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Using Maple to Evaluate Two Types of Special Integrals

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ABSTRACT

This article uses the mathematical software Maple for the auxiliary tool to study two types of special integral problems. We can obtain the infinite series forms of the two types of integrals by using integration theory of complex variables functions. The research method adopted in this paper is to find solutions through manual calculations and verify our results using Maple. This method can not only let us find the calculation errors but also help us to revise the original thinking direction because we can verify the correctness of our ideas from the consistency of manual calculations and Maple calculations. Therefore, Maple can bring us inspiration and guide us to find the problem-solving method.

Keywords: Integral Problems, Complex Variables Functions, Infinite Series Forms, Maple.

I. INTRODUCTION

As is The computer algebra system (CAS) has been widely employed in mathematical and scientific studies. The rapid computations and the visually appealing graphical interface of the program render creative research possible. Maple possesses significance among mathematical calculation systems and can be considered a leading tool in the CAS field. The superiority of Maple lies in its simple instructions and ease of use, which enable beginners to learn the operating techniques in a short period. In addition, through the numerical and symbolic computations performed by Maple, the logic of thinking can be converted into a series of instructions. The computation results of Maple can be used to modify our previous thinking directions, thereby forming direct and constructive feedback that can aid in improving understanding of problems and cultivating research interests. To get a better understanding of Maple can use the online system of Maple, or browsing Maple's website www.maplesoft.com. On the other hand, for the books on Maple can refer to [1-3].

There are many methods appeared in calculus and engineering mathematics courses to solve the integral problems, which include change of variables method, integration by parts method, partial fractions method, trigonometric substitution method, etc. In addition, Adams et al. [4], Nyblom [5], and Oster [6] provided some other methods to solve the integral problems. In addition, Yu [7-33], Yu and Chen [34], and Yu and Sheu [35-37] used some techniques, for example, complex power series, integration term by term theorem, area mean value theorem, and generalized Cauchy integral formula to solve some types of integrals. In this paper, we consider the following two types of special integral problems which are not easy to obtain their answers using the methods mentioned above.

$$\int \frac{\sin\left[(n+1)\theta - m\tan^{-1}\left(\frac{\theta}{\ln r}\right)\right]}{[(\ln r)^2+\theta^2]^{m/2}}d\theta, \qquad (1)$$

and

$$\int \frac{\cos\left[(n+1)\theta - m\tan^{-1}\left(\frac{\theta}{\ln r}\right)\right]}{[(\ln r)^2+\theta^2]^{m/2}}d\theta, \qquad (2)$$

where r,θ are real numbers, $r>1$, and m,n are positive integers. The infinite series forms of the two types of integrals can be determined by using integration theory of complex variables functions; these are the major results in this paper (i.e., Theorems

1 and 2). On the other hand, two examples are provided to demonstrate the manual calculations, and we verify the results using Maple. This method can not only let us find the calculation errors but also help us to revise the original thinking direction because we can verify the correctness of our ideas from the consistency of manual calculations and Maple calculations.

II. METHODS AND MATERIAL

First, we introduce some definitions and formulas used in this paper.

Definitions:

The complex logarithmic function $\ln z$ is defined by $\ln z = \ln|z| + i\phi$, where z is a complex number, ϕ is a real number, $z = |z| \cdot e^{i\phi}$, and $-\pi < \phi \le \pi$.

Formulas:

1) Euler's formula:
$e^{i\theta} = \cos\theta + i\sin\theta$, where θ is any real number.

2) DeMoivre's formula :
$(\cos\theta + i\sin\theta)^p = \cos p\theta + i\sin p\theta$, where p is any integer, and θ is any real number.

To obtain the major results, a lemma is needed which is the complex integral formula used in this study.

Lemma *Suppose that z is a complex number, $z \ne 0,1$, and m,n are positive integers, then*

$$\int \frac{z^n}{(\ln z)^m} dz$$

$$= \sum_{\substack{k=0 \\ k \ne m-1}}^{\infty} \frac{(n+1)^k}{k!(k-m+1)} (\ln z)^{k-m+1}$$

$$+ \frac{(n+1)^{m-1}}{(m-1)!} \ln(\ln z) + C_1, \quad (3)$$

where C_1 is a constant.

