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A Multi-Item Inventory Model with Demand-Dependent Unit Cost : A Geometric Programming Approach with GA-SVM

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ABSTRACT

Multi-item inventory models are developed with and without back-orders where demand is related to the unit price as price is inversely proportional to demand. The models are associated with infinite/finite storage capacities. In total, there are four multi-item inventory models which are formulated with the cost functions and with/without constraints in the form of signomilas and solved by both a modified geometric programming technique and gradient-based non-linear programming method. For each model, sensitivity analysis with respect to the degree of economies of scale and invariant cost parameters are also presented. Each case is illustrated with numerical example and the results from two methods are compared.

Keywords: Genetic Algorithms, Support Vector machines, Geometric Programming, Inventory models, EOQ and Demand dependent.

I. INTRODUCTION

Since the development of economic order quantity model by Harris[1], the researchers have formulated and solved different types of inventory models by several methods. Detailed reviews on the development of this area can be obtained in Arrow, Karlin and Scarf[2], Hadley and Within[3], Naddor[4] and others.

In most of the inventory models, demand and unit cost of a product are assumed to be independent. But, in reality, this is not true. When demand of a product is high, the product is manufactured in large quantities and fixed cost of production are spread over large number of items. Ultimately, this process results in lower average unit production cost. For this reason, demand and unit cost are assumed to be inversely related to each other. Cheng [8, 9 and 11] developed some inventory models with this assumption and solved them using geometric programming technique [12].

Geometric Programming is a class among available amongst available non-linear programming techniques. It has certain advantages over other optimization methods. In some cases, it reduces to the unique solution of the simultaneous equations. For this reason, G.P. has been very popular and effectively used to engineering design and other areas. Duffin et al [5] first showed that geometric programming technique could be used with some advantages for particular type of problems. Even though geometric programming is an excellent method of solve non-linear decision making problems, the use of geometric programming in inventory models has been relatively infrequent. Kotchenberger[6] was the first to solve the basic economic order quantity model using geometric programming. In Worral and Hall [7], geometric programming techniques were utilized to solve a multi-item inventory model under several constraints. Cheng [8 and 9] applied geometric programming to solve modified economic order quantity models. Harri Abou-el-ata [12] and Abou-el-ata and Kotb[13] solved multi-item inventory models under some restrictions with varying inventory costs using geometric programming and presented a sensitivity analysis based on the geometric programming approach. Here, we propose to solve some multi-item modified EOQ models with varying inventory costs and demand dependent unit cost using geometric programming technique. Some of these models are under constraints also[13,14 and 15].

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In this paper, multi-item modified EOQ inventory models with demand dependent unit cost and infinite replenishment has been formulated in crisp environment with/without storage space constraint. Here, shortages are allowed and fully backlogged[16, 17 and 18]. With the above assumptions, expressions for the total average cost are derived in signomial form and minimized via both a modified geometric programming technique and gradient-based non – linear optimization method[19, 20 and 21]. As a particular case, the results of the models without shortages are derived. The models are illustrated with numerical examples and the results from two minimization techniques are compared. It is seen that in some cases, especially for constrained inventory models, G.P. produces slightly better results. A sensitivity analysis is presented for both the models with respect to the cost parameters and degree of economies of scale [22].

II. MODEL ANALYSIS

The following notations and assumptions are being made in developing the mathematical model.

2.1 Model Formulation

W= maximum allowable available storage space,
n= number of items,
Parameters for the i^{th} (i=1,2,3,…,n) item,
D_i = Demand per unit time,
Q_i= shortage level,
S_i= shortage level,
C_{oi}= cost per unit item,
C_{1i} = holding cost per unit item,
C_{2i} = shortage cost per unit item,
C_{3i} = set up cost per cycle,
W_i= storage area required per unit time,
W_{ji} = dual variable.

2.2 Assumptions

(i) Production is instantaneous with zero lead time
(ii) When the demand of an item increases then the total purchasing cost spread all over the items and hence the unit purchasing cost reduces and varies inversely with demand.

$$c_{oi} = \alpha_i D_i^{-\beta_i} \quad (1)$$

Where

$$\alpha_i > 0 \;\; and \;\; \beta_i > 1 \quad (2)$$

Equation 2 holds good as purchasing cost and demand for an are non-negative.

III. MATHEMATICAL FOR MULATION AND ANALYSIS

3.1 Model 1:

The EOQ model with shortages and demand – dependent unit cost.

