An Analytical Study to Realize Soliton Based Optical and Logic Operation using Coupling Behavior of Optical Coupler

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ABSTRACT

Due to some extra-ordinary character of optical soliton pulse, it is very important to introduce such pulse in optical communication system in place of ordinary optical soliton pulse. Here, optical soliton based AND logic operation implementation is reported in an analytical way by using the coupling technique of optical soliton pulse in coupled waveguide. In such case the electro-optic behavior of optical waveguide is extensively used. For high secure communication system this technique may play a very crucial role. Here ordinary optical pulse is converted to optical soliton pulse before introducing into coupling region. The proposed logic operation is realized by controlling the energy exchange from one waveguide to other waveguide in a coupling region of a waveguide coupler.

Keywords: Evanescent Wave, Directional Coupler, Electro-Optic Effect, Optical Soliton Pulse

I. INTRODUCTION

When a plane wave undergoes total internal reflection at the interface of two dielectrics then there is an wave in the rarer medium, which is termed as evanescent wave. Thus evanescent wave is a near field wave exhibiting exponential decay with distance from the interface. Evanescent wave coupling is a term mainly used in optics. In this coupling evanescent waves are transmitted from one medium to another. When two waveguides are placed in such a way that the evanescent field waves of one waveguide is insignificant at the other waveguide then the interaction between the two waveguides becomes very small and as a result the energy in each waveguide propagates independently even in presence of other waveguides. But if the separation between the two waveguides becomes so small that the evanescent waves of the two waveguides overlap to a considerable extent, then there is an exchange of energy between the two waveguides. In this situation, the evanescent field associated with the propagating modes in the two waveguides interact and lead to a periodic exchange of energy between the two waveguides. Such type of waveguides where coupling between two waveguides occurs is termed as directional coupler. The strength of interaction between two waveguides in a directional coupler depends upon the waveguides parameters, the separation between the waveguides and the wave length of operation. There is an complete transfer of energy from one to other waveguide will occur if the propagation constant of the modes in the two waveguides are identical and otherwise there is an incomplete transfer of energy. This type of energy exchange can be used in building an optical modulator or optical switch. Optical directional couplers are used in signal routing, in time division multiplexing etc. [1,2,3].

Here we propose an all-optical remote controlled solitonic AND logic operations at any distance from the input end of the waveguide by using the evanescent wave coupling phenomena. To achieve
the proposed logic, directional couplers and electro optic effect of wave guide media are used actively.

II. METHOD AND MATERIALS

Optical wave coupling directional coupler

In a directional coupler (as shown in figure 1) evanescent field will be appeared in the wave guide 2 when optical pulse propagates through wave guide 1. If b(z) represent the amplitude of the evanescent wave in the wave guide-2 due to incident of a pulse of amplitude a(z) in wave guide-1, then we may write the couple mode equation as

\[ \frac{da(z)}{dz} = -i\beta_1 a(z) - ik_{12}b(z) \quad \text{(1)} \]

And

\[ \frac{db(z)}{dz} = -i\beta_2 a(z) - ik_{21}b(z) \quad \text{(2)} \]

Where, \( k_{12}, k_{21} \) are the strength of interaction between two modes referred as coupling constant. \( \beta_1 \) and \( \beta_2 \) are the propagation constants of the mode in the wave guide-1 and wave guide-2 respectively. The general solution shows that in the coupled wave guides, one has two independent set of modes of propagation constant \( \beta_1 \) and \( \beta_2 \). Where s and a stands for symmetric modes and antisymmetric modes respectively. The values of \( \beta_1 \) and \( \beta_2 \) are given as

\[ \beta_{s,a} = \frac{1}{2} \left( \beta_1 + \beta_2 \right) \pm \left[ \frac{1}{4} \left( \beta_1 - \beta_2 \right)^2 + k^2 \right]^{1/2} \quad \text{(3)} \]

Where, \( k = (k_{12}k_{21})^{1/2} \quad \text{(4)} \)

The general solution of equations 1 and 2 can be written respectively as

\[ a(z) = a_s \exp(-i\beta_1 z) + a_a \exp(-i\beta_2 z) \quad \text{(5)} \]

And

\[ b(z) = \left( \frac{\beta_1 - \beta_2}{k_{12}} \right) a_s \exp(-i\beta_1 z) + \left( \frac{\beta_1 - \beta_2}{k_{21}} \right) a_a \exp(-i\beta_2 z) \quad \text{(6)} \]

