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A Mathematical Model of Two Phases Newtonian Layer Renal Blood Flow in Capillaries

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ABSTRACT

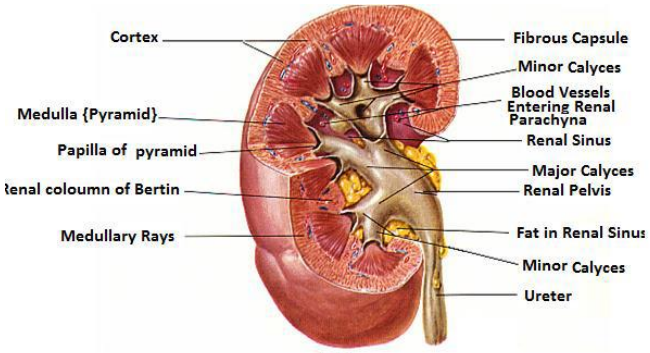
In this paper we have formulated the mathematical modelling for the renal blood flow along the capillaries in case of renal disease Diabetes, considering the nature of renal circulatory system in human body. Here we have considered the blood flow has two phases, one of which is red blood cell and other is plasma. Further analysis of a clinical data in case of a Diabetic patient for Hematocrit v/s Blood pressure is shown. The graphical presentation for particular parametric value is much closer to the clinical observation. The overall presentation is in tensorial form and solution technique adapted is analytical as well as numerical.

Keywords: Pressure Drop, Hematocrit, Renal Circulation, Glomerular Capillary, Diabeties, Peritubular Capillary.

I. INTRODUCTION

A. Anatomy of Kidney

Kidneys are bean shaped organs, about the size of a fist. They lie on either side of the spine at the back of the abdominal cavity. A kidney has an outer fibrous renal capsule and is supported by adipose tissue. The kidney has two main parts, which are cortex and the inner medulla. The outer cortex is reddish brown and is the part where fluid is filtered from blood. The inner medulla is a paler in colour and is made up of conical shaped sections called renal pyramids. This is the area where some materials are selectively reabsorbed into the bloodstream. There is a large area in the centre of the kidney called renal pelvis, which is a funnel shaped cavity that collects urine from the renal pyramids in the medulla and drains it into the ureters. [1][2] The medial border of the kidney is called the hilus and is the area where the renal blood vessels leave and enter the kidney.The normal adult kidney is about 10-12 cm long, 5-7 cm wide, and 2-3 cm thick, and it weighs 125–170g [3][4][5][24]



[7]

B. Function of kidney

The main function of kidneys are processing the blood and remove waste and excess water through the urine. The kidney is not one big filter. Each kidney is comprised of about a million filtering systems called nephrons. Each nephron filters a small amount of blood. The nephron consists of a filter, called the glomerulus, and a tubule. The nephrons overcome a two-step procedure. The glomerulus lets

fluid and waste items pass through it, however, it prevents blood cells and large molecules, mostly proteins, from passing.The filtered fluid then passes through the tubule, which sends out required minerals back to the blood stream and eliminates wastes. The final product ends up being urine.[6][7]

C. Nephron a functional unit of kidney

Each kidney contains around 1 million individual nephrons, the kidneys microscopic functional units that filter blood to produce urine. The nephron is made of 2 main parts: the renal corpuscle and the renal tubule. Responsible for filtering the blood our renal corpuscle is formed by the capillaries of the glomerulus and the glomerular capsule (also known as Bowman's capsule). The glomerulus is a bundled network of capillaries that increases the surface area of blood in contact the blood vessel walls. Surrounding the glomerulus is the glomerular capsule, a cup-shaped double layer of simple squamous epithelium with a hollow space between the layers. Special epithelial cells known as podocytes form the layer of the glomerular capsule surrounding the capillaries of the glomerulus. Podocytes work with the endothelium of the capillaries to form a thin filter to separate urine from blood passing through the glomerulus. The outer layer of the glomerular capsule holds the urine separated from the blood within the capsule. At the far end of the glomerular capsule, opposite the glomerulus, is the mouth of the renal tubule.[8][24]