The total average cost of multi-item for an infinite replenishment problem is

$$Min\;\; TC(D_i, Q_i, S_i) = \sum_{i=1}^{n} \alpha_i D_i^{-\beta_i} + c_3 D_i \big/ Q_i \quad (3)$$

The above objective function is an unconstrained signomial function with one degree of difficulty and is now solved by G.P. The corresponding dual problem is

$$Max\;\; d(w) = \prod_{i=1}^{n} \left(c_{3i} \big/ w_{2i} \right)^{w2i} \left(\alpha_i \big/ \beta_i \right)^{w_{1i}} \quad (4)$$

subject to the normality, orthigonality and non-negativity conditions. According to Hariri and Abou-el-ata and Kotb [13] these conditions are

$$w_{1i} + w_{2i} + w_{3i} + w_{4i} - w_{5i} = 1 \quad (5)$$

There are four linear equations in five unknowns having an infinite number of solutions. However the optimal values of the weights in terms of w_{3i} are

$$w_{1i} = 1 \big/ (2\beta_i - 1) \quad (6)$$

$$w_{2i} = (\beta_i - 1)/(2\beta_i - 1) \quad (7)$$

$$w_{4i} = (\beta_i - 1)/(2\beta_i - 1) + w_{3i} \quad (8)$$

substituting the values of W_{ji}'s in the equation (2) we get,

$$d(w_{3i}) = \sum_{i=1}^{n} \alpha_i D_i^{-\beta_i} + c_3 D_i \big/ Q_i \quad (9)$$

To find the optimal value of W_{3i} which maximizes $d(W_{3i})$, we take logarithm on equation (4), differentiate with respect to W_{3i} and then the result is set to zero. This yields

$$w_{3i}^{*} = \left(c_{1i} \big/ c_{2i} \right) (\beta_i - 1)/(2\beta_i - 1) \quad (10)$$

The other optimal values of the weights are

$$w_{1i}^* = 1 \big/ \left(2bi - 1\right) \quad (11)$$

$$w_{2i}^* = c_{3i}\left(bi - 1\right)/\left(2bi - 1\right) \quad (12)$$

$$w_{4i}^* = \left(1 + c1i\right)\left(bi - 1\right)/\left(2bi - 1\right) + w_{3i} \quad (13)$$

$$w_{5i}^* = 2.c_{1i}.\left(bi - 1\right)/\left(2bi - 1\right) + w_{3i} \quad (14)$$

Substituting these optimal values of the weights in the equation (4) we get the optimal values of $d(w_{3i})$ as $d(w_{3i}^*)$.

The minimum total average cost will be obtained from relation

$$TC^* = n\left(d(w_{5i})^*\right)^{1/n} \quad (15)$$

To obtain the optimal values D_i^*, Q_i^* and S_i^* we use the following relations according to Duffin and Peterson[5],

$$w_{5i}^* = 2.c_{1i}.\left(bi - 1\right)/\left(2bi - 1\right) \quad (16)$$

Solving the relations we get the optimal values of the decision variables as

$$D_i^* = 2.c_{1i}.\left(bi - 1\right)/\left(2bi - 1\right) + 2bi \quad (17)$$

$$Q_i^* = \left(1 + c1i\right)\left(bi - 1\right)/\left(2bi - 1\right) \quad (18)$$

$$S_i^* = \left(1 + c1i\right)/\left(2bi - 1\right) \quad (19)$$

2.3 Model 2:

The EOQ model with shortages and demand – dependent unit cost under storage capacity restriction.

The total average cost of multi items for an infinite replenishment problem is

$$Min\ \ TC\left(D_i, Q_i, S_i\right) = \sum_{i=1}^{n} a_i D_i - bi + c_3 D_i \big/ Q_i + \prod_{i=1}^{n} \left(c_{3i} \big/ w_{2i}\right)^{w2i} \left(a_i \big/ bi\right)^{w_{1i}} \quad (20)$$

subject to

$$\sum_{i=1}^{n} w_i Q_i \le w \quad (21)$$

Following Hariri and Abou-el-ata[12] or Abou-el-ata and Kotb [13], the primal problem is a constrained signomial function with degree of difficulty two and it is solved by G.P. method. The corresponding dual function is

$$Max\ \ d(w) = \sum_{i=1}^{n} \left(ai \big/ w_{4i}\right)^{w_{1i}} \left(c_{3i} \big/ w_{2i}\right)^{w_{2i}} \left(c_{1i} + c_{2i}\right)/2w_{3i} \quad (22)$$

$$g_i = \sum_{i=1}^{n} w_{6i} \quad (23)$$

Proceeding as in model-1, the normality and orthogonality conditions are

$$w_{1i} + w_{2i} + w_{3i} + w_{4i} - w_{5i} = 1 \quad (24)$$

$$\left(1 - bi\right)w_{1i} + w_{2i} = 0 \quad (25)$$

Solving the above equations in terms of w_{1i} and w_{2i} we get

$$w_{2i} = \left(1 - bi\right)w_{1i} \quad (26)$$

substituting the above weights in the equation 5 we get

$$d(w_{1i}, w_{3i}) = \prod_{i=1}^{n} {}^{w_{1i}}\left(c_{3i} \big/ w_{2i}\right)^{w_{2i}} \left(c_{1i} + c_{2i}\right)/2w_{3i} \quad (27)$$

To find the optimal value of Wji's that minimizes (6) we take logarithm of (6) differentiate with respect to W_{1i} and W_{3i} and then set to zero.

