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# Cube Difference Labeling of Star Related Graphs

**R. Uma[1] , S. Divya[2]**

[1]Assistant Professor, PG Dept. of Mathematics, Sree Saraswathi Thyagaraja College, Pollachi, Tamil Nadu, India

[2]M. Phil Scholar, PG Dept. of Mathematics, Sree Saraswathi Thyagaraja College, Pollachi, Tamil Nadu, India

## ABSTRACT

Let $G = (V(G), E(G))$ be a graph. $G$ is said to be cube difference labeling if there exist a injection $f: V(G) \rightarrow \{0,1,2, \ldots, p-1\}$ such that the induced function $f^*: E(G) \rightarrow N$ given by $f^*(uv) = |[f(u)]^3 - [f(v)]^3|$ is injective. A graph which satisfies the cube difference labeling is called the cube difference graph. The cube difference labeling of merge graph $(C_3 * K_{1,n})$, Bistars and the subdivision of the edges of the star graph $K_{1,n}$ are discussed in this paper.

**Keywords :** Cube difference labeling, Cycle, Merge graph, bistars and the subdivision of the edges of the stars graph $K_{1,n}$ .

## I. INTRODUCTION

Graph theory is one of the branches of Mathematics with many applications in different disciplines. Labeling of graphs is the assignment of values to vertices or edges or both subject to certain conditions. Labeling plays a vital role in cryptography, X-rays, Coding – Decoding, Networking, Chemical engineering etc.. In 1960's Rosa initiated the concept of labeling in the name of β – valuation. Since then many types of labeling came into existence and a detailed report is given in Gallian's survey[2].

Cube difference labeling was introduced by J.Shiama[4] in 2013. A.Solairaju and R. Raziya Begum [5] proved that Merge graph $(T_6 * S_n)$ is super edge-magic graph. Sunny Joseph Kalayathakal and C. Susanth [6] have analysed bistar graph under coloring of graphs. The subdivision of the edges of the star graph $K_{1,n}$ satisfies square graceful labeling and this was proved by T.Tharmaraj and P.B.Sarasija [7]. In this paper, a discussion is made on cube difference labeling of Merge graph, bistars and the subdivision of the edges of the star graph $K_{1,n}$.

## II. PRELIMINARIES

**Cube difference labeling:** Let $G = (V(G), E(G))$ be a graph. $G$ is said to be cube difference labeling if there exist a injection $f: V(G) \rightarrow \{0,1,2, \ldots, p-1\}$ such that the induced function $f^*: E(G) \rightarrow N$ given by $f^*(uv) = |[f(u)]^3 - [f(v)]^3|$ is injective.

**Cycle :** A closed walk $v_0, v_1, \ldots, v_n = v_0$ in which $n \geq 3$ and $v_0, v_1, \ldots, v_{n-1}$ are distinct is called a cycle of length n.

**Star:** A complete biparitite graph $K_{1,n}$ is called a star and it has $n+1$ vertices and $n$ edges.

**Bistar:** A bistar graph $B(m,n)$ is a graph obtained by attaching $m$ pendant edges to one end point and $n$ pendant edges to the other end point of $K_2$ .

**Merge graph:** A merge graph $G_1 * G_2$ can be formed from two graphs $G_1$ and $G_2$ by merging a vertex of $G_1$ with a vertex of $G_2$.

**Subdivision of a graph:** A subdivision of a graph $G$ is a graph that can be obtained from $G$ by a sequence of edge subdivisions.

## III. CUBE DIFFERENCE LABELING OF STAR RELATED GRAPHS

**Theorem 1:** The Merge graph $(C_3 * K_{1,n})$ is a cube difference graph.

**Proof:** Let G be the merge graph$(C_3 * K_{1,n})$. Then the graph G has an order $p = 3 + n$ and the size $q = 3 + n$. The vertex set $V(G) = \{u_0, u_1, u_2, v_1, v_2, \ldots, v_n\}$, where $u_0$ is an apex vertex, $v_i's$ are adjacent vertices of $u_0$, $u_i$'s are the vertices of the cycle as shown in the fig 1.

Now define the function $f: V(G) \rightarrow \{0,1,2, \ldots, p-1\}$ as follows:

$f(u_i) = i$, for $0 \leq i \leq 2$;

$f(v_i) = i + 2$, for $1 \leq i \leq n$.
Then the induced function $f^*: E(G) \rightarrow N$ is defined by $f^*(uv) = |[f(u)]^3 - [f(v)]^3|$ are distinct and satisfies cube difference labeling.