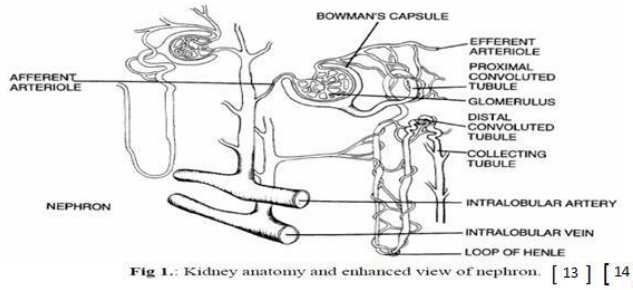


Fig 1.: Kidney anatomy and enhanced view of nephron. [13] [14]

D.Blood supply to kidney

Kidneys is one of the most vital functions of the circulatory system. Despite their relatively small size, the kidneys receive about 20% of the heart's blood output for filtration. The kidneys function is dependent on a constant blood supply. In the abdomen, the renal arteries branch from the abdominal aorta inferior to the superior mesenteric artery and extend laterally toward the kidneys. Just before reaching the kidney, each renal artery divides into five segmental arteries, which provide blood to the various regions of the kidney. Each segmental artery enters the hilus of the kidney and divides into several interlobar arteries, which pass through the renal columns between the renal pyramids and carry blood toward the exterior of the kidney. At the junction between the renal cortex and renal medulla, the interlobar arteries form the arcuate arteries, which turn to follow the contours of the renal pyramids. From the arcuate arteries several branches, known as interlobular arteries, separate at right angles and extend through the renal cortex toward the exterior of the kidney. Each interlobular artery forms several afferent arterioles, which end in a bed of capillaries known as glomeruli where blood is filtered to form urine. In resting adult kidney receive 1.2 to 1.3 l blood per minut or 25% of cardiac output . Renal Blood flow canbe measured with electromagnetic or other type of flow meter or it canbe determined by applying the Fick principle [9] From renal plasma flow , the renal blood flow can be calculated by dividing by one minus the hematocrit : Hematocrit (HCT) - 45% .The renal Blood flow = RPF ×1/(1-HCT) → 700×1/(1-0.45) = 1273 ml/ Minut [10]

E. Pressure in Renal Vessal

Blood is a liquid tissue composed of roughly 55% fluid plasma and 45% cells. The three main types of cells in blood are red blood cells, white blood cells and platelets. 92% of blood plasma is composed of water and the other 8% is composed of proteins, metabolites and ions. The density of blood plasma is approximately 1025 kg/m^3 and the density of blood cells circulating in the blood is approximately

1125 kg/m^3. Blood plasma and its contents is known as whole blood. The average density of whole blood for a human is about 1060 kg/M^3.[11] The pressure in glomerular capillary has been measured directly in the rate and has been found to be considerably lower than the predicted on thye basis of indirect measurement . When the mean systolic arterial pressure is 100 mmhg , then glomerular capillary pressure is about 45 mmhg . The pressure drop across the glomerulas is only 1 to 3 mmhg , but further drop occurs in the efferent arteriole so that the pressure in the peritubular capillary is about 8 mmhg . The pressure in renal vein is about 4 mmhg .The pressure gradient are similar to squirrel monkey end presumably in Human with glomerular capillary pressure that is about 40% of systolic arterial pressure [9]

F. Blood

Blood is a body fluid in humans that delivers necessary substances such as nutrients and oxygen to the cells and transports metabolic waste products away from those same cells. Blood is a two-phase liquid exhibiting non-Newtonian rheological behavior. Viscosity of blood depends on the acting shear forces and is determined by hematocrit value, plasma viscosity and the mechanical properties of red blood cells (RBC) under given shear conditions. RBC are highly deformable bodies and this property significantly contributes to blood flow both under bulk flow conditions and in microcirculation.[12][15]. Approximately 8% of an adult's body weight is made up of blood .Females have around 4-5 litres, while males have around 5-6 litres. This difference is mainly due to the differences in body size between men and women. Blood is classified as a connective tissue and consists of three main components(i), Erythrocytes also known as red blood cells (RBCs). (ii) Leukocytes, also known as white blood cells (WBCs).(iii) Platelets. Blood has three main functions: transport, protection and regulation. Blood has several roles in inflammation: white blood cells, destroy invading microorganisms and cancer cells. Antibodies and other proteins destroy pathogenic substances. Platelet factors initiate blood clotting and help minimise blood loss. [16][17][18][19][20] [21].