We have

$$\left(1 - bi\right)w_{1i} + w_{2i} + \left(c_{1i} + c_{2i}\right)/2w_{3i} \quad (28)$$

substituting the value of w_{3i} in equation (7) we have

$$^{w_{1i}}\left(c_{3i} \big/ w_{2i}\right)^{w_{2i}} \left(c_{1i} + c_{2i}\right)/2w_{3i} + \left(1 - bi\right)w_{1i} \quad (29)$$

Solving the above non-linear eqation by any trial and error method, we get the optimal value w^*_{1i} which maximizes the dual function and the corresponding optimal values of the weights are

$$W_{2i}^* = \left(1 - bi\right)w_{1i} \quad (30)$$

$$W_{3i}^* = \left(1 - bi\right)c_{1i} \quad (31)$$

The optimal values of D_i^*, Q_i^*, S_i^* and TC^* are obtained from the relations as in model-1

$$D_i^* = \sum_{i=1}^{n} \left(ai \big/ w_{4i}\right)^{w_{1i}} \left(c_{3i} \big/ w_{2i}\right)^{w_{2i}} \left(c_{1i} + c_{2i}\right) \quad (32)$$

$$Q_i^* = \left(c_{1i} + c_{2i}\right)/2w_{3i} + \left(1 - bi\right)w_{1i} \quad (33)$$

$$S_i^* = \left(1 - bi\right)w_{1i} + w_{2i} \quad (34)$$

and

$$TC^* = n\{d(w^*)\}^{1/n} \quad (35)$$

IV. RESULTS

Now we will illustrate the model for some numerical data. We consider two items with the following values

Table 1. The optimal values of the variables and cost function with SVM

i	a	b	C1	C2	C3
1	1500	1.7	1.2	30	70
2	1700	1.9	1.5	26	80

Table 2. The shortage optimal values of the variables and cost function with GA-SVM

i	D_i*	Q_i*	S_i*	TC*
1	480.45	243	9.24	831
2	223.32	232	8.32	

Table 3. The without shortage optimal values of the variables and cost function with GA.

i	D_i*	Q_i*	S_i*	TC*
1	480.45	243	9.24	831
2	223.32	232	8.32	

Here, the results placed at the top are due to G.P. and the results from non-linear optimization techniques are placed within the brackets.
For two items we assume the following data

Table 4. The optimum values with shortages

i	a	b	C_{1i}	C_{2i}	C_{3i}	W
1	170	2.8	1.7	30	44	6
2	150	2.9	2.2	35	35	7

Table 5. The optimum values without shortages

i	D_i*	Q_i*	S_i*	TC*
1	48	32	3	
2	32	19	4	246

Table 6. The mixed optimum values without shortages

i	D_i*	Q_i*	S_i*	TC*
1	58	42	4	
2	38	29		346

From the above tables, it is observed that for both models, G.P. gives better results than the gradient – based non-linear optimization technique. For the present formulation, this improvement is noticeable for the inventory models under constraints.

V. SENSITIVITY ANALYSIS

Following Cheng [8,9, 11], the sensibility analysis has been developed with the percentage change of degree of economies of scale and cost parameters. The primal decision variables, dual objective function and primal objective function are changed due to change of c_{1i}, c_{2i} and c_{3i} .

Table 6.

%	i	Di*	Qi*	Si*	TC*
2	1	453	240	10	840
	2	239	160	9	
4	1	453	240	8	845
	2	235	231	9	
6	1	431	254	10	847
	2	236	157	11	
8	1	453	239	11	855
	2	235	156	10	

5.1 Sensitivity Analysis for Model-1 with GA-SVM

Now, for the numerical data in 6.1 of model-1, the changes in the decision parameters and cost function are presented due to the percentage variation in inventory costs and the exponent.

Table 7. Effect of changing holding cost

%	i	Di*	Qi*	Si*	TC*
2	1	468	240	9.01	850
	2	230	160	8.95	840
4	1	475	240	7.951	873
	2	230	158	8.01	750
6	1	472	233	8.8	790
	2	230	156	7.92	800
8	1	470	230	7.45	790
	2	225	231	6.98	810

From the above table we see that the optimal cost increases with the increase of the cost parameters, but the optimal cost decreases with increase in the degree of economies of scale.

VI. CONCLUSION

In this paper we developed multi-item demand-dependent EOQ models with or without space capacity restriction. We investigated also the cases when the shortages are not allowed with GA-SVM. We solved such problems by revised geometric programming method. Finally, we discussed about the sensitivity analysis with respect to cost parameter and degree of economy of scale. This analysis is different from conventional GP and quite general in nature. Hence, this technique can be applied to solve the different decision making problems in inventory and other areas.

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