce an edge, therefore videos get viral when reached to certain connections. In sociology, economics, political science, medicine, social biology, psychology, anthropology, history, and related fields, one often wants to study a society by examining the structure of connections within the society. This could befriend networks in a high school or Facebook, support networks in a village or political/business connection networks. For these sorts of networks, some basic questions are: how do things like information flow or wealth flow or shared opinions relate to the structure of the networks, and which players have the most influence? In medicine, one is often interested in physical contact networks and modeling/preventing the spread of diseases. In some sense, even more, basic questions are how do we collect the data to determine these networks, or when infeasible, how to model these networks.

The modern semantic search engine, which is called Facebook Graph Search acquaint by Facebook in March 2013. In general, all search engine gives result in list of link, but Facebook Graph Search gives the answer to user in nature language rather than a list of links. In Facebook Graph Search engine graph Search

feature combining external data into a search engine allow user particular search results and the big data acquired from its over one billion users. In Facebook Graph Search, engine search algorithm is same, as Google search engine algorithm so searching will very faster in Facebook site.

4.4. To clear road blockage:

When roads of a city are blocked due to ice. Planning is needed to put salt on the roads. Then Euler paths or circuits are used to traverse the streets in the most efficient way.

4.5. While using Google to search for Webpages, Pages are linked to each other by hyperlinks. Each page is a vertex and the link between two pages is an edge.

4.6. The matching problem:

In order to assign jobs to employees (servers) there is an analogue in software to maximize the efficiency.

4.7. Traveling Salesman Problem (TSP):-

TSP is a very much intimate problem, which is founded on Hamilton cycle. The problem statement is: Given a number of cities and the cost of traveling from any city to any another city, find the cheapest round-trip route that visits every city precisely once and returns to the starting city. In graph terminology, where the vertices of the graph present cities and the edges present the cost of traveling between the connected cities (adjacent vertices), traveling salesman problem is almost trying to find out the Hamilton cycle with the minimum weight. This problem has been exhibited to be NP-Hard. Even though the problem is computationally very difficult, a large number of heuristic program and accurate methods are known, so that some instances with tens of thousands of cities have been solved. The most direct solution would be to try all permutations and see which one is cheapest (using brute force search). The running time for this approach is $O(V!)$, the factorial of the number of cities, thus this solution gets visionary even for only 24 cities. A dynamic programming solution solves the problem with a runtime complexity of $O(V^2 2^V)$ by considering $dp[end][state]$ which means the minimum cost to travel from start vertex to end vertex using the vertices stated in the state (start vertex may be any vertex chosen at the start). As there are $V 2^V$ subproblems and the time complexity to solve each sub-problems is $O(V)$, the overall runtime complexity $O(V^2 2^V)$.

4.8. Timetable scheduling:

Allotment of classes and subjects to the teachers is one of the major issues if the constraints are complex. Graph theory plays a significant role in this problem. For m teachers with n subjects, the usable number of p periods timetable has to be prepared. This is done as follows. A bipartite graph (or bigraph) is a graph whose vertices may be divided into two disjoint sets U and V such that every edge connects a vertex in U to one in V ; that is, U and V are independent sets G where the vertices are the number of teachers say $m_1, m_2, m_3, m_4, \dots, m_k$ and n number of subjects say $n_1, n_2, n_3, n_4, \dots, n_m$ such that the vertices are connected by p_i edges. It is assumed that at any one period, each teacher may teach at almost one subject and that maximum one teacher may teach each subject. Consider the first period. The timetable for this single period consistent to a matching in the graph and conversely, each matching consistent to a imaginable assignment of teachers to subjects taught during that period. So, the solution for the timetabling problem will be obtained by partitioning the edges of graph G into a minimum number of matching. Also, the edges have to be colored with a minimum number of colors. This problem may also be solved by vertex coloring algorithm. "The line graph $L(G)$ of G has an equal number of vertices and edges of G and two vertices in $L(G)$ are connected by an edge if the corresponding edges of G have a vertex in common. The line graph $L(G)$ is a simple graph and a proper vertex coloring of $L(G)$ gives a proper edge coloring of G by the same number of colors. So, the problem may be solved by finding minimum proper vertex coloring of $L(G)$." For example, Consider there are 4 teachers namely m_1, m_2, m_3, m_4 and 5 subjects say n_1, n_2, n_3, n_4, n_5 to be taught. The teaching requirement matrix $P = [P_{ij}]$ is given below.

