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Topological Structure in Digital Image Processing : A Survey

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ABSTRACT

Image processing is a technique with the help of this we can perform some operations on image. Digital image processing is the use of computer algorithms to perform image processing on digital images. Digital topology deals with properties and features of two-dimensional (2D) or three-dimensional (3D) digital images that correspond to topological properties (e.g., connectedness) or topological features (e.g., boundaries) of objects. This article is an overview of recent research of generalized topological property in the field of digital image processing. In this study we present the general framework on topology and digital image processing.

Keywords: Digital Image Processing, Topology, Digital Topology.

I. INTRODUCTION

An image is a physical likeness or representation of a object made visible. Digital image is an image that has been converted into binary array which is readable on a computer or some other digital device. Digital image analysis involves ``segmenting" the image into parts and find the relationship between these parts and studies various properties among them. More abstractly, want to separate out the connected components of an image subset, to determine the adjacency relationships among those components, to track and encode their borders without changing their connectedness properties. Since all these approaches are related to the nearness concept, which is the basic idea of a topological property. Therefore it was thought to establish the notion of topology on digital space, known as digital topology. Thus digital topology is deal with the topological properties of digital image. Its result provides a sound in mathematical basis for digital image processing operation such as:

1. Connected components labelling (or region extraction) which is scan an image and groups its pixels into components based on pixel connectivity.
2. Boundary following is a technique that is applied to digital images in order to extract their boundary.
3. Contour filling.
4. Object counting.
5. Thinning is the transformation of a digital image into a simplified, but topologically equivalent image.

In digital image processing, an object in the 2D or 3D-space is approximated digitally by a set of pixels or voxels respectively. Digital topology studies the properties of this set of pixels or voxels that correspond to topological properties of the original object. At the beginning Rosenfeld (In 1979) give the systematic study on digital topology regarding a digital image as on graph based whose nodes are the pixels and whose edges are linking the adjacent pixels to each other. Which divides the digital space into two graphs using pair adjacency-relations to represents the foreground and background of a binary digital image. It is called the neighbourhood graph.

The digital topology framework, graph-based approaches was first proposed by Rosenfeld which divides the digital space into two graphs using pair adjacency-relations to represents the foreground and background of a binary digital image. Further various theories on digital topological frameworks have been proposed by many authors (Like Khalimsky, Eladio Domínguez, Angel Francés, Alberto Márquez,etc). The feature of digital topology lies in analysis of the properties like connectedness, adjacency, digital homotopy etc. with different fields like pattern recognition, medical imaging, image processing, neuroscience, geo-science etc.

II. Literature Review

With respect to the nearness concept, digital topology provides a suitable answer for image processing algorithm to computing structural representation of an image.

In 1935 digital topology related work call ``grid cell topology" was proposed by P. Alexandroff and H. Hopf [1], for 2D-space where an axiomatic basis was given for the theory of cell complex(called combinatorial topology). Further, in 1989, V. Kovalevsky [12] extended the notion of grid cell topology into higher dimensions digital space by means of cellular topology. In 1979 Rosenfeld pioneered graph theoretic tools were used for structuring Z^2, namely the well-known binary relations of 4-adjacency and 8-adjacency [6],[7]. He agree that an adjacency relation is not consistent if there is no topological space whose connectedness relation is analog to that of an image with the adjacency relation. Unfortunately, neither 4-adjacency nor 8-adjacency itself allows for analogue of the Jordan curve theorem [15]. He proved that Jordan's curve theorem is indeed true for digital curves if the curve and its complement are equipped with different topologies. To overcome this disadvantage, E.Khalimsky et al.(In 1990) give an approach based in purly topological space [14]. Their topological approach to computer graphics utilizes a connected topology on a finite order set which arises from a natural generalization of the classical approach to connected linearly ordered topological space(also called Khalimsky topology [2]), for structuring digital plane. In 1984, V. A. Kovalevsky [8], present for all kinds of plane grids only the 6-neighborhood is consistent. It admits the definition of a digital curve, in particular a contour, as a set of pixel pairs. Thus curves become objects of zero thickness and may be coded with one bit per grid step. T. Y. Kong and A.W. Roscoe (In 1985)[9], define a binary digital picture to be a pair whose components are a set of lattice-points and an adjacency relation on the whole lattice which show that digital pictures have natural ``continuous analogs". In the 3D case consider the possibility of using a uniform relation on the whole lattice. In 1989, Kovalevsky's [12] approach based on abstract cell complexes topology of finite set. Under this topology defined connected subsets and their boundaries. E.D.Khalimsky et.al(in 1990) [13], gives a topological proof of the non-topological Jordan curve theorem which requires two different definitions, 4-connectedness, and 8-connectedness, one for the curve and the other for its complement. In 1992, Eladio Domínguez, Angel Francés, Alberto Márquez [16], they developed the functional architecture of a framework for Digital Topology. The authors in [16], established that the curves that can be embedded in the graph $E_8$are the only curves satisfying the Jordan Curve Theorem. In the year 1997, Pavel Ptak, Helmut kofler, Walter Kropatsch, illustrated the point-neighborhood definition of topology. They proof the results on 4-connectedness and on 8-connectedness in Z^2. They also show that there is no topology compatible with 6-connectedness in Z^2, with respect to graph-theoretical approach. In the year 1999, Gilles Bertrand and Michel Couprie [43], developed two different adjacency relations that are used for structuring the discrete space Z^n. They propose a model for digital topology based on the notion of order and discrete topology and give the different possible configurations that appear in 2D and 3D spaces in Z^4. In 2000, G.J.F.Banon [24], define connectedness in terms of a bounded sub-collection of sets and to analyze the topological aspect of a binary image in an expanded domain in which it is sufficient to consider only one kind of connectedness. In 2001 Eladio Domynguez and Angel R. Frances [26], provides an approach based on two foundations. One is multilevel architecture which is a bridges the gap between the discrete world of digital objects and the Euclidean world of their continuous interpretations and another one is axiomatic definition of the notion of digital space.