G. Structures and Function of renal capillaries

The kidneys have an extensive vascular supply and receive about 20% of the cardiac output. The renal vascular pattern is unusual in that blood flows through two capillary beds, one with high pressure (glomerular) and the second with low pressure (peritubular), connected in series. Blood enters the kidney via the renal artery and, after a series of divisions, arrives at the glomerulus. Blood entering the glomerular capillaries must first pass through an afferent arteriole. Blood exiting the glomerular capillaries passes through a second arteriole, the efferent arteriole. Blood then flows through the peritubular capillaries, which include the vasa recta that extend into the renal medulla. Blood leaves the peritubular capillaries, collects in progressively larger venules and veins, and then exits the kidney via the renal vein.[22][23][24]

II. METHODS AND MATERIAL

A. Disease (Diabetes)

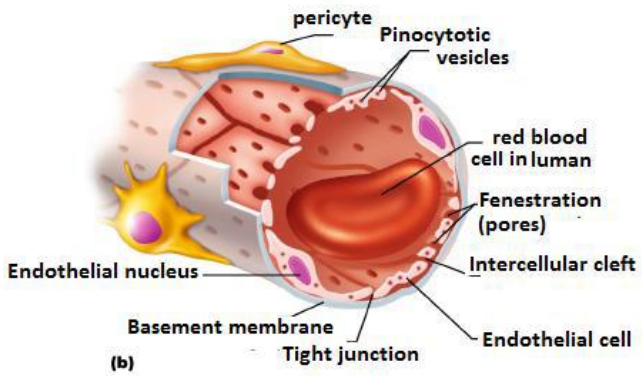
Diabetes is the condition in which the body does not properly process food for use as energy. Most of the food we eat is turned into glucose, or sugar, for our bodies to use for energy. The pancreas, an organ that lies near the stomach, makes a hormone called insulin to help glucose get into the cells of our bodies. When you have diabetes, your body either doesn't make enough insulin or can't use its own insulin as well as it should. This causes sugars to build up in your blood. This is why many people refer to diabetes as "sugar." Diabetes can cause serious health complications including heart disease, blindness, kidney failure, and lower-extremity amputations.[25][26]

B.Hematocrit

This is the ratio of the volume of red cells to the volume of whole blood. Normal range for hematocrit is different between the sexes and is approximately 45% to 52% for men and 37% to 48% for women. This is usually measured by spinning down a sample of blood in a test tube, which causes the red blood cells to pack at the bottom of the tube. The hematocrit (expressed as percentage points) is normally about three times the hemoglobin concentration (reported as grams per deciliter).[24][32][33][34][35]

C. Description of the problem

How the blood flow in capillaries is possible as we know that these vessels are far enough from the heart as well as thin. It's a natural question because the blood flows very slowly in arterioles where there is high viscosity. The satisfactory answer of this problem is given by Fahreaus-Lindqvist effect. According to this effect the blood flows in two separated layers while passing through capillaries. The plasma layer containing almost no blood cells. The second layer is that of blood cells which float in plasma on the axis of the capillary. In this process the effective blood viscosity depends upon radius of the capillary. That's why the effective viscosity decreases, as the radius and thus the blood flow becomes possible.[24]



D.Real model

Blood viscosity is the thickness and stickiness of blood. It is a direct measure of the ability of blood to flow through the vessels. It is defined as the inherent resistance of blood to flow. Normal adult blood viscosity is 40/100, which is read as "forty over one hundred" and reported in units of millipoise.[27] Blood is a vigorous organ in so far as it behaves as a non-Newtonian fluid, which means that its viscosity changes as a function of shear rate. Think of shear rate as velocity. When blood moves quickly as in peak-systole, it is physically thinner; when it moves slowly during end-diastole, it is thicker and stickier. This is because red cells aggregate. The phenomenon is known as the shear-thinning, non-Newtonian behaviour of whole blood [28].[29][30][31].In this paper we selected generalized three dimensional orthogonal curvilinear co-ordinate system, which is prescribed as E3 called as 3-dim Euclidean space. Here we have some quantities related to moving blood in cylindrical vessels: blood velocity $V^k = V^k(x^i, t)$,k=1,2,3 blood pressure P = p(x^i,t) and density $\boldsymbol{\rho}$=$\boldsymbol{\rho}$(x^i,t) where x^i be the co-ordinates of any point in space and i-1,2,3.