P	n_1	n_2	n_3	n_4	n_5
m_1	2	0	1	1	0
m_2	0	1	0	1	0
m_3	0	1	1	1	0
m_4	0	0	0	1	1

Table- 1: The teaching requirement matrix for four teachers and five subjects

The bipartite graph is constructed follows as,

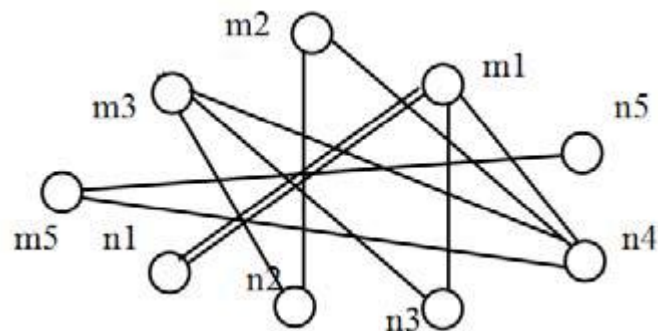


Figure 8- Bipartite graph with Four Teachers and five Subjects

Finally, the authors found that proper coloring of the above-mentioned graph could be done by four colors using the vertex-coloring algorithm, which leads to the edge coloring of the bipartite multigraph G . Four colors are explained to four periods.

V. APPLICATIONS OF GRAPH THEORY IN TECHNOLOGY

5.1 Graphs Applications in Computer Science:-

Computer networks are too much of the most popular in today's real life. In computer networks, nodes are connected to each other with the help of links. This final network of nodes forms a graph. In computer network graph is used to form a network of nodes and enable useful packet routing in the network. This comprehends determination the shortest paths between the nodes, analyze the current network traffic and search fasted root between the nodes, searching cost-efficient route between the nodes. Standard algorithms like that Dijkstra's algorithm, Bellman-Ford algorithm are used to in the different ways with graph to get the solutions.

5.1.1. Data Mining:-

Data mining is a process of comprehending needed data from immense data with the help of different methods. Mostly the data we deal with in data science can be shaped as graphs. These graphs may be mined applying known algorithms and various techniques in graph theory to realise them in the better system, e.g. in social networks, every person in the network could be divinatory as a vertex and any connection between them is so-called as an edge. Any problem-related logistics could be modeled as a network. The Graph is an entrancing model of data backed with a strong theory and a set of quality algorithms to solve related problems.

5.1.2. GSM Mobile Phone Networks and Map Coloring:-

Groups Special Mobile (GSM) is a mobile phone network where the geographical region of this network is divided into hexagonal regions. All mobile phones link up to the GSM network by searching for cells in the neighbors. Since GSM operate only in four discrete frequency ranges, it is clear by the conception of graph theory that only four colors can be utilised to the color of the cellular regions. These four different colors are applied for the proper coloring of the regions. The vertex-coloring algorithm can be used to allocate at almost four discrete frequencies for any GSM mobile phone network.

Given a map drawn on the plane or on the surface of a sphere, the four color theorem resources that it is forever potential to color the regions of a map properly using at almost four different colors such that no two adjacent regions are assigned the similar color. Now, a dual graph is created by setting a vertex inside each region of the map and connect two different vertices by an edge if their respective regions share all section of their limits in ordinary. Where the appropriate coloring of the dual graph gives the suitable coloring of the original map. Since coloring of the regions of a planar graph G is equivalent to coloring the vertices of its dual graph and contrarily. By coloring the map regions using four color theorem, the four frequencies may be assigned to the regions consequently.

5.1.3. Web Designing:-

Website designing may be structured as a graph, where the web pages are entitled by vertices and the hyperlinks between them are entitled by edges in the graph. This concept is called as a web graph. Which enquire the interesting information? Other implementation areas of graphs are in the web community. Where the vertices constitute classes of objects, and each vertex symbolizing one type of objects, and each vertex is connected to every vertex symbolizing another kind of objects. In graph, theory such as a graph is called a complete bipartite graph. There are many benefits graph theory in website development like Searching and community discovery, Directed Graph is used in website usefulness evaluation and link structure. Also searching all connected component and providing easy detection.

5.1.4. Language Processing:-

In the Computer language processing in the tools corresponding compiler parse tree are applied to

identify if the input is having correct syntactic structure or not. This parse tree is make from directed acyclic graph make on lexical entities. Graph is here put upon to identify correct structure of input and to assist entire processing of language.

5.1.5. Code Decoding:-

Usually, in modern coding theory Bipartite graph is applied for decoding the code words, which are, receives from the channel. For example Factor graph and Tanner graph is mainly applied for decoding the code. Tanner graph is an application of bipartite graph therefore, vertices are divided into two parts in which the first bipartition represent the digit of the code word, and the another side bipartition represent the combination of digits that are expected to sum zero in a code word except for errors.

5.1.6. Electronic Chip Design:-

In electronic chip design for each, one component is well thought out as a vertex of the graph. The machine that makes the connection between these components a printed circuit board gets input in the form of a graph where edges refer that there is a connection between the pair of components. The head that makes this connection on the board then get the optimal to moves across the chip to get the in-demand resultant circuit.

