International Journal of Scientific Research in Computer Science, Engineering and Information Technology
© 2017 IJSRCSEIT | Volume 2 | Issue 1 | ISSN : 2456-3307

Mathematical Models in Mechanical and Biomedical Tribology with Computational Simulations/Optimization Methods

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ABSTRACT

Surface damage, wear, corrosion, and erosion-corrosion in bioengineering artificial implants, interior, exterior, or partlially-interior/exterior biomedical devices causes significant operational bioengineering/biomechanical difficulties—the same phenomena that occurs classically in a large number of mechanical systems/machinery. Additionally, this kind of deterioration could also involve prostheses, temporary prostheses or orthopaedic supplies, surgical permanent devices, and even surgery theatre tools, causing a series of important associated functional difficulties. This usually happens during surgery and the post-operation stage. The consequences of this industrial-biomedical design complexity are extent, from re-operation, failure of medical devices, or post-surgical discomfort/pain to complete malfunction of the device or prostheses. In addition to all these hurdles, there are economic loss and waste of operation-surgical time, re-operations and manoeuvres carried out in modifications or repair. The wear is caused mainly by solid surfaces in contact, with important participation of the lubrication physiological/artificial conditions. Corrosion of protective coatings also constitute a number of significant mechanical and bioengineering difficulties. Mathematical modelling through optimization methods, initially mostly developed for industrial mechanical systems, overcome these engineering/bioengineering complications/difficulties, and reduce the experimental/tribotesting period in the rather expensive manufacturing process. In this contribution we provide a brief review of the current classified wear, erosion and/or corrosion mathematical models in developed for general mechanics, and based on our recent modelling international publications in tribology, as a introductory research. Subsequently the aim focus on specific tribology for biomedical applications and, additionally, a brief of optimization methods for precise modelling of given appliances with computational series, programming presentation, and numerical-software practical recipes. Results comprise an initial review of tribological/wear/erosion/corrosion models with further simulations, computational nonlinear optimization programming and graphical data/examples both in mechanical and biomechanical engineering. 3D computational imaging series are sharply shown with extent explanations.

Keywords: Tribology, Wear, Biomedical Devices, Biomedical Implants, Erosion, Corrosion, Erosion-Corrosion, Mathematical Modelling, Nonlinear Optimization, Tribotest

I. INTRODUCTION

Although the common technical concept of wear, erosion, and corrosion is usually related to materials in mechanical engineering/physics, these physical-chemical phenomena are widely extended both in nature and artificial world—Table I sets definitions a number of types with examples. In the nature, the earth planet surface and several of its fundamental structures have been configured by erosion and corrosion, that is, interaction among natural components/phenomena with subsequent erosion and corrosion, during millions of years. In the artificial world, erosion and corrosion are not only linked to specific machinery materials engineering. They are found also, for example, related to textile-design manufacturing industry, mixed natural-artificial human implants, wear of human physical body interacting with machines or age increase in Health Sciences, special aerospace engineering design, or extensive branches of mechanical tools, footwear, jewelry, or defence industry -in other words, artificial material-material interaction(s). Pure natural and biological/medical wear, erosion and corrosion belongs to any kind of compounds of earth materials that change in structure or superficial constitution along the decades/centuries, i. e., wind particles with rocks, humid air with stones, or plants whose roots create chemical corrosion in rocks before the subsequent erosion of them commences . Between both groups, (Tables 1,4), we find a mixed type of natural-artificial erosion-corrosion phenomena, such as wear of buildings structures caused by natural compounds, automotive erosion-corrosion with natural wind impact, [1,2, 3,4], or changes of air chemical composition in the environment, etc. Biomedical Tribology and Tribocorrosion constitutes a mixed up branch whose specialization shares parts of every group defined previously. In other words, Biomedical Tribology involves artificial wear of medical implants and devices but also natural biochemical corrosion or wear of any medical device which is set into the human biomechanical system. This fact implies that there are special and difficult mathematical, physical, and chemical/biochemical conditions when the mechanical device of interest is biomedical and is surrounded by human/animal tissues.

Therefore, according to all these conditions/constraints, it is straightforward to guess and estimate the importance of the study/research of wear, erosion, and corrosion in technology and science as industrial-material, biomedical tribology essentials and environmental-geophysical factors. For built-up mechanized purposes, pure mechanical or biomedical, given the economic loss caused by erosion and corrosion in an extensive range of engineering/technology areas, the selection of materials became a must. As a result, a large number of technical approaches to deal this question have been put in practice, mainly since the beginning of the industrial era.

"Trial and error" methods, that is, the Forward Problem technique, was found expensive, imprecise and time consuming [5]. In consequence, applications of the Inverse Problem methods were used to determine, *a posteriori*, the validation/refinement of theoretical mathematical models previously approximated [6,7,8,6,9]. In doing so, the modelling optimization time arose, in order to carry out an initial mathematical approximation for a subsequent experimental choice of the most convenient materials [5,10]. Since the optimization task has become a routinary/compulsory task at daily research routine, and not necessarily all the investigators got used to work with optimization programming and tools, graphical optimization, among several optional-practical methods arose in recent years—for instance, see section with images focused on graphical optimization with Freemat and Matlab, or F#.

Table I

CLASSIFICATION OF EROSION AND CORROSION FOR ORIGIN/CAUSE [50]	
TYPE	**EXAMPLES**
Natural	Geophysical earth changes, rocks corrosion-erosion, human body wear for ageing and biomechanical movement
Artificial	Coatings damage with particles in gas/vapor or gas/vapor, wear in machinery parts, corrosion of coatings after erosion
Natural - Artificial	Degradation of concrete caused by natural impact, metal corrosion for natural air humidity. In biotribology this kind of mixed wear, erosion and corrosion is frequent in medical implants whose physical contacts are both natural and artificial, e. g., hip arthroplasty implants.

In terms of general mechanics/machinery/devices, material coatings erosion,corrosion,deformation and stress cracks are considered an industrial hurdle that creates loss of budget, energy, reparation-time, and operating time. Material substrate, although important also and chemically/physically linked to these processes, does not constitute the primary problem. Statistically, a rate higher than 90%, of mechanical-machine failures are linked to fatigue,friction,and wear. Succintly, according to [11], the aggressive environments that cause degradation in general are, wear, corrosion, oxidation, temperature, gas-particle size/velocity [12], and any combination of these factors. In biomedical tribology the degradation is more specific, chemical factors take a fundamental role, and biomechanical forces that cause wear are also essential for durability of artificial implants. Hence, the practical objective to find out engineering/bioengineering solutions is to use new/improved optimal materials for the technical design, in such a way according to precision of durability and functional operation of the mechanical system/device or group of any kind of apparatus/prostheses. Actually there is a number of mathematical models for tribilogy, biotribology, wear, erosion, corrosion, and combined erosion-corrosion or tribocorrosion. The objective of these modelling algorithms is to design accurate theoretical optimization models for initial search of optimal material characteristics, before passing on to the type of material testing/tribotesting with (approximated) those previous parameters- given as a solution of the theoretical model. In such a way, that mainly the coatings of the device, could be improved in durability, tribology/biotribology capabilities, and erosion-corrosion resistance.

Engineering solutions, as said, for these problems that cause economic loss, together with a waste of, e.g., functioning time and expensive reparations in mechanical structures, power plants, biomedical and mechanical apparatus/equipment are based on precision-design of both coating materials resistant to abrasion-erosion, and/or friction [1,3], and mechanical optimization of the operational structