



A Brief Introduction to Portfolio Optimization Using Genetic Algorithm

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ABSTRACT

A portfolio can be said as a group of financial assets such as stocks bonds and even cash and funds. Portfolio optimization refers to the allocation of the investment in such a way among assets so as to maximize the overall profit and minimize the risk. The problem is obtaining the risk and expected return for each of the individual assets, further computations involving how to divide the basic wholesome amount of investment into different assets so as the entire weight of the assets remain one is ensured. Portfolio Optimization problem is an important and hard optimization problem that, with the addition of necessary realistic constraints,becomes computationally intractable, in the area of economics and finance. Genetic Algorithm (GA) is an optimization technique which mimics the natural evolution that has the optimization features. GA has been increasingly used during the last decades to support complex decision-making in a number of fields, such as image processing, logistics and transportation, telecommunication networks, bioinformatics, finance, and many more. In recent years, much work has been done in finding optimum solution in solving portfolio problem with the use of GA. This paper gives a brief introduction about how to use Genetic Algorithm for solving portfolio optimization problem. This study focuses on optimization of Markowitz model using GA.

Keywords : Portfolio Optimization,Markowitz Model, Genetic Algorithm.

I. INTRODUCTION

Most real-life problems can be reduced to some kind of optimization. Thus, optimization is one of the most applicable areas of mathematics and computer science. The difficulty of an optimization problem depends on the type of the objective function that is optimized, constraints and decision

variables [1].Keeping in mind our today's fast complicated and technologically advanced life considering only a single optimization function is not enough, thus we need to work with multi objective functions. Multi-objective optimization is much more complicated than single-objective problems. The problem becomes even harder when some variables can take real, while other can take only

integer values. Such mixed continuous/discrete problems usually require problem specific search techniques in order to generate optimal or near-optimal solution [2]. Apart from this numerical optimization problems are of huge concern. Numerical optimization problems can be combinatorial (discrete, where variables can take only integer values) or continuous (global optimization), where continuous problems can be constrained or unconstrained (bound constrained) [3]. Portfolio optimization problem is a very important and widely researched problem in the areas of finance and economy and it belongs to the group of numerical optimization problems, with or without constants, with real and sometimes mixed variables [4]. Portfolio optimization problem is generally also known as portfolio selection problem. It is a well-known problem of the financial domain which is still getting a great deal of attention. Portfolio is a collection of financial instrument even known as assets owned by an individual or organization. Portfolio usually consists of stocks, bonds, futures, options, mutual funds, etc [5]. This problem actually belongs to the group of hard optimization technique and traditional, deterministic methods could not handle it successfully. Thus, it is better to adopt some metaheuristic algorithms to solve these problems even more efficiently [6]. The overall main motive of the portfolio optimization problem is to maximize the profit and minimize the risk. An attempt to obtain these criteria was initiated by Markowitz in the year 1952, he proposed a model which came to be known as the Markowitz mean variance model [7]. It was in fact the very first mathematical formulation to solve the portfolio optimization problem [8]. Despite of Markowitz formulations there are some reasons due to which we don't obtain good solutions and thus to obtain even more efficient and optimized results we apply various metaheuristic algorithm [9].

II. PORTFOLIO OPTIMIZATION

Portfolio is nothing but a collection of papers contained in a bag. But the portfolio which is of our

concern is related to finance. So as far as economics is considered, Portfolio is nothing but a collection of bonds or stocks or funds or assets. The main motive of this portfolio problem is to divide an investment among various assets. Each of those assets would be associated with some return and risk value. The allocation should be such that the profit obtained is maximized [10]. We cannot complete portfolio without mentioning Markowitz portfolio theory, he is known as the father of portfolio theory. He proposed the “mean-variance model”, where he has stated various assumptions and criteria’s. [11] But the basic motive is to allocate the investment such that return is maximized and risk is reduced. Even return and risk go side by side and if u expect to get maximum return the chances of risk also increases. Thus, the main aim of the portfolio problem can be stated by the following equation, which in our case is the working formula or objective function. [12]

$$\min \{ \lambda [\sum_{i=1}^N \sum_{j=1}^N w_i w_j \sigma_{ij}] + (1 - \lambda) [\sum_{i=1}^N w_i \mu_i] \} \quad (1)$$