1)Equation of Continuity : When there is absence of source and sink in any region of flowing fluid, the fluid mass is conserved in that region. As we observed that there is no source or sink in the whole circuit of the human blood circulatory system, the heart behaves merely like a pumping station, so the law of conservation of mass can well be applied to hemodynamic [36]. Since, whole blood flow circuit of the kidney is called a Renal Circulatory System. Hence renal circulatory system is a sub system of human circulatory system. Blood enter in kidney by arteries and out by veins and in a kidney no source or sink.

Mass of enter the blood = mass of outer the blood Therefore law of conservation of mass can also be applied for renal circulatory system.

The flow of blood is affected by the presence of blood cells. This effect is directly proportional to the volume occupied by blood cells.

Let X is the volume portion covered by the blood cells in unit volume. And X can be replaced by H/100, where H is the hematocrit the volume percentage of blood cells. Then the volume portion covered by plasma will be 1-X. if the mass ratio of blood cells to plasma is r, then clearly

$r = \frac{x\rho_c}{(1-x)\rho_p}$ Where ρ_c and ρ_p and are densities of blood cells and blood plasma respectively. Usually this mass ratio is not constant; even then this may be supposed to be constant in present context [37].

The both phase of blood, i.e., blood cells and plasma move with a common velocity. Campbell and Pitcher have presented a model for this situation. According to this model we consider the two phases of blood separately [38]. Hence according to principle of conservation of mass, the equations of continuity for the two phases are as follows [39].

$\frac{\partial(X\rho_c)}{\partial t}$ +(X$\rho_c V^i$), $i = 0$

$\frac{\partial(1-X)\rho_p}{\partial t}$ +((1-X)$\rho_p V^i$), $i = 0$

Where v is the common velocity of the two phases blood cells and plasma and (X$\rho_c V^i$), i is co-variant derivative of (X$\rho_c V^i$) with respect to X^i.

In the same way ((1-X)$\rho_p V^i$) with respect to X^i.

If we define uniform density ρ_m as follows:

$\frac{1+r}{\rho_m} = \frac{r}{\rho_c} + \frac{1}{\rho_p}$ …….(1)

Then the equations can be combined together as follows,

$\frac{\partial(\rho_m)}{\partial t}$ +($\rho_m c$), $i = 0$

As we know that blood is incompressible fluid,hence ρ_m will be a contant constant quantity. Thus the equation of continuity for blood flow takes the following form:

$V_{,i}^{\ i}$=0

$\frac{\partial V^i}{\partial X^i} + \frac{V^i \partial\sqrt{g}}{\sqrt{g}\partial X^i} = \frac{1}{\sqrt{g}}\left(\sqrt{g}V^i\right)_{,i}$=0

2) Equation of Motion: According to this principle, the total momentum of any fluid system is conserved in absence of external force. So the law of conservation of momentum can well apply to renal circulatory system. In other words, the rate of change of momentum of a fluid particle with respect to time equals to external force exerted on it. This is also called Newton's 2nd law of motion.

So, the rate of change of momentum is equal to sum of about two mentioned forces, which may be symbolically presented as follows.

$\frac{dp}{dt}$ =-P+F where ,$\frac{dp}{dt}$ =rate of change of momentum

P=Internal pressures

F=viscous force

The hydro dynamical pressure p between the two phases of blood can be supposed to be uniform because the both phases i.e. blood cells and plasma are always in equilibrium state in blood [40]. Taking viscosity coefficient of blood cells to be η_c and applying the principle of conservation of momentum, we get the equation of motion for the phase of blood cells as follows:

$X\rho_c \frac{\partial v^i}{\partial t} + \left(X\rho_c v^i\right)v_{,j}^{\ i} = -Xp_{,j}\, g^{ij} + X\eta_c\left(g^{jk}v_{,k}^{\ i}\right)_{,j}$…….(2)

Similarly ,taking the viscosity coefficient of plasma to be η_p the equation of motion of plasma will be as follows:

$(1-X)\rho_p \frac{\partial v^i}{\partial t} + \left((1-X)\rho_p v^j\right)v_{,j}^{\ i} = -(1-X)p_{,j}\, g^{ij} + (1-X)\eta_p\left(g^{jk}v_{,k}^{\ i}\right)_{,j}$ ..(3)

Now adding equation (2) and (3) and using relation (1),the equation of blood flow with the both phases will be as follows:

$\rho_m \frac{\partial v^i}{\partial t} + \left(\rho_m v^j\right)v_{,j}^{\ i} = -p_{,j}\, g^{ij} + \eta_m\left(g^{jk}v_{,k}^{\ i}\right)_{,j}$

Where $\eta_m = X\eta_c + (1-X)\,\eta_p$ is the viscosity coefficient of blood as a mixture of two phases.