Proof $\int \frac{z^n}{(\ln z)^m} dz$

$$= \int \frac{e^{nw}}{w^m} e^w dw \quad (\text{let } z = e^w)$$

$$= \int w^{-m} e^{(n+1)w} dw \circ$$

$$= \int w^{-m} \sum_{k=0}^{\infty} \frac{1}{k!}(n+1)^k w^k dw$$

$$= \int \sum_{k=0}^{\infty} \frac{1}{k!}(n+1)^k w^{k-m} dw$$

$$= \sum_{\substack{k=0 \\ k \ne m-1}}^{\infty} \frac{1}{k!}(n+1)^k \frac{1}{k-m+1} w^{k-m+1}$$

$$+ \frac{1}{(m-1)!}(n+1)^{m-1} \ln w + C_1$$

$$= \sum_{\substack{k=0 \\ k \ne m-1}}^{\infty} \frac{(n+1)^k}{k!(k-m+1)} (\ln z)^{k-m+1}$$

$$+ \frac{(n+1)^{m-1}}{(m-1)!} \ln(\ln z) + C_1. \qquad \text{q.e.d.}$$

III. RESULTS AND DISCUSSION

Main Results

Next, we use Lemma to determine the infinite series forms of the integrals (1) and (2).

Theorem 1 *Let r,θ be real numbers, $r > 1$, m,n are positive integers, and C is a constant, then*

$$\int \frac{\sin\left[(n+1)\theta - m\tan^{-1}\left(\frac{\theta}{\ln r}\right)\right]}{[(\ln r)^2 + \theta^2]^{m/2}} d\theta$$

$$= \frac{-1}{r^{n+1}} \sum_{\substack{k=0 \\ k \ne m-1}}^{\infty} \left[\begin{array}{c} \frac{(n+1)^k [(\ln r)^2 + \theta^2]^{(k-m+1)/2}}{k!(k-m+1)} \\ \times \cos\left[(k-m+1)\tan^{-1}\left(\frac{\theta}{\ln r}\right)\right] \end{array} \right]$$

$$- \frac{1}{r^{n+1}} \frac{(n+1)^{m-1}}{(m-1)!} \cdot \frac{1}{2} \ln[(\ln r)^2 + \theta^2] + C. \quad (4)$$

Proof In Eq. (3), let $z = re^{i\theta}$, then

$$\int \frac{(re^{i\theta})^n}{(\ln r + i\theta)^m} ire^{i\theta} d\theta$$

$$= \sum_{\substack{k=0 \\ k \ne m-1}}^{\infty} \frac{(n+1)^k}{k!(k-m+1)} (\ln r + i\theta)^{k-m+1}$$

$$+ \frac{(n+1)^{m-1}}{(m-1)!} \ln(\ln r + i\theta) + C_1.$$

Using Euler's formula, DeMoivre's formula Euler, and Definitions yields

$$r^{n+1}\int \frac{\exp\left[i\left((n+1)\theta+\pi/2-m\tan^{-1}\left(\frac{\theta}{\ln r}\right)\right)\right]}{[(\ln r)^2+\theta^2]^{m/2}}d\theta$$

$$=\sum_{\substack{k=0\\k\neq m-1}}^{\infty}\frac{(n+1)^k}{k!(k-m+1)}\left[\begin{array}{l}[(\ln r)^2+\theta^2]^{(k-m+1)/2}\\ \times\exp\left[i(k-m+1)\tan^{-1}\left(\frac{\theta}{\ln r}\right)\right]\end{array}\right]$$

$$+\frac{(n+1)^{m-1}}{(m-1)!}\left[\frac{1}{2}\ln[(\ln r)^2+\theta^2]+i\tan^{-1}\left(\frac{\theta}{\ln r}\right)\right]+C_1. \tag{5}$$

By the equality of real parts of both sides of Eq.(5), the desired result holds.

q.e.d.

