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Computational Modelling and Analysis of Transmission dynamics of Zika Virus Based on Treatment

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ABSTRACT

Zika virus is caused to be spread among human population by the Aedes aegypti pathogen. As per our information, no treatment is available to control this disease. In this paper, a basic SEIR mathematical model for transmission dynamics of Zika virus is presented and analyzed to understand the in-depth study of this deadly disease. A threshold parameter is also called reproduction number obtained, by using next generation technique, to analyze the stability of the disease free equilibrium (DFE) and endemic point. It has been seen that if R_0<1, DFE is locally asymptotically stable and unstable if R_0>1. Lyapunov functional is constructed to assess the global stability of the present model.

Keywords: Zika Virus, Reproduction Number, Next Generation Techniques and Stability Analysis

I. INTRODUCTION

Diseases have a close relation to mankind since its inception. Many developed and developing nations have been spending enormous amount of money to counter theses disease. But new and new diseases Zika virus is one of the deadly viruses emerging in both the developing and developed nations. The outbreak of Zika virus is a matter of serious concern for the entire world. Zika virus (ZIKV) is named after the Zika forest in Uganda. Prior to 2007, there were few cases of Zika virus which had been seen in some parts of African and Asian countries; however some cases were not reported. Due to the similarity of symptoms of Zika virus with many other diseases (Hennessey *et al.* 2016), many cases may have not been recognized. Between 2013 and 2014, other Zika virus outbreaks were documented in different regions of the Pacific islands (cf. Musso *et al.*2014). It is a vector-borne disease generally transmitted by infected mosquitoes, *Aedes Aegypti,* which also causes dengue, chikungunya and yellow fever. Yakob and Clements (2013) formulated a mathematical model of chikungunya dynamics and control. Agusto *et al.* (2017*)* presented a mathematical model for Zika virus dynamics with sexual transmission route.

The first vector-borne disease that is caused by both mosquitos bites and directly from human to human is Zika virus. Many researchers and biologists have been working in this field with the help of mathematical models to get more knowledge about the virus. Watmough *et al.* (2002) developed techniques to find reproduction numbers and sub-threshold endemic equilibria by next generation techniques for compartmental models of diseases transmission. Korobeinikov and Wake (2002) developed Lyapunov function and global properties for SIR and SEIR epidemiological models. Brauer and

Castillo-Chavez (2001) formulated a Mathematical model in population biology and epidemiology.

It has been observed that using of some indoor medicinal spray can reduce the impact of mosquitoes' population which is often found in corners. This work explores the impact of indoor spraying in lessening the impact of Zika virus we presented a mathematical model involving the transmission dynamics of an ordinary differential equation using a Susceptible-Exposed-Infected-Recovered (SEIR) model for the human interacting with a Susceptible-Exposed-Infected (SEI) for the mosquito. The rest of the paper is organized as follows:

In sec. 2, the mathematical model is formulated and presented. Mathematical analysis is carried out in sec. 3. Finally conclusions are drawn in section 4.

II.Model Description

The total human population is divided into four different classes namely the susceptible class$S_h(t)$, exposed class $E_h(t)$, infected class $I_h(t)$ and recovered by$R_h(t)$. The total population is given by:

$$N_h(t) = S_h(t) + E_h(t) + I_h(t) + R_h(t)$$

And total mosquitoes population is given by:

$$N_m(t) = S_m(t) + E_m(t) + I_m(t)$$

Following notations are used in the formation of mathematical models:

$S_h(t)$: Number of susceptible humans at time t.
$E_h(t)$: Number of exposed humans at time t.
$I_h(t)$: Number of infected humans at time t.
$R_h(t)$: Number of recovered humans at time t.
$S_m(t)$: Number of susceptible mosquitoes at time t.
$E_m(t)$: Number of exposed mosquitoes at time t.
$I_m(t)$: Number of infected mosquitoes at time t.
$R_m(t)$: Number of recovered mosquitoes at time t.

Following are some parameters that we used in the formation of mathematical models.

Λ : Recruitment rates of population in susceptible classes.
γ_1 : Rate of deduction for individuals in latent Zika virus.
γ_2 : Treatment Rate for individuals in active Zika virus.