3) Different constitutive equations for blood: Generally blood is non-homogeneous mixture of plasma and blood cells. Though for practical purposes it may be considered to be homogeneous two-phase mixture of plasma and blood cells. The constitutive equations proposed for whole blood mixture are as follows:

(i) Newtonian equation

$\tau=\boldsymbol{\eta}e,$

where **η** is the viscosity coefficient This is found to hold good in the broad blood vessels where there is low hematocrit [41].

(ii) The non-Newtonian power law equation

$\tau=\boldsymbol{\eta}e^{n}$

This is found to be conformable for strain rate between 5 and 200

$0.68\leq n\leq 0.80$ [42]

The non -Newtonian Herschel-Bulkley equation [43]

$\tau=\boldsymbol{\eta}e^{n}+\tau_0(\tau\geq\tau_0)$

$e=0(\tau<\tau_0)$

It holds good when blood shows yield stress τ_0.

We notice that the yield stress arise because blood cells form aggregates in the form of rouleaux at low strain rate.

If $\tau<\tau_0$,no blood flow takes place.It is found that yield stress is given by the following formula

$$\tau_0^{\frac{1}{3}}=\frac{A(H-H_m)}{100}$$

Where,

$A=(.008\pm 0.002\text{dyne}/cm^2)^{\frac{1}{3}}$

H is normal hematocrit and H_m is the hematocrit below which there is no yield stress.

4) Boundary Conditions:

(i)The velocity of blood flow on the axis of capillaries at r=0 will be maximum and finite, say V_0 = maximum velocity.

(ii)The velocity of blood flow on the wall blood vessel at r=R, where, R is the radius of capillary, will be zero. This condition is well known as no-slip condition.

5) Mathematical Modeling: We consider the two layer blood flow to be Newtonian. The first layer is that of plasma while second one is core layer. Let the viscosity of plasma layer be η_p and that of core layer η_m

Where $\eta_m=X\eta_C+(1-X)\eta_p$ where

η_C = viscosity of blood cells η_p = Viscosity of Plasma layer

η_m =Viscosity of core layer

and X is portion of blood cells in unit .$X=\frac{H}{100}$

SOLUTION:

Now we describe the basic equations for Power law blood flow as follows:

(i)Equation of Continuity:

In tensorial form as follows:

$$\frac{1}{\sqrt{g}}\left(\sqrt{g}v^i\right)_{,i}=0$$

(ii)Equation of motion:

$$\rho_m\frac{\partial v^i}{\partial t}+\rho_m v^j v^i_{,j}=-\rho_{,j}g^{ij}+\eta_m\left(g^{jk}v^i_{,k}\right)_{,j}$$

$$\rho_m=X\rho_C+(1-X)\rho_p$$

Where

ρ_m =density of mixture blood

ρ_C = density of plasma

ρ_p = density of blood cells

The blood flow in capillary is symmetrical w.r.t. axis. Hence, v_θ, v_z, v_r and p do not depend upon θ. Since only one component of velocity which is along axis is effective.

We have,

$v_\theta = 0, v_z = V, v_r = 0$

Since, flow is steady,

$$\frac{\partial p}{\partial t} = \frac{\partial v_r}{\partial t} = \frac{\partial v_\theta}{\partial t} = \frac{\partial v_z}{\partial t} = 0$$

$\frac{\partial v_z}{\partial z} = 0$

$v_z = V(r)$

r-component

$\rho_m(0) = -\frac{\partial p}{\partial r} + \eta_m(0)$

$\frac{\partial p}{\partial r} = 0$

P = p(z)

θ-component

$\rho_m(0) = 0 + \eta_m(0)$

0=0

z- component

$$\rho_m v_z \frac{\partial v_z}{\partial t} = -\frac{\partial p}{\partial z} + \eta_m \left[\frac{1}{r}\frac{\partial}{\partial r}\left\{r\frac{\partial v_z}{\partial r}\right\} + \frac{\partial^2 v_z}{\partial z^2}\right]$$

$$\rho_m v_r \frac{\partial V(r)}{\partial t} = -\frac{\partial p}{\partial z} + \eta_m \left[\frac{1}{r}\frac{\partial}{\partial r}\left\{r\frac{\partial V(r)}{\partial r}\right\} + \frac{\partial^2 V(r)}{\partial z^2}\right]$$

and pressure p depends on z.