Theorem 2 *If the assumptions are the same as Theorem1,then*

$$\int \frac{\cos\left[(n+1)\theta-m\tan^{-1}\left(\frac{\theta}{\ln r}\right)\right]}{[(\ln r)^2+\theta^2]^{m/2}}d\theta$$

$$=\frac{1}{r^{n+1}}\sum_{\substack{k=0\\k\neq m-1}}^{\infty}\left[\begin{array}{l}\frac{(n+1)^k[(\ln r)^2+\theta^2]^{(k-m+1)/2}}{k!(k-m+1)}\\ \times\sin\left[(k-m+1)\tan^{-1}\left(\frac{\theta}{\ln r}\right)\right]\end{array}\right]$$

$$+\frac{(n+1)^{m-1}}{r^{n+1}(m-1)!}\tan^{-1}\left(\frac{\theta}{\ln r}\right)+C. \tag{6}$$

Proof Using the equality of imaginary parts of both sides of Eq.(5) yields the desired result holds.

q.e.d.

Examples

For the special integrals discussed in this article, we provide two examples and use Theorems 1 and 2 to obtain their infinite series forms. Moreover, Maple is used to calculate the approximations of some definite integrals and their infinite series forms to verify our answers.

Example 1 Let $r=5, m=7, n=9$ in Theorem 1, then by Eq. (4), we have

$$\int \frac{\sin\left[10\theta-7\tan^{-1}\left(\frac{\theta}{\ln 5}\right)\right]}{[(\ln 5)^2+\theta^2]^{7/2}}d\theta$$

$$=\frac{-1}{5^{10}}\sum_{k=0}^{k=5}\left[\begin{array}{l}\frac{10^k[(\ln 5)^2+\theta^2]^{(k-6)/2}}{k!(k-6)}\\ \times\cos\left[(k-6)\tan^{-1}\left(\frac{\theta}{\ln 5}\right)\right]\end{array}\right]$$

$$-\frac{1}{5^{10}}\sum_{k=7}^{\infty}\left[\begin{array}{l}\frac{10^k[(\ln 5)^2+\theta^2]^{(k-6)/2}}{k!(k-6)}\\ \times\cos\left[(k-6)\tan^{-1}\left(\frac{\theta}{\ln 5}\right)\right]\end{array}\right]$$

$$-\frac{10^6}{5^{10}6!}\cdot\frac{1}{2}\ln[(\ln 5)^2+\theta^2]+C. \tag{7}$$

Therefore, the definite integral

$$\int_0^{\pi} \frac{\sin\left[10\theta-7\tan^{-1}\left(\frac{\theta}{\ln 5}\right)\right]}{[(\ln 5)^2+\theta^2]^{7/2}}d\theta$$

$$=-\frac{1}{5^{10}}\sum_{k=0}^{k=5}\left[\begin{array}{l}\frac{10^k[(\ln 5)^2+\pi^2]^{(k-6)/2}}{k!(k-6)}\\ \times\cos\left[(k-6)\tan^{-1}\left(\frac{\pi}{\ln 5}\right)\right]\end{array}\right]$$

$$-\frac{1}{5^{10}}\sum_{k=7}^{\infty}\left[\begin{array}{l}\frac{10^k[(\ln 5)^2+\pi^2]^{(k-6)/2}}{k!(k-6)}\\ \times\cos\left[(k-6)\tan^{-1}\left(\frac{\pi}{\ln 5}\right)\right]\end{array}\right]$$

$$-\frac{10^6}{5^{10}6!}\cdot\frac{1}{2}\ln[(\ln 5)^2+\pi^2]+\frac{10^6}{5^{10}6!}\cdot\ln(\ln 5)$$

$$+\frac{1}{5^{10}}\sum_{k=0}^{k=5}\frac{10^k(\ln 5)^{k-6}}{k!(k-6)}+\frac{1}{5^{10}}\sum_{k=7}^{\infty}\frac{10^k(\ln 5)^{k-6}}{k!(k-6)}. \tag{8}$$

We employ Maple to verify the correctness of Eq. (8) below.

```
>evalf(int(sin(10*theta-
7*arctan(theta/ln(5)))/((ln(5))^2+(theta)^2)^(7/2),theta
=0..Pi),23);
```