μ : Natural death rate.
ξ : Rate of indoor spraying of medicine
k : Natural rate of progression to active Zika virus.
d : Natural death rate.
p : Newly infected individual.
q : Modification parameter

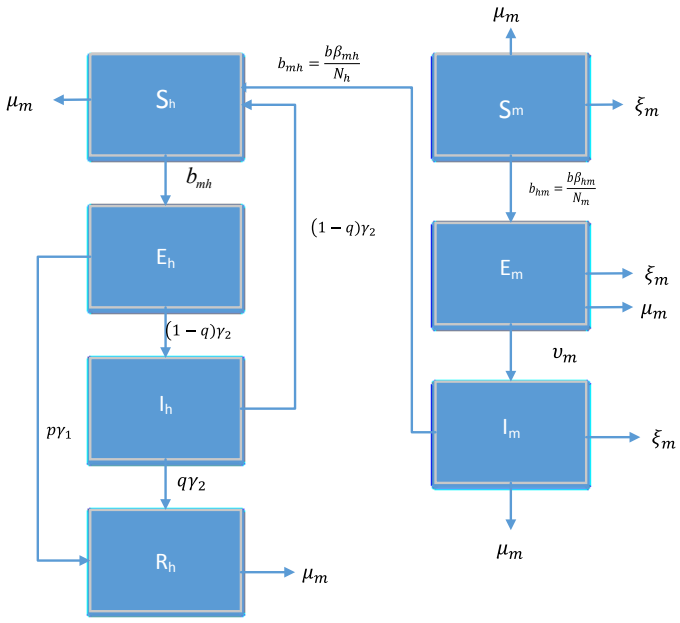

Figure 1. Transition diagram of transmission dynamics of Zika Virus

A. Model equations

The system of governing equations is as follows:

Human Population

$$S_h(t) = \Lambda_1 - b_{mh}S_hI_m - \mu S_h + \gamma_2(1-q)I_h(t) \quad (1)$$

$$E_h(t) = b_{mh}S_hI_m - \mu E(t) - KE(t) - \gamma_1 E(t) \quad (2)$$

$$I_h(t) = KE(t) - \mu I_h(t) - dI_h(t) + (1-p)\gamma_1 E(t) - \gamma_2 I_h(t) \quad (3)$$

$$R_h(t) = p\gamma_1 E(t) - \mu R_h(t) + \gamma_2 q I_h(t) \quad (4)$$

B. Mosquito Population

$$S_m(t) = \Lambda_2 - \mu_m S_m - b_{mh} S_h I_h - \xi S_m(t) \quad (5)$$

$$E_m(t) = b_{mh} S_m I_h - \mu_m E_m(t) - V_m E_m(t) + \xi E_m(t) \quad (6)$$

$$I_m(t) = V_m E_m(t) - \mu_m I_m(t) - \xi I_m(t) \quad (7)$$

With initial conditions:

$$S_h(0) = 0, E_h(0) = 0, I_h(0) = 0, R_h(0) = 0, S_m(0) = 0, E_m(0) = 0, I_m(0) = 0$$

III.ANALYSIS

Here we compute the equilibrium states, namely the Zika free equilibrium (ZFE) and the endemic equilibrium (EE) and provide stability analysis by determining the basic reproduction number.

Lemma 1: The feasible region τ defined by

$$\tau = \left\{ S_h(t), E_h(t), I_h(t), R_h(t), S_m(t), E_m(t), I_m(t) \in R^7 N(t): \leq \frac{\Lambda}{\mu} \right\}$$

With initial conditions

$$S_h(0) \geq 0, E_h(0) \geq 0, I_h(0) \geq 0, R_h(0) \geq 0, S_m(0) \geq 0, E_m(0) \geq 0, I_m(0) \geq 0$$

is positively invariant of system of equations (1) to (7)

Proof: Adding the equations of system (1) to (7), we get

$$\frac{dN}{dt} \leq \Lambda - \mu N. \quad (8)$$

On solving this differential equation (8), we obtain $0 \leq N(t) \leq \frac{\Lambda}{\mu} + N(0)e^{-\mu t}$,

Where $N(0)$ represents the initial values of the total population. Thus $\lim_{t\to\infty} \sup N(t) \leq \frac{\Lambda}{\mu}$

It implies that the region

$$\tau = \left\{ S_h(t), E_h(t), I_h(t), R_h(t), S_m(t), E_m(t), I_m(t) \in R^7 N(t): \leq \frac{\Lambda}{\mu} \right\}$$

is positively invariant set of system (1) to (7). Therefore, existence and uniqueness of system (1) to (7) on the region given by set.