$p = -\frac{\partial p}{\partial z}$

By using first and second boundary condition, we get

$V = \frac{p}{4\eta_m}(R^2 - r^2)$

The velocity of plasma layer is obtained by replacing η_m by η_p in formula of Newtonian model, which is as follows:

$V_p = \frac{p}{4\eta_P}(R^2 - r^2)$; R-$\delta \le r \le R$

The velocity of core layer can also be obtained in a similar way as follows:

$Vm = \frac{p}{4\eta_m}(R^2 - r^2) + \frac{p}{4\eta_m}[R^2 - (R - \delta)^2]\left(\frac{\eta_m}{\eta_p} - 1\right)$; $0 \le r \le R$-δ

Where R is the radius of the capillary and δ is the thickness of the plasma layer. δ Is supposed to be independent of R. [15]

III. RESULTS AND DISCUSSION

A.Clinical data:

S. no.	H.B.	Hematocrit	Blood Pressure(mmhg)
1	14.9	44.7	130/80=17331.6/10665.6p
2	14.8	44.4	120/80=15998.4/10665.6p
3	15.9	47.7	130/80=17331.6/10665.6p
4	14.7	44.1	120/85=15998.4/11332.2p

B. Bio-Physical Interpretation:

The blood flow in capillary is

$$Q = \int_0^{R-\delta} v_m 2\pi r dr + \int_{R-\delta}^{R} v_p 2\pi r dr$$

$$Q = \int_0^{R-\delta} \left[\frac{p}{4\eta_m}(R^2 - r^2) + \frac{p}{4\eta_m}[R^2 - (R-\delta)^2]\left(\frac{\eta_m}{\eta_p} - 1\right)\right] 2\pi r dr \ ..$$

$$\ldots + \int_{R-\delta}^{R} \frac{p}{4\eta_P}(R^2 - r^2) 2\pi r dr$$

$$Q = \frac{\pi p R^4}{8\eta_p}\left[1 - \left(1 - \frac{\delta}{R}\right)^4\left(1 - \frac{\eta_P}{\eta_m}\right)\right]$$

In this model we can find directly the relationship between the hematocrit and blood pressure drop.

$$p = -\frac{dp}{dz}$$

$$p\int_{z_i}^{z_f} dz = -\int_{p_i}^{p_f} dp$$

$$p(z_f - z_i) = (p_i - p_f)$$

$$p = \frac{(p_i - p_f)}{(z_f - z_i)} = \frac{\Delta p}{length\ of\ capillary}$$

$$Q = \frac{\pi p R^4}{8\eta_p}\left[1 - \left(1 - \frac{\delta}{R}\right)^4\left(1 - \frac{\eta_P}{\eta_m}\right)\right]$$

$$\Delta p = \frac{8Q\,\eta_p \times length\ of\ capillary}{\pi R^4\left[1-\left\{1-\frac{\delta}{R}\right\}^4\left\{1-\frac{\eta_p}{\eta_m}\right\}\right]} \quad \ldots\ldots\ldots(4)$$

Now putting all standard values in equation (4)

$\eta_m = X\eta_C + (1 - X)\eta_p$

η_p = Viscosity of Plasma layer=1.2× 10^{-3} ps

R= radius of capillary= 0.0965 m

$\delta = Thickness\ of\ plasma = 10^{-6}$ m

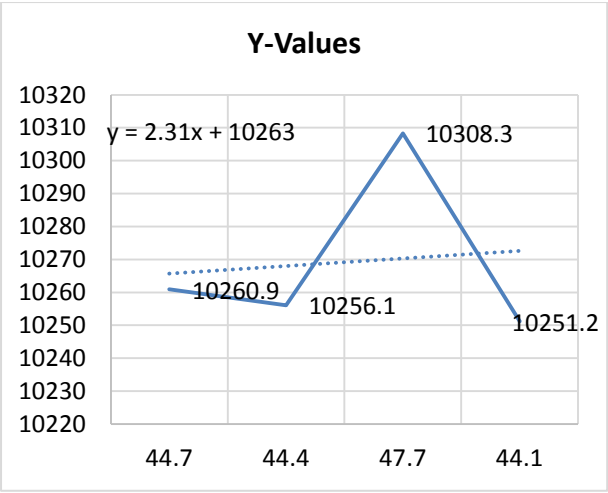
Q = flow flux of blood in renal capillary = 0.01833 pa

Length of capillary = 19000

$$\Delta p = \frac{(1.939\times10^{-4}\times H)+0.0117}{(1.5766\times10^{-8}\times H)+(1.2802\times10^{-6})} \quad \ldots..(5)$$

Using equation (5),pressure drop (PD) is calculated in result….