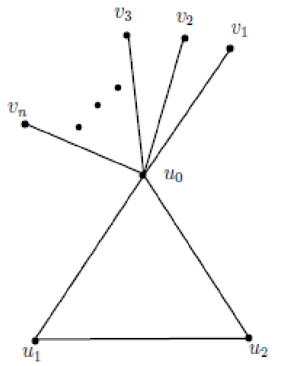


**Figure 1.** Cube difference labeling of the merge graph $(C_3 * K_{1,n})$.

**Example 1:**

Cube difference labeling of the merge graph $(C_3 * K_{1,5})$ is given in the fig .2. The order and size are 8. The vertex set $V(G) = \{u_0, u_1, u_2, v_1, v_2, v_3, v_4, v_5\}$. Then the vertex labels are $f(u_0) = 0$ , $f(u_1) = 1, f(u_2) = 2$, $f(v_i) = i + 2$, for $1 \leq i \leq 5$; $f(v_1) = 3, f(v_2) = 4, f(v_3) = 5, f(v_4) = 6, f(v_5) = 7$.Then the edge labels are given by $f^*(uv) = |[f(u)]^3 - [f(v)]^3|$. $f^*(u_0u_1) = 1$ , $f^*(u_1u_2) = 7$ , $f^*(u_2u_0) = 8$ , $f^*(u_0v_1) = 27$ , $f^*(u_0v_2) = 64$ , $f^*(u_0v_3) = 125$ , $f^*(u_0v_4) = 216$ , $f^*(u_0v_5) = 343$. Hence the edges are distinct. Hence the merge graph $(C_3 * K_{1,5})$ is a cube difference graph.

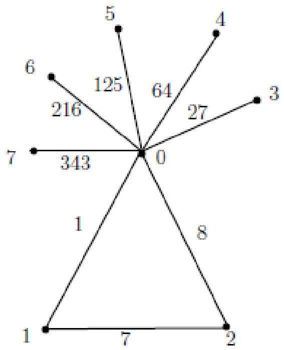


**Figure 2.** Cube difference labeling of the merge graph $(C_3 * K_{1,5})$.

**Theorem 2:** The Bistar graphs $B(m, n)$ is a cube difference graph.

**Proof:**

**Case (i): For $B(m, n)$**

Let G be bistar graph $B(m, n)$. By the definition of $B(m, n)$, the order is $p = m + n + 2$ and the size is $q = m + n + 1$ . The vertex set $V(G) = \{u_0, u_1, u_2, \ldots, u_m, v_0, v_1, v_2, \ldots, v_n\}$ , where $u_1, u_2, \ldots, u_m$ are $m$ pendant vertices adjacent to $u_0$ and $v_1, v_2, \ldots, v_n$ are $n$ pendant vertices adjacent to $v_0$. Now define the function $f: V(G) \rightarrow \{0,1,2, \ldots, p - 1\}$ as follows:

$f(u_0) = 0, f(v_0) = 1$.
$f(u_i) = i + 1$,for $1 \leq i \leq m$;
$f(v_i) = m + i + 1$, for $1 \leq i \leq n$.

Then the induced function $f^*: E(G) \rightarrow N$ defined by $f^*(uv) = |[f(u)]^3 - [f(v)]^3|$ are distinct. Hence the bistar graph $B(m, n)$ is a cube difference graph. The general graph $B(m, n)$ is shown in fig 3.

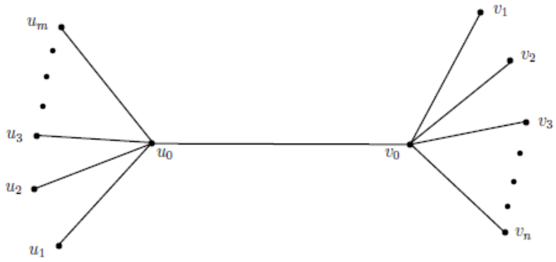


**Figure 3.** Cube difference labeling of bistar graph $B(m, n)$.

**Case (ii): For $B(n, n)$**

Let G be bistar graph $B(n, n)$. The graph given in fig 4 is $B(n, n)$ of order $2n + 2$ and size $2n + 1$.The vertex set $V(G) = \{u_0, u_1, u_2, \ldots, u_n, v_0, v_1, v_2, \ldots, v_n\}$, where $u_1, u_2, \ldots, u_n$ are $n$ pendant vertices adjacent to $u_0$ and $v_1, v_2, \ldots, v_n$ are $n$ pendant vertices adjacent to $v_0$. Now define the function $f: V(G) \rightarrow \{0,1,2, \ldots, p - 1\}$ as follows: $f(u_0) = 0$, $f(v_0) = 1$;
$f(u_i) = i + 1$, for $1 \leq i \leq n$;
$f(v_i) = n + i + 1$, for $1 \leq i \leq n$.