0.0071993620615605001108 42

```
>evalf(-1/5^10*sum(10^k*((ln(5))^2+Pi^2)^((k-
6)/2)/(k!*(k-6))*cos((k-6)*arctan(Pi/ln(5))),k=0..5) -
1/5^10*sum(
10^k*((ln(5))^2+Pi^2)^((k-6)/2)/(k!*(k-6))*cos((k-
6)*arctan(Pi/ln(5))),k=7..infinity)-
10^6/(5^10*6!)*1/2*ln((ln(5))^2+Pi^2)+10^6/(5^10*6
!)*ln(ln(5))+1/5^10*sum(10^k*(ln(5))^(k-6)/(k!*(k-
6)),k=0..5)+1/5^10*sum(10^k*(ln(5))^(k-6)/(k!*(k-
```

6)),k=7..infinity),23);

0.0071993620615605001108 43

Example 2 In Theorem 2, if $r=2, m=4, n=3$, then by Eq. (6), we obtain

$$\int \frac{\cos\left[4\theta - 4\tan^{-1}\left(\frac{\theta}{\ln 2}\right)\right]}{[(\ln 2)^2+\theta^2]^2} d\theta$$

$$= \frac{1}{16}\sum_{k=0}^{2}\left[\begin{array}{l}\frac{4^k[(\ln 2)^2+\theta^2]^{(k-3)/2}}{k!(k-3)} \\ \times\sin\left[(k-3)\tan^{-1}\left(\frac{\theta}{\ln 2}\right)\right]\end{array}\right]$$

$$+\frac{1}{16}\sum_{k=4}^{\infty}\left[\begin{array}{l}\frac{4^k[(\ln 2)^2+\theta^2]^{(k-3)/2}}{k!(k-3)} \\ \times\sin\left[(k-3)\tan^{-1}\left(\frac{\theta}{\ln 2}\right)\right]\end{array}\right]+\frac{2}{3}\tan^{-1}\left(\frac{\theta}{\ln 2}\right)+C. \tag{9}$$

Thus, we obtain the following definite integral

$$\int_0^{\pi/3} \frac{\cos\left[4\theta - 4\tan^{-1}\left(\frac{\theta}{\ln 2}\right)\right]}{[(\ln 2)^2+\theta^2]^2} d\theta$$

$$= \frac{1}{16}\sum_{k=0}^{2}\left[\begin{array}{l}\frac{4^k[(\ln 2)^2+\pi^2/9]^{(k-3)/2}}{k!(k-3)} \\ \times\sin\left[(k-3)\tan^{-1}\left(\frac{\pi}{3\ln 2}\right)\right]\end{array}\right]$$

$$+\frac{1}{16}\sum_{k=4}^{\infty}\left[\begin{array}{l}\frac{4^k[(\ln 2)^2+\pi^2/9]^{(k-3)/2}}{k!(k-3)} \\ \times\sin\left[(k-3)\tan^{-1}\left(\frac{\pi}{3\ln 2}\right)\right]\end{array}\right]+\frac{2}{3}\tan^{-1}\left(\frac{\pi}{3\ln 2}\right). \tag{10}$$

Also, we use Maple to verify the correctness of Eq. (10).

```
>evalf(int(cos(4*theta-4*arctan(theta/ln(2)))/((ln(2))^2+(theta)^2)^2,theta=0..Pi/3),23);
```

2.0418366947648703186136

```
>evalf(1/16*sum(4^k*((ln(2))^2+Pi^2/9)^((k-3)/2)/(k!*(k-3))*sin((k-3)*arctan(Pi/(3*ln(2)))),k=0..2)+1/16*sum(4^k*((ln(2))^2+Pi^2/9)^((k-3)/2)/(k!*(k-3))*sin((k-3)*arctan(Pi/(3*ln(2)))),k=4..infinity)+2/3*arctan(Pi/(3*ln(2))),23);
```

2.0418366947648703186136

IV.CONCLUSION

From the discussion above, we know that the integration theory of complex variables functions is the main technique to solve two types of special integral problems. In fact, the application of this method is extensive, and can be used to easily solve many difficult problems; we endeavor to conduct further studies on related applications. On the other hand, Maple also plays a vital assistive role in problem-solving. In the future, we will extend the research topic to other calculus and engineering mathematics problems and solve these problems using Maple.

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