A. Zika virus free equilibrium and Reproduction Number

We derived the equilibrium states and the investigate their stability by using the reproduction number. Set of equation (1) to (7) has a Zika virus free equilibrium given by,

$$E_0 = (S_h^0, E_h^0, I_h^0, R_h^0, S_m^0, E_m^0, I_m^0) = \left(\frac{\Lambda}{\mu}, 0,0,0,0,0,0\right)$$

The basic reproduction number, R_0, is calculated by using the next generation matrix (cf. ven Den and Watmough (2002).We consider the $F^* \; and \; V^*$ which are defined for the appearance of new infection and transfer of individuals out of the infective compartments, respectively, we obtain

$$F^* = \begin{bmatrix} b_{mh} S_h I_m, \\ ((1-p)\gamma_1 + K)E(t), \\ b_m h_m S_m I_m, \\ V_m E_m \end{bmatrix} \quad \text{and}$$

$$V^* = \begin{bmatrix} (\mu + K - \gamma_1)E(t), \\ (\mu + d + \gamma_2)I_h(t), \\ (\mu_m + V_m - \xi)E_m(t), \\ (\mu_m + \xi)I_m(t) \end{bmatrix}$$

The Jacobian matrices after taking the partial derivatives of $F^* \; and \; V^*$ at E_0 are

$$F = \begin{bmatrix} 0 & 0 & 0 & b_{mh} S_h \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & b_m \, h_m s_m \\ 0 & V_m & 0 & 0 \end{bmatrix} \quad (9)$$

And

$$V = \begin{bmatrix} \mu + k - \gamma_1 & 0 & 0 & 0 \\ 0 & \mu + d + \gamma_2 & 0 & 0 \\ 0 & 0 & \mu + V_m - \xi & 0 \\ 0 & 0 & 0 & \mu_m + \xi \end{bmatrix} \quad (10)$$

$$|V| = (\mu + k - \gamma_1)\begin{vmatrix} \mu + d + \gamma_2 & 0 & 0 \\ 0 & \mu + V_m - \xi & 0 \\ 0 & 0 & \mu_m + \xi \end{vmatrix}$$

$$= (\mu + k - \gamma_1)\{(\mu + d + \gamma_2)(\mu + V_m - \xi)(\mu_m + \xi)\}$$

Now

$$V^{-1} = \frac{1}{|V|} Adj\, V = \begin{bmatrix} \frac{1}{\mu+k-\gamma_1} & 0 & 0 & 0 \\ 0 & \frac{1}{\mu+d+\gamma_2} & 0 & 0 \\ 0 & 0 & \frac{1}{\mu+V_m-\xi} & 0 \\ 0 & 0 & 0 & \frac{1}{\mu_m+\xi} \end{bmatrix}$$

$$\therefore FV^{-1} = \begin{bmatrix} 0 & 0 & 0 & b_{mh}S_h \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & b_m\, h_m s_m \\ 0 & V_m & 0 & 0 \end{bmatrix} * \begin{bmatrix} \frac{1}{\mu + k - \gamma_1} & 0 & 0 & 0 \\ 0 & \frac{1}{\mu + d + \gamma_2} & 0 & 0 \\ 0 & 0 & \frac{1}{\mu + V_m - \xi} & 0 \\ 0 & 0 & 0 & \frac{1}{\mu_m + \xi} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & \frac{b_{mh}S_h}{\mu_m + \xi} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{b_m\, h_m s_m}{\mu_m + \xi} \\ 0 & \frac{V_m}{\mu + d + \gamma_2} & 0 & 0 \end{bmatrix}$$