$$\sum_{i=1}^N w_i E(r_i) = E(r_p^*) \quad (2)$$

$$\left[\sum_{i=1}^N w_i = 1 \right] \quad (3)$$

$$\varepsilon_i z_i \leq w_i \leq \delta_i \quad i = 1, \dots, N \quad (4)$$

$$0 \leq w_i \leq 1 \quad i=1, \dots, N \quad (5)$$

$$0 \leq \varepsilon_i \leq \delta_i \leq 1 \quad i = 1, \dots, N \quad (6)$$

Eq. (1): Minimizes risk

Eq. (2): Maximizing expected return

Eq. (3): The sum of asset weights is equal to

Eq. (4): Impose the lower and upper limit restrictions for weight of each asset

Eq. (5) and Eq. (6): Define variable domain

III. GENETIC ALGORITHM

Nature has always been a great source of inspiration to all mankind. GA is search based algorithms based on the concepts of natural selection and genetics. Genetic Algorithm is a subset of a much larger branch of computation known as Evolutionary Computation. Genetic Algorithm was developed by John Holland and his students and colleagues at the University of Michigan, most notably David E. Goldberg and has since been tried on various optimization problems with a high degree of success [13]. In GAs, we have a pool or a population of possible solutions to the given problem. These solutions then undergo recombination and mutation (like in natural genetics), producing new children, and the process is repeated over various generations. Each individual (or candidate solution) is assigned a fitness value (based on its objective function value) and the fitter individuals are given a higher chance to mate and yield more “fitter” individuals. This is in line with the Darwinian Theory of “Survival of the Fittest”. In this way we keep “evolving” better individuals or solutions over generations, till we reach a stopping criterion [14]. Genetic algorithms are one of the most important parts of a wider set of computer science instruments known as evolutionary computation. As the name suggests, such tools are constructed based on the Darwinian evolutionary principles observed in nature: the same ideas are applied and the same patterns are mimicked in order to improve and specialize the individuals according to the specific problem being faced. The field of application for evolutionary computation (and for all its subsets) is the one of search problems and optimization, with particular attention to the so-called black box problems and, in general, to all those cases in which the solver has no idea about how a solution would look like. More specifically, genetic algorithms belong to the subgroup of evolutionary algorithms. Those algorithms, genetic

ones included, can be defined as heuristic (or metaheuristic) optimization methods. With the term heuristic we denote a whole set of strategies devoted to problem solving, applied to cases in which a complete search in the entire space of feasible solutions would be impossible, or even just not efficient. This choice, of course, sacrifices completeness, so the risk is that of finding a resolution which is not optimal. On the other hand, the resulting process will turn out to be more rapid and the result will still be widely acceptable. Famous heuristic method examples are the well-known rule of thumb, trial and error, stereotyping and even common sense. Genetic algorithms are constructed applying biological evolutionary principles such as organism’s reproduction, natural selection, genes mutation and recombination and genetic inheritance. The first step is to randomly create an initial population from the set of possible values admitted as a solution. This first generation is tested by means of a fitness function, which must always be present in order to constantly determine the adequateness of the candidate solutions, somehow mimicking the way nature tests organisms to find out which ones are more suitable to survive. At this point, a second generation arises from the application of the above-mentioned genetic processes to the individual that will have their features mutated, cross-combined and bequeathed. The whole process is then iterated until a predetermined level of fitness is reached by an individual or until a given number of generations is created [15].

Fitness function: The population of candidate solutions, then, is plugged into such function in order to check their strength and to consequently perform the adequate evolutionary operations.

$$f=x^2 + y^2 \text{ where } 0 \leq x \leq 3, 0 \leq y \leq 3$$

(7)

Encoding: All the required elements, in order to be plugged into genetic algorithm, need to be encoded in binary or real values. Following is the example of binary encoding of Eq -7.

Chromosome A

0	1	1	0
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Chromosome B

0	0	1	1
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Fig-1:Encoded Chromosome

Selection: Based on some arbitrary principle, the individuals that are better than the initial problem will be rewarded with the possibility to breed a new generation, bequeathing their traits to the new born individuals; Tournament selection is tournament consist in randomly selecting two or more candidate solutions and comparing their goodness [16].