S.no.	1	2	3	4
H	44.7	44.4	47.7	44.1
PD	10260.9	10256.1	10308.3	10251.2



IV. CONCLUSION

In Bio physical Interpretation ,We have taken clinical data regarding with Blood Pressure and Hematocrit of Diabetic Patient and we get the relation $\Delta p = \frac{(1.939\times10^{-4}\times H)+0.0117}{(1.5766\times10^{-8}\times H)+(1.2802\times10^{-6})}$ by using the Two phase-Newtonian Model (Newtonian Power low flow) and draw the graph between Blood pressure drop and Hematocrit in renal Capillary in Newtonian flow . And trend of graph shows the relation between Blood Pressure drop and Hematocrit as linear as

y= 2.31x+10263. This linear relation approves the two phase relation $\eta_m = X\eta_c + (1 - X)\,\eta_p$ where X= H/100 And slope of trend line is … **2.31**

V. ACKNOWLEDGEMENT

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VI. REFERENCES

[1]. Structure and Function of the Kidney Essay. Submitted by: ppaul11 on March 14, 2015

[2]. Reddi AS. Structure and function of the kidney. In: Reddi AS. Essentials of Renal Physiology. New Jersey: College Book Publishers, 1999:21-43.

[3]. Madsen KM , Tisher CC . Anotomy of kidney . In Brenner BM ,ed. Brenner and Rector's .The kidney , 7th ed. Vol.- 1Pheladelphia :Saunders, 2004,3-72

[4]. Kriz W ,Elgar M Renal anatomy .In Johnson RJ, Feehally J, eds. Comprehenshive clinical nephrology ,2nd ed. Edinburgh Mosby ; 2003;1-11

[5]. Cotran , RS S ; Kumar , Cotran, RS S.; Kumar, Vinay; Fausto, Nelson; Robbins, Stanley L.;Abbas, Abul K. (2005). Robbins and Cotran

pathologic basis of disease. St. Louis, MO: Elsevier Saunders.ISBN 0-7216-0187-1

[6]. Nutrition and Health ; Nutrition Kidney disease ; edited by LD Byham -Grey , JD . Burrowes and GM Chertow Humana Press ;Totowa ; NJ

[7]. Essential of Clinical Nephrology ; 1st edition ; Published by ; Dar EI Shorouk, 8 Sebawieh Al masry,Nasr City , Cairo, Egypt ; post box : 33 Pnorama : dar@shrouk.com

[8]. Renal physiology and Body Fluids ; chapter-22 Kidney Function ;George A Tanner ; Ph.D. ,pp-393

[9]. Review of Medical physiology ; 23 edition , Kim Barrett ,Headwen , Brooks Scott Boitano Susan Baman ; pp-644

[10]. Kapur J.N., Mathematical Models in Biology & Medicine, EWP New Delhi, 354, 1992.

[11]. Shmukler , Michael (2004) " Density of the blood " The physics Factbook . Retrieved October 4 2006

[12]. Physiology and maintenance ; Vol-III ;Hemorheology and hemodynamics-oguz k Buskurt, Herburt.J Meiselman

[13]. " Human Kidney . Diagram, with nephron ," Pearson publishing .Lab Art Library .96L,12001. wwwlabartlibrary.com/symbiosis/96/96L12001.pdf.juli7, 2003

[14]. International journal of Engineering Research and development e-ISSN:227 067X,p-ISSN :2278-800X , www.ijerd.com Vol-5 Issue 5 (Dec.2012) pp-23-30

[15]. ISO 9001:2008 Certified International Journal of Engineering and Innovative Technology (IJEIT) Volume 4, Issue 4, November 2014

[16]. Journal of physiology and pathophysiology) ; DOI; 10.5897/JPAP2013.0075 ISSN 2141-260X Academic Journal vol-4(2) pp 23-28 march 2013 www.academic journals.org/JPAP