$$R_0 = \sqrt{\frac{V_m}{\mu+d+\gamma_2}} \quad (11)$$

B. Endemic Equilibrium

The Model system of equations (1) to (7) has a endemic equilibrium point

$$E_1 = (S_h^*, E_h^*, I_h^*, R_h^*, S_m^*, E_m^*, I_m^*)$$

$$S_h^* = \frac{\Lambda_1 + \mathrm{bm_h S_h I_m} + \mu \mathrm{S_h}}{\gamma_2(1-q)} \qquad E_h^* = \frac{\mathrm{bm_h S_h I_m}}{(\mu+k+\gamma_1)} \qquad I_h^* = \frac{\mathrm{k}+\mu+\mathrm{d}+\gamma_2}{\gamma_1(1-p)}$$

$$R_h^* = \frac{\mu+\gamma_1 \mathrm{p}}{\gamma_2 q} \qquad S_m^* = \frac{\Lambda_2 + \mu_m \mathrm{s_m} + \mathrm{b_{mh} S_h I_h}}{\gamma_2 q}$$

$$E_m^* = \frac{\mathrm{bm_h S_m I_h} + \mu_m \mathrm{E_m} + \mathrm{V_m E_m}}{\xi S_m} \qquad I_m^* = \frac{\mu_m \mathrm{I_m} + \mathrm{V_m E_m}}{\xi I_m}$$

$$\lambda^* = \frac{\beta(I_h^* + \theta + R_h^*)}{S_h^* + E_h^* + I_h^* + R_h^* + S_m^* + I_m^* + E_m^*}$$

We shall resolve the above endemic equilibrium points E_1 to the Centre Manifold theory and investigate their stability of E_1.

Lemma 2: The developed system (1) to (7) is uniformly persistent on Ψ.

Proof: Uniform persistent system of (1) to (7) implies that there exist a constant $\delta > 0$ such that any solution of (1) to (7) which starts in ψ^0, the interior of ψ., satisfies

$$\delta \le \lim_{t\to\infty} inf S_h(t), \qquad \delta \le \lim_{t\to\infty} inf E_h(t),$$
$$\delta \le \lim_{t\to\infty} inf I_h(t),$$
$$\delta \le \lim_{t\to\infty} inf R_h(t), \qquad \delta \le \lim_{t\to\infty} inf S_m(t),$$
$$\delta \le \lim_{t\to\infty} inf E_m(t),$$
$$\delta \le \lim_{t\to\infty} inf I_m(t),$$

To prove global stability of endemic equilibrium considers a Lyapunov function V as:

$$\begin{aligned} V = {} & \left(S_h(t) - S_h^*(t)\, lnS_h(t)\right) \\ & + \left(E_h(t) - E_h^*(t)\, lnE_h(t)\right) \\ & + \left(I_h(t) - I_h^*(t)\, lnI_h(t)\right) \\ & + \left(R_h(t) - R_h^*(t)\, lnR_h(t)\right) \\ & + \left(S_m(t) - S_m^*(t)\, lnS_m(t)\right) \\ & + \left(E_m(t) - E_m^*(t)\, lnE_m(t)\right) \\ & + \left(I_m(t) - I_m^*(t)\, lnI_m(t)\right) \end{aligned}$$

Which is continuous for all $S_h(t), E_h(t), I_h(t), R_h(t), S_m(t), E_m(t), I_m(t) > 0$ and satisfies

$$\frac{\partial V}{\partial S_h(t)} = \left(1 - \frac{S_h^*(t)}{S_h(t)}\right), \quad \frac{\partial V}{\partial E_h(t)} = \left(1 - \frac{E_h^*(t)}{E_h(t)}\right), \ \ldots\ldots\ldots \frac{\partial V}{\partial I_m(t)} = \left(1 - \frac{I_m^*(t)}{I_m(t)}\right).$$