Crossover:The idea is to choose two or more individuals which will be manipulation and mixed in order to give birth to a child solution.Here is example of one-point crossover:

Before Crossover

Parent A

0	1	1	0
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Parent B

0	0	1	1
---	---	---	---

After Crossover

Child A

0	1	1	1
---	---	---	---

Child B

0	0	1	0
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Fig-2:Crossover operation between Chromosomes

Mutation: Inspired by biological mutation, this process is aimed at ensuring genetic diversity

throughout the generations of candidate solutions. [17]

Child A:

Before Mutation

0	1	1	0
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After mutation

1	1	1	0
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Fig-3:Mutation operation on Chromosome

Termination: Commonly, the algorithm terminates when either a maximum number of generations has been produced, or a satisfactory fitness level has been reached for the population [13]. If the algorithm has terminated due to a maximum number of generations, a satisfactory solution may or may not have been reached [14].

IV. GA BASED APPROACH FOR PORTFOLIO OPTIMIZATION

In our work, the methodology used is initially the collection of assets is done. From the selected asset pool, a fixed number of assets are chosen based on some specific criteria. Then the initial population is generated from the assets selected. The initial population forms the stepping stone for the calculation or overall optimization of the portfolio problem. On the entire population genetic algorithm is applied. Here the genetic algorithm serves as the optimization tool. The three basic steps of which the genetic algorithm is comprised of that is selection, crossover and mutation operations are applied consecutively. The fitness is calculated using the above-mentioned Eq 1. If the termination condition is not satisfied the process is repeated. After the process terminates, the chromosomes are taken into consideration and whichever chromosome gives the optimum percentage of asset allocation with maximum profit and minimum risk is considered to

be our final solution. Different steps of the algorithm are shown in the Fig 4.

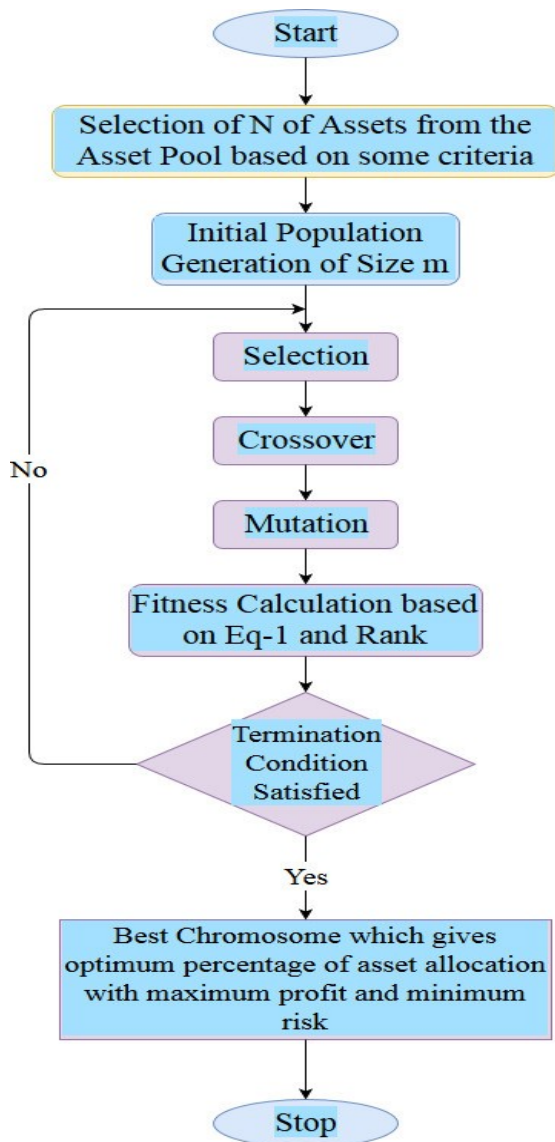


Fig-4:Flowchat for GA based Portfolio Optimization

V. CONCLUSION

As previously told this paper focuses only the over view of implementation of GA for optimizing portfolio. Hence other variation of this problem can be solvable using GA. Multi Objective approach can be adopted to handle more complex version of this problem.