[17]. Journal of Global Research in Computer Science ISSN- 2229-37IX; (www.jgrcs.info)

[18]. Vol -2 ; No.-4 April 2011

[19]. J Blood Disord Transfus; ISSN2155-9864 JDBT ; open access journal ; vol. 3 Issue -2 http//dx .doi.org /10 .4172/2155-9864.1000118

[20]. Aubuchnon JP , Kruskall MS (1997) Transfusion safely ;Realigning efforts with risk . Transfusion 37: 1211-1216

[21]. J Blood Disord Transfus; ISSN2155-9864 JDBT ; open access journal ; vol. 3 Issue -2 http//dx .doi.org /10 .4172/2155-9864.1000119

[22]. Metz J. MacGrath KM Copperchini ML, Haeusler M, Haysom HE , et al.(1995) Appropriateness of transfusion of red blood cells , platelets and fresh frozen plasma . An audit in a teritarycare teaching hospital , MedJ Aust 162;572-577

[23]. A.C.Guyton; Medical Physiology; Chapter 26; Urine Formation by the kidneys; I. Glomerular Filtration, Renal Blood flow and their control; PP 283-285

[24]. Shmukler, Michael ; Density of Blood ; The Physics Factbook ; Retrived 4 October 2006

[25]. Physiology and Anatomy . Elixir Physio. & Anatomy 89 (2015) 36723-36729

[26]. Shoback, edited by David G Garner , Dolores (2011) " chapter 17" . Greenspan's basic and clinical endocrinology (9th Edit.) . Newyork : McGraw-Hill Medical . ISBN 0-07 -162243-8

[27]. Walter F. Boron (2004) . Medical physiology : A cellular and Molecular Aproach . Elsevier / Saunders. ISBN1-4160-2328

[28]. Sloop GD. A Unifying Theory of Atherogenesis. Med Hypothesis. 1996;47:321-5

[29]. Cokelet GR, Meiselman HJ. Blood rheology. In : Baskurt OK, et al.Handbook of Hemorheology and Hemodynamics. IOS Press, 200; 45-71

[30]. ISSN: 2277-3754 ISO 9001:2008 Certified International Journal of Engineering and Innovative Technology (IJEIT) Volume 4, Issue 4, November 2014

[31]. Sherman, I.W. & Sherman, V.G.; Biology- a human approach oxford univ. press New York, oxford; 276-277; 1989.

[32]. Sapna Ratan Shah; Mathematical analysis of blood flow throw atherosclerotic arterial segment having non-symmetric and mild stenosis; International journal of research in pure & applied physics; 21 april, 2011.

[33]. Purves, William K.; Sadava, David; Orians, Gordon H.; Heller, H. Craig (2004). Life: The Science of Biology (7th ed.). Sunderland, Mass: Sinauer Associates. p. 954. ISBN 0-7167-98565.

[34]. Stuart J, Kenny MW: Blood rheology. J din Pathol 19803:417-429

[35]. Chien S: Blood rheology in hypertension and cardiovascular disease. Cardiovasc Med 1977;2:356-360 [35] Berkow, Robert, ed. Merck Manual of Medical Information. Whitehouse Station, NJ: Merck Research Laboratories, 1997

[36]. Fogelson, A.L.; A mathematical model and numerical method for studying platelet adhesion and aggregation during blood clotting; J.comput. physics; 56; 1984.

[37]. Singh P. and Upadhyay K.S. ; A New approach for the shock propagation in two phases system; Nat.Acad.Sci.Letters, Vol 8, No 2, 1985

[38]. Compbell, I.J. and Pitcher, A.S.; Shock waves in a liquid containing gas bubbles, proc.Roy.Soc; A 243, 1958.

[39]. Kapur,J.N; and Gupta, R.c ; Power-law fluid flow in the inlet length of a circular pipe; The math, seminar; 3, 55-67; 1963.

[40]. Ruch, T.C. and H.D., Patton, (Eds); Physiology and Biophysics-volsii and iii, W.B.S.; 1973.

[41]. Taylor, M.G. ; Hemodynamics. Ann.Rev. Physiol; 35; 87,1973.

[42]. Kapur J.N., Mathematical Models in Biology & Medicine, EWP New Delhi, 353, 1992.

[43]. Kapur J.N., Mathematical Models in Biology & Medicine, EWP New Delhi, 354, 1992.