Where, \( a_s = \left( \frac{\beta_1 - \beta_2}{\beta_1} \right) \) \quad \text{(7)}

\[ a_a = \left( \frac{\beta_1 - \beta_2}{\beta_2} \right) \quad \text{(8)} \]

Putting the values of \( a_s \) and \( a_a \) from equations 7 and 8, in equation 6 and after some simplification we obtained as

\[ b(z) = \frac{k}{2} \left[ \frac{1}{4} \left( \beta_1 \right)^2 + k^2 \right]^{1/2} \exp(-i\beta_1 z) + \exp(-i\beta_2 z) \]

\[ \text{--------- (9)} \]

Where, \( \Delta \beta = \beta_1 - \beta_2 \quad \text{(10)} \)

Now putting the values of \( \beta_1 \) and \( \beta_2 \) from equation (3) into equation (9) and after some simplification we obtained as

\[ b(z) = k \frac{\cos \theta}{\theta} \exp\left[ -i \frac{\beta_1 + \beta_2}{2} \right] z \quad \text{(11)} \]

Where, \( \theta = \left[ \frac{1}{4} \left( \Delta \beta \right)^2 + k^2 \right]^{1/2} \quad \text{(12)} \)

As the power in the wave guide is proportional to the square of the amplitude of the pulse in the corresponding wave guide, therefore, the power of the evanescent wave in wave guide-2 is proportional to \( |b(z)|^2 \).

\[ \text{Where, } |b(z)|^2 = k^2 \frac{\cos^2 \theta}{\theta^2} \quad \text{(13)} \]

Similarly using equation (3), (5), (7) and (8), we can show that

\[ a(z) = \left[ \frac{\cos \theta + i \frac{\Delta \beta}{2 \theta} \sin \theta}{\frac{\Delta \beta}{2 \theta} \sin \theta} \right] \exp\left[ -i \frac{\beta_1 + \beta_2}{2} \right] z \quad \text{(14)} \]

And the power in wave guide 1 is becomes as,

\[ |a(z)|^2 = 1 - \frac{k^2}{\theta^2} \sin^2(\theta) \quad \text{--------- (15)} \]
From equations (13) and (15), it is clear that, there is a periodic exchange of energy between two adjacent wave guides. The period of exchange of this energy is

\[ T = \frac{\Pi}{1 \frac{(\Delta\beta)^2 + k^2}{2}} \]

(16)

The minimum interaction length of two wave guide for which maximum energy will be transferred from one wave guide to other is referred as coupling length of the directional coupler. The coupling length is obtained as,

\[ L_c = \frac{\Pi}{2 \frac{(\Delta\beta)^2 + k^2}{2}} \]

(17)

Now for two identical wave guides of equal propagation constant ie \( \Delta\beta = 0 \), we obtain a directional coupler of coupling length \( L_{CO} \).

\[ L_{CO} = \frac{\Pi}{2k} \]

(18)

In this case, energy incident in the input of one wave guide then it comes out through the other. Now if some how we introduce \( \Delta\beta \) between two wave guides such that the coupling length becomes equal to \( \frac{1}{2} \cdot L_{CO} \), then the energy incident on one wave guide will comes out through the same wave guide. The first case where light incident on one wave guide and comes out through the other is referred as cross state and the case where light incident on one wave guide and comes out through the same is referred as parallel state. The minimum \( \Delta\beta \) required for going over from the cross state to parallel state is

\[ \Delta\beta = \sqrt{\frac{\Pi}{L_{CO}}} \]

(19)

The propagation constant of a wave guide may be changed by applying electro-optic effect. Oppositely directed electric fields are applied to the two wave guides, so that the effective index of one wave guide increases where as that of the other decreases. The electric field is applied along the c-axis as the light is polarized along the c-axis.

### III. RESULT AND DISCUSSION

#### Implementation of AND logic

The schematic diagram of the proposed logic implementation is as shown in figure (2). Here two directional coupler is used in cascade.

The first coupler is designed by coupling wave guide 1 and wave guide 2, where as the second coupler is shaped by pairing the outlet of wave guide 2 from first coupler and an another similar wave guide 3. Oppositely focused electric field having a pre-specified shape are applied in both the couplers in coupling section, such that the situation as stated in equation (19) have been fulfilled in case of each couplers. \( I_1 \), \( I_2 \) and \( I_3 \) symbolize the one-to-one optical inputs at the input side of the wave guide 1, wave guide 2 and wave guide 3. Here, optical soliton pulse is fed into the input face of the wave guide 1, wave guide 2 and wave guide 3. However, optical soliton pulse is fed into the input face of the wave guide 2 only to realize the propose AND logic. In this case only \( I_2 \) is present and \( I_1 \), \( I_3 \) are absent. To achieve optical soliton pulse we may use the SPM and non-linearity character of the non-linear optical fiber media. Biased electro-optic modulator are widely used for attaining the right situation to transfer the normal optical pulse to the solitonic one.\[4-7 \] \( Y_1 \), \( Y_2 \)
and $Y_3$ symbolize the output pulse at the outlet side of the wave guide 1, wave guide 2 and wave guide 3 respectively. 