VI. REFERENCES

- [1] F M Engels , Portfolio Optimization: Beyond Markowitz Master's Thesis by Marnix Engels ,January 13, 2004
- [2] CB KALAYCI , An Application Of Artificial Bee Colony Algorithm To Cardinality Constrained Portfolio Optimization Problem Department of Industrial Engineering, Pamukkale University, Turkey
- [3] E ElSeidy, A new particle swarm optimization based stock market prediction technique Department of Mathematics, Faculty of Science, Ain Shams University Cairo, Egypt
- [4] H Soleimani,H.R.G Mohammed and H Salimi, journal homepage: www.elsevier.com/locate/eswa
- [5] K Dahal, N Harnporanchai, Portfolio optimization using multiobjective genetic algorithm,
- [6] M Tuba and N Bacanin Arti?cial Bee Colony Algorithm Hybridized with Fire?y Algorithm for Cardinality Constrained Mean-Variance Portfolio Selection Problem
- [7] Markowitz, H.M. (March 1952). "Portfolio Selection". *The Journal of Finance*. 7 (1): 77–91. doi:10.2307/2975974. JSTOR2975974.
- [8] Markowitz, H.M. (1959). *Portfolio Selection: Efficient Diversification of Investments*. New York: John Wiley & Sons. (reprinted by Yale University Press, 1970, ISBN978-0-300-01372-6; 2nd ed. Basil Blackwell, 1991, ISBN978-1-55786-108-5)
- [9] Merton, Robert. September 1972. "An analytic derivation of the efficient portfolio frontier," *Journal of Financial and Quantitative Analysis* 7, 1851-1874
- [10] H. Soleimani, H.R.G Mohammed and H Salimi, Markowitz-based portfolio selection with minimum transaction lots, cardinality constraints and regarding sector capitalization using genetic algorithm in 2009 , Volume 36, Issue 3, Part 1, April 2009, Pages 5058-50.
- [11] T.J Chang, S.C Yang and K.J Chang, Portfolio optimization problems in different risk measures using genetic algorithm risk measures using genetic algorithm in 2009, Volume 36, Issue 7, September 2009, Pages 10529-10537.
- [12] Y. Crama and M Schyns, Simulated annealing for complex portfolio selection problems in 2003, Volume 150, Issue 3, 1 November 2003, Pages 546-571.

- [13] H. Zhu, Y Wang, K Wang and Y Chen, Particle Swarm Optimization (PSO) for the constrained portfolio optimization problem in 2011, Volume 38, Issue 8, August 2011, Pages 10161-10169.
- [14] H.R Golmakani and M Fazel, Constrained Portfolio Selection using Particle Swarm Optimization in 2011, Volume 38, Issue 7, July 2011, Pages 8327-8335.
- [15] R.T Zhang, W Chen and Y.M Cai, Particle Swarm Optimization for Constrained Portfolio Selection Problems in 2006, <http://ieeexplore.ieee.org/abstract/document/4028471/>
- [16] T.C.S Cheng and R Bai, A Combinatorial Algorithm for The Cardinality Constrained Portfolio Optimization Problem in 2014, <http://ieeexplore.ieee.org/abstract/document/6900357/>
- [17] N Becanin and M Tuba, Firefly Algorithm for Cardinality Constrained Mean-Variance Portfolio Optimization Problem with Entropy Diversity Constraint in 2014, <http://ww.w.naturalspublishing.com/files/published/8p86f17u4wsfu5.pdf>
- [18] T Cui, S Cheng and R Bai, A Combinatorial Algorithm for The Cardinality Constrained Portfolio Optimization Problem in 2014, <http://ieeexplore.ieee.org/abstract/document/6900357/>
- [19] N Becanin and M Tuba, Artificial Bee Colony Algorithm Hybridized with Firefly Algorithm for Cardinality Constrained Mean-Variance Portfolio Selection Problem in 2014, Volume 143, 2 November 2014, Pages 197-207
- [20] H. Kamili and M.E. Riff, Portfolio Optimization Using the Bat Algorithm, <https://link.springer.com/article/10.1007%2Fs10479-006-0145-1>
- [21] I Strumberger, N Bacanin and M Tuba, Constrained Portfolio Optimization by Hybridized Bat, <http://uksim.info/isms2016/CD/data/0665a083.pdf>