5.1.7.

The computer has a lot of hardware as well as the software component. Of that, one of the components is a compiler. It is computer programme that translates the one computer language into other languages. One of the compiler optimization technique for register allotment to improve the performance time is register allotment method, in which most frequently used values of the compiled programme are kept in fast processor registers. Generally, register gets real value when they used for operations. In the textbook, the register allocation method is to model as graph coloring model. The compiler makes an intervention graph, where vertices are symbolical registers and an edge may be colored with k -colors then the variables may be stored in k -registers.

5.2. Graphs in Operation Research:-

Graph theory is a really natural and powerful tool in combinatorial operations research. Some important Operation Research problems, which may be explicate using graphs, are given here. The transport network is used to model the transportation of commodity from one destination to another destination. The objective is to maximize the flow or minimize the cost within the

recommended flow. The graph theory is established as more than competent for these types of problems though they have more than restraints. In this operation research, directed graph is known as the network, the vertices are known as a node and the edges are known as arcs. There are many applications of the network flow model, as some of them are a picture a series of water pipes fitting into a network, Kirchoff's current law, ecology, food web, information theory, thermodynamics, Robert Ulanowicz. The one of simplest and common approach, which is used network flow, is maximum network flow. In which find out a path from source to sink (destination) that is carried out the maximum flow capacity. Figure 6 is an example of maximum flow, in which 11 is maximum flow in the network.

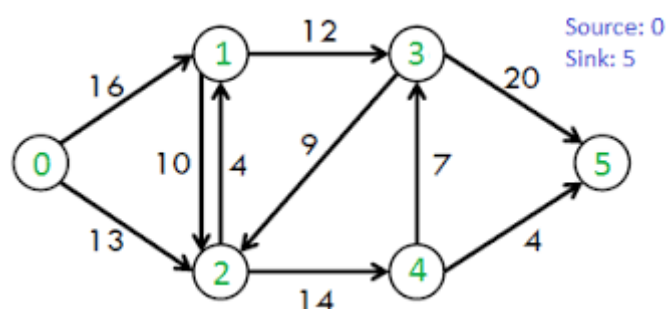


Figure nine- Example of maximum flow network.

5.3. Graphs in Chemistry:-

The structural formulae of covalently bonded compounds are graphs; they are known as constitutional graphs. Graph theory provides the basis for definition, enumeration, systematization, codification, nomenclature, correlation, and computer programming. The chemical information is associated with structural formulae and that structural formulae may be consistently and uniquely exponent and redeemed. One does translate chemical structures into words by nomenclature rules. Graphs are important for the polymer. The grandness of graph theory for chemistry stems mainly from the existence of the phenomenon of isomerism, which is rationalized by chemical structure theory. This theory calculates for all constitutional isomers by using purely graph-theoretical methods.

5.4. Graphs in Biology:-

Graph Theory is a really large subject; it is also extensively used for the analysis in biological networks. In biology analysis, the number of components of the system and their fundamental interactions is differentiating as network and they are usually represented as graphs where lots of nodes are connected with thousands of vertices. Graphs are

widely used in following biological analysis; Protein-protein interaction (PPI) networks, Regulatory networks (GRNs), Signal transduction networks, and Metabolic and biochemical networks. If we analysis above components than it will be generated the structure network which is similar to one of the graph component in graph theory. Graph isomorphism method may be used for matching two components in the biological analysis. If two graphs are isomorphic to each other than, we may conclude that the following biological component like protein interaction, biochemical have same molecular property in the biological component. Similarly isomorphism there is subgraph may be also applied for the biological analysis method. If among two graphs, one of the graphs is sub-graph than in biological analysis the sub-graph component formula may be calculated from main biological graph component. According to above example, we must have knowledge about graph theory then only we may realize the concept of biological analysis in the real field.

5.5. Geography:-

Consider a map, say of India. Let each country be a vertex and connect two vertices with an edge if those countries share a border. A famous problem that went unsolved for over a hundred years was the four color problem. Roughly this states that any map can be colored with at most four colors in such a way that no two adjacent countries have the same color. This problem motivated a lot of the development of graph theory and was finally proved with the aid of a computer in 1976.

VI. Conclusion

The main aim of this paper is to present the importance of graph theoretical ideas in various areas of the real field for the researcher that we may use graph theoretical concepts for the research. An overview is presented particularly to project the concept of graph theory. Thus, the graph theory section of each paper is given importance than to the other sections. This paper is useful for students and researchers to get an overview of graph theory and its application in various real fields like everyday life, Computer Science, Operation Research, Chemistry, Biology, and Geography. There are many problems in this area, which are eventually to be examined. Researchers may get some information related to graph theory and its

applications in the real field and may get some ideas related to their field of research.

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