Then the induced function $f^*: E(G) \rightarrow N$ is defined by $f^*(uv) = |[f(u)]^3 - [f(v)]^3|$ are distinct. Hence the bistar graph $B(n, n)$ is a cube difference graph. The general graph $B(n, n)$ is shown in fig 4.

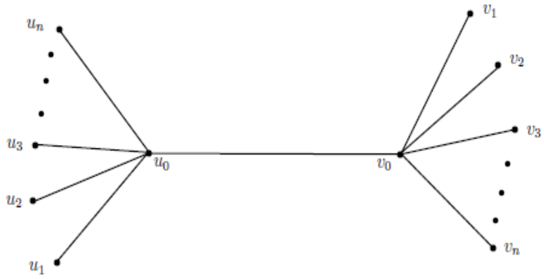


**Figure 4.** Cube difference labeling of bistar graph $B(n, n)$.

**Example 2:**

**For Case (i):**

Cube difference labeling of the bistar graph $B(6,5)$ is given in the fig 5. The order is 13 and the size is 12.

The vertex set

$V(G) = \{u_0, u_1, u_2, u_3, u_4, u_5, u_6, v_0, v_1, v_2, v_3, v_4, v_5\}$. Then the vertex labels are $f(u_0) = 0, f(v_0) = 1$ .$f(u_i) = i + 1$,for $1 \le i \le 6$;$f(u_1) = 2$, $f(u_2) = 3, f(u_3) = 4, f(u_4) = 5$, $f(u_5) = 6, f(u_6) = 7$.$f(v_i) = m + i + 1$ ,for $1 \le i \le 5$.$f(v_1) = 8, f(v_2) = 9, f(v_3) = 10, f(v_4) = 11, f(v_5) = 12$. Then the edge labels of the bistar graph $B(6,5)$ is given by $f^*(uv) = |[f(u)]^3 - [f(v)]^3|$. $f^*(u_0u_1) = 8, f^*(u_0u_2) = 27, f^*(u_0u_3) = 64, f^*(u_0u_4) = 125, f^*(u_0u_5) = 216, f^*(u_0u_6) = 343, f^*(v_0v_1) = 511, f^*(v_0v_2) = 728, f^*(v_0v_3) = 999, f^*(v_0v_4) = 1330, f^*(v_0v_5) = 1749, f^*(u_0v_0) = 1$. Hence the edges are distinct. Hence the bistar graph $B(6,5)$ is a cube difference graph.

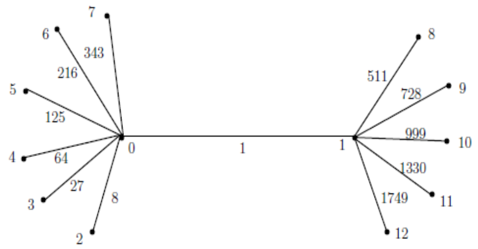


**Figure 5.**Cube difference labeling of bistar graph $B(6,5)$.

**For Case (ii):**

Cube difference labeling of the bistar graph $B(5,5)$ is given in the fig 6. The order is 12 and the size is 11.

The vertex set

$V(G) = \{u_0, u_1, u_2, u_3, u_4, u_5, v_0, v_1, v_2, v_3, v_4, v_5\}$ . Then the vertex labels are $f(u_0) = 0$ , $f(v_0) = 1$ . $f(u_i) = i + 1$, for $1 \le i \le 5$ ; $f(u_1) = 2$ , $f(u_2) = 3$, $f(u_3) = 4$, $f(u_4) = 5$, $f(u_5) = 6$. $f(v_i) = n + i + 1$ ,for $1 \le i \le 5$. $f(v_1) = 7$ , $f(v_2) = 8$ , $f(v_3) = 9$, $f(v_4) = 10$, $f(v_5) = 11$. Then the edge labels of the bistar graph $B(5,5)$ is given by $f^*(uv) = |[f(u)]^3 - [f(v)]^3|$ . $f^*(u_0u_1) = 8$ , $f^*(u_0u_2) = 27$ , $f^*(u_0u_3) = 64$ , $f^*(u_0u_4) = 125$ , $f^*(u_0u_5) = 216$ , $f^*(v_0v_1) = 342$ , $f^*(v_0v_2) = 511$ , $f^*(v_0v_3) = 728$,$f^*(v_0v_4) = 999$,$f^*(v_0v_5) = 1330$,$f^*(u_0v_0) = 1$. Hence the edges are distinct and accepts cube difference labeling.