The state (present or absent) of the external electric field applied at the coupling region are considered as inputs of the proposed logic. The state of the biasing field applied to the first coupler is considered as input A and that for coupler 2 is considered as input B. The input state is considered as state zero (0) only when the external voltage is applied and the input state is considered as state one when the external triggering field is absent. The optical soliton signal ($Y_3$) emerges from wave guide 3 is considered as output of the proposed AND logic. The output is considered as state one only when a considerable high intense optical soliton pulse is come out from the optical wave guide. At the same time the output state is considered as state zero when a very low intense or zero intense optical pulse is obtained at the outlet of the wave guide 3 i.e. at $Y_3$.

Case 1: When $A=1, B=1$: In this situation both the biasing field at both the coupling region are absent. Then a portion of the input solitonic optical pulse will be transferred to wave guide 1 from wave guide 2 through the first coupling region. Then a portion of the transferred pulse will be conveyed to the wave guide 3 through second coupler. Therefore, a portion of the input optical pulse will be acquired at $Y_3$. The output state in this occasion is considered as state one.

Case 2: When $A=1, B=0$, i.e. the triggering electric field is applied in the coupling region of second coupler but not in the first coupler. In this situation, a little part of the input soliton pulse will be shifted to the wave guide 1 from wave guide 2 through the first coupling region. This part of the pulse will remain in the wave guide 1 and no part of this will be moved to wave guide 3. Therefore, zero intense optical pulse will be achieved at $Y_3$ which is defined as state zero output.

Case 3: When $A=0, B=1$, then external field is applied only in the coupling section of the first coupler not in the second coupler. Here, almost total optical soliton pulse will be remain in the wave guide 2 and no part of input will be transferred to wave guide 1 through the coupling area of the first coupler. Therefore, almost zero intense pulse will be realized at $Y_3$, which is defined as state zero output.

Case 4: When $A=0, B=0$, then biasing field are applied in both the coupling portion. In this situation, no part of the input soliton pulse will be transferred to the wave guide 1 from wave guide 2 through coupling region 1. Hence, zero intense output will be noticed at the outlet of waveguide 3 i.e. at $Y_3$.

The above different grouping of inputs and corresponding output as as shown in tabular form in table 1.

<table>
<thead>
<tr>
<th>Input optical soliton pulse present at Input side of the second wave guide ($I_2$)</th>
<th>State of A</th>
<th>State of B</th>
<th>Intensity at $Y_3$</th>
<th>State of $Y_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero</td>
<td>Zero</td>
<td>Measurable amount</td>
<td>Zero</td>
<td></td>
</tr>
<tr>
<td>Zero</td>
<td>One</td>
<td>zero</td>
<td>Zero</td>
<td></td>
</tr>
<tr>
<td>One</td>
<td>Zero</td>
<td>zero</td>
<td>Zero</td>
<td></td>
</tr>
</tbody>
</table>

The output optical intensity at $Y_3$ for different combination of inputs A, B are, therefore, follow the truth table of AND logic operation.

IV. CONCLUSION

The evanescent wave coupling between wave guides and electro-optic effect of a wave guide are used here actively to achieve the proposed remote controlled logic. The main advantage of the proposed technique over the other exist techniques is that it is all optical
in nature. The proposed logic operation may be achieved at any distance from the input end of the wave guide as the optical pulse is transferred to solitonic one before introducing it. It is to be mentioned that voltage required to obtain the required values of $\Delta \beta$ can be reduced either by increasing the length or by reducing the electrode gap. It should be mentioned that if the intensity of the optical pulse introduced into the optical fiber is very high valued then the coupling length will be adjusted according to the non-linearity of the wave guide. For proper functioning of the system, it is very crucial to take perfectly same type waveguide and same coupling length of the both coupling region. Also exact same biasing field should be used to triggered both the attached region. As the ordinary optical pulses is replaced by solitonic one, therefore, this technique is very much effective for ultra high speed remote controlled operation. Not only this, the proposed scheme is also suitable for more sealed data transfer.

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VI. REFERENCES