Another approach was introduced by J.Slapal [27] in 2003, based on closure operation which is generalized topological structures. These are associated with $\alpha - ary$, which are well-behaved with respect to connectedness and for application in digital topology. In particular, for any natural number $n > 1$, find an appropriate closure operation on Z, which is associated with a special n-ary relation on Z. In the case n = 2 this closure operation coincides with the known Khalimsky topology [2]. In 2007,Ying Bai, Xiao Han, Jerry L. Prince [32], define a rigorous extension of the digital topology framework for adaptive octree grids, including the characterization of adjacency, connected components, and points. Guobin Zhu, Xiaoli Liu, Zhige Jia, Qinggquan (In 2007)Li [33], proposed hierarchical framework, a progressive region growing method is proposed that incorporates spatial information related to adjacency between pixels. Particularity of this method is that connected regions and their topology generate objects in different scales, furthermore constructing a tree-object structure reflecting their spatial relationships. Ulrich Eckhardt and Longin Latecki (In 2008) [35], demonstrated that the connectedness relation in a topological space, which is identical to that of a 2D image with the 8-adjacency, corresponds to a non-planar graph. But they have not considered pairs of adjacencies in 3D.

A drawback of the Khalimsky topology is that the Jordan curves with respect to it can never turn at an acute angle. To overcome this deficiency another topology was introduced by J.Slapal [36] in 2008, and established that Khalimsky topology and Marcus topology are quotient topologies of Slapal's topology. In [39],Vladimir Kovalevsky, developed a new set of axioms of digital topology. They define a locally finite (LF) topological spaces which is satisfying the axioms is that the neighbourhood relation is antisymmetric and transitive. The (a, b)-adjacency relations commonly used in computer imagery can be brought into accordance with the connectedness of a topological space. It was demonstrated that in spaces of any dimension ``n" only those pairs (a, b) of adjacencies are consistent, in which exactly one of the adjacencies is the ``maximal" one corresponding to $(3^n - 1)$ - neighbors. In 2012, Loic Mazo, Nicolas Passat, Michel Couprie, Christian Ronse [40], illustrated all the standard pairs of adjacencies (namely the (4, 8) and (8, 4)-adjacencies in Z^2, the (6, 18), (18, 6), (6, 26), and (26, 6)-adjacencies in Z^3, and more generally the (2n, 3n - 1) and (3n - 1, 2n)-adjacencies in Z^n) correctly modeled in F^n. Moreover, they established that the digital fundamental group of a digital image in Z^n is isomorphic to the fundamental group of its corresponding image in F^n. From these results, it becomes possible to establish links between topology-oriented methods developed either in classical digital spaces (Z^n) or cubical complexes (F^n). J.Slapal(In 2013) [42], discuss an Alexandroff topology on Z^2 having the property that its quotient topologies include the Khalimsky and Marcus-Wyse topologies. Also introduced a further quotient topology and prove a Jordan Curve Theorem for it.

III. CONCLUSION

This study we present general framework on digital topology at where result of the different approaches, like graph theoretic, axiomatic and embedding can be developed simultaneously. Therefore, the present research work may be treated as study and developed a generalized framework, which is familiar in all the approaches of digital topology.

Here we have some approaches for digital topology. These various approaches have been made to study geometrical and topological properties of binary digital images. The graph theoretic approach directly related to connectedness. However, to develop a consistent topology of 2D images by means of graphs have failed due to the connectivity paradox, e.g., the Jordan curve theorem claims that a simple closed curve in the Euclidean plane separates the remainder of the plane into two connected components (the inside and the outside of the curve). It may be easily shown that, when removing a point from the curve, the remainder of the plane becomes connected. It is also becomes very difficult to handle more complicated concepts of topology such as continuity, homotopy etc. The axiomatic approach is mathematically very elegant. The axiom have to be chosen in such a way that the digital structure gets properties which are close as possible to the properties of usual topology. But it does not directly provide the languages which are wanted in applications. For example, the objects which are not particularly open set but rather connected sets, the sets which are contained other sets. The Embedding approach is adequate for structures which can be related to an Euclidean space . For example the digital plane Z^2 is consider as a subset of R^2. This leads to the embedding approach for digital topology. The problem is that one has to find for each question an appropriate embedding.

All these approaches, however, not turned out to be fulfil for all the requirement. Which is not perfectly fulfilling the gap between digital topology and digital image.

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