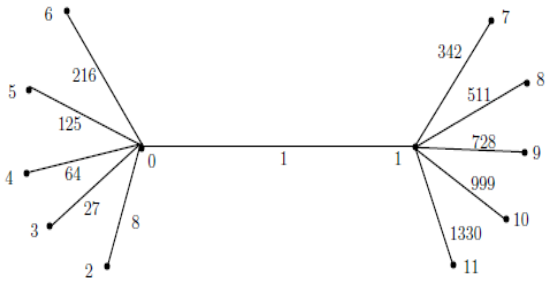


**Figure 6.** Cube difference labeling of bistar graph $B(5,5)$

**Theorem 3:** The subdivision of the edges of the star graph $K_{1,n}$ is cube difference graph.

**Proof:** Let $G$ be a graph obtained by the subdivision of the edges of the star graph $K_{1,n}$ . Since G being a star graph has an order $p = 2n + 1$ and size $q = 2n$ where the vertex set

$V(G) = \{v, u_0, u_1, \dots, u_{n-1}, w_0, w_1, \dots, w_{n-1}\}$ , where $u_0, u_1, \dots, u_{n-1}$ are adjacent vertices of $v$ and $w_0, w_1, \dots, w_{n-1}$ are adjacent vertices of $u_0, u_1, \dots, u_{n-1}$ respectively. Now, let us define the edge set $E(G) = \{E_1, E_2\}$, where $E_1 = (v, u_i)$ and $E_2 = (u_i, w_i)$where $i = 0,1, \dots, n-1$. Now define the function $f: V(G) \to \{0,1,2, \dots, p-1\}$ as follows:

$f(v) = 0$.

$f(u_i) = i + 1$, for $0 \le i \le n-1$.

$f(w_i) = (n+1) + i$, for $0 \le i \le n-1$.

Then the induced function $f^*: E(G) \to N$ is defined by $f^*(uv) = |[f(u)]^3 - [f(v)]^3|$ are distinct. Hence the subdivision of the edges of the star graph $K_{1,n}$ is a cube difference graph. The general graph of the subdivision of the edges of the star graph $K_{1,n}$ is shown in fig 7.

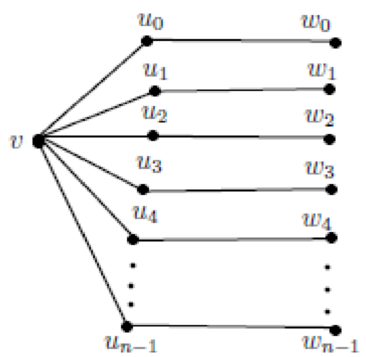


**Figure 7.** Cube difference labeling of thesubdivision of the edges of the star graph $K_{1,n}$.

**Example 3:**

Cube difference labeling of subdivision of the edges of the star graph $K_{1,5}$ is given in the fig 8. The order and the size are 11 and 10 respectively. The vertex set $V(G) = \{v, u_0, u_1, u_2, u_3, u_4, w_0, w_1, w_2, w_3, w_4\}$. Then the vertex labels are $f(v) = 0$ . $f(u_i) = i + 1$, for $0 \leq i \leq 4$; $f(u_0) = 1$, $f(u_1) = 2$, $f(u_2) = 3$, $f(u_3) = 4$ , $f(u_4) = 5$ . $f(w_i) = (n + 1) + i$ , for $0 \leq i \leq 4$ . $f(w_0) = 6$ , $f(w_1) = 7$ , $f(w_2) = 8$ , $f(w_3) = 9$, $f(w_4) = 10$. Then the edge labels of the subdivision of the edges of the star graph $K_{1,5}$ is given by $f^*(uv) = |[f(u)]^3 - [f(v)]^3|$ . $f^*(vu_0) = 1$ , $f^*(vu_1) = 8$ , $f^*(vu_2) = 27$ , $f^*(vu_3) = 64$ , $f^*(vu_4) = 125$ , $f^*(u_0w_0) = 215$ , $f^*(u_1w_1) = 335$ , $f^*(u_2w_2) = 485$ , $f^*(u_3w_3) = 665$ , $f^*(u_4w_4) = 875$ . Hence the edges are distinct. Hence the subdivision of the edges of the star graph $K_{1,5}$ is a cube difference graph.

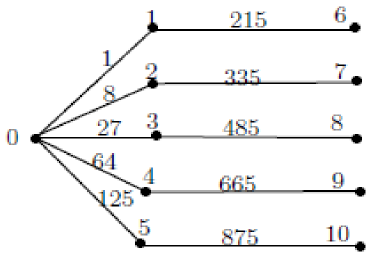


**Figure 8.** Cube difference labeling of the subdivision of the edges of the star graph $K_{1,5}$.

## IV. CONCULSION

In this paper, cube difference labeling of star related graphs are successfully discussed. The above information may be helpful for researchers to get some idea related to labeling.

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