(cf. Korobeinikov and Wake [11]) .Consequently, the endemic equilibrium E_1 is the only extremum and the global minimum of the function $V \in R_+^7$. Also

$$\big(S_h(t), E_h(t), I_h(t), R_h(t), S_m(t), E_m(t), I_m(t)\big) > 0$$

and

$$V\big(S_h(t), E_h(t), I_h(t), R_h(t), S_m(t), E_m(t), I_m(t)\big) = 0$$

only at E_1 .Thus $\big(S_h(t), E_h(t), I_h(t), R_h(t), S_m(t), E_m(t), I_m(t)\big)$ is a Lyapunov function. At equilibrium $\Lambda = \lambda^* S_h^*(t) + \mu S_h^*(t)$, substituting this into the time derivative of V along the solution path of the model system (1) to (7), we have

$$\begin{aligned}
&V' \\
&= \big(S_h(t) - S_h^*(t)\big)\frac{S_h(t)'}{S_h(t)} + \big(E_h(t) - E_h^*(t)\big)\frac{E_h(t)'}{E_h(t)} \\
&+ \big(I_h(t) - I_h^*(t)\big)\frac{I_h(t)'}{I_h(t)} + \big(R_h(t) - R_h^*(t)\big)\frac{R_h(t)'}{R_h(t)} \\
&+ \big(S_m(t) - S_m^*(t)\big)\frac{S_m(t)'}{S_m(t)} \\
&+ \big(E_m(t) - E_m^*(t)\big)\frac{E_m(t)'}{E_m(t)} + \big(I_m(t) - I_m^*(t)\big)\frac{I_m(t)'}{I_m(t)} \\
&\le -\mu\frac{\big(S_h(t) - S_h^*(t)\big)^2}{S_h(t)} \\
&+ c\big(S_h(t), E_h(t), I_h(t), R_h(t), S_m(t), E_m(t), I_m(t)\big)
\end{aligned}$$

Thus the function c, can be shown to be non-positive using lemma Barbalat[9]. Hence, $V' \le 0$ with inequality only at E_1. The only invariant set in ψ^0, is the set consisting endemic equilibrium E_1. Thus all solutions of model system (1) to (7) which intersect ψ^0, limit to an invariant set, the singleton $\{E_1\}$. Therefore, from the LaSalle invariance principle, model system (1) to (7) is uniformly persistent.

Theorem 1: The Zika virus free equilibrium state E_0 of the system (1) to (7) is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.

Theorem 2: The virus free equilibrium point E_0, is globally asymptotically stable if $R_0 < 1$ and unstable otherwise.

Proof: The equation of the infected components in system (1) to (7) can be written as

$$\begin{bmatrix} E_h' \\ E_m' \\ I_h' \\ I_m' \end{bmatrix} = [F - V]\begin{bmatrix} E_h \\ E_m \\ I_h \\ I_m \end{bmatrix} - \left[1 - \frac{S_h(t)}{N}\right]\begin{bmatrix} 0 & 0 & 0 & b_{mh}S_h \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & b_m\, h_m s_m \\ 0 & V_m & 0 & 0 \end{bmatrix}\begin{bmatrix} E_h \\ E_m \\ I_h \\ I_m \end{bmatrix}$$

Where F and V are defined in (9) and (10) for all $t \ge 0$ in τ. Thus we have

$$\begin{bmatrix} E_h' \\ E_m' \\ I_h' \\ I_m' \end{bmatrix} = [F - V]\begin{bmatrix} E_h \\ E_m \\ I_h \\ I_m \end{bmatrix} \qquad (12)$$

Since all Eigen values of the matrix $F - V$ have negative real parts, the linearized differential inequality (12) is stable whenever $R_0 < 1$. Consequently $[E_h, E_m, I_h, I_m] \to (0,0,0,0)\ as\ t \to \infty$. Thus following the comparison theorem, we have $[E_h, E_m, I_h, I_m] \to (0,0,0,0)$ & $S_h(t) \to \frac{\Lambda}{\mu}$ as $t \to \infty$. Hence the DFE (E_0) is globally asymptotically stable for $R_0 < 1$.

IV.CONCLUSION

In this paper, we have considered a mathematical model for transmission dynamics of zika virus by using indoor spraying. We derived the basic reproduction number for proposed system of coupled equations. Our work predicts that the Zika virus can be controlled by applying spraying and by reducing. It has been seen that if R_0<1, DFE is locally asymptotically stable and unstable if R_0>1. Lyapunov functional is constructed to assess the global stability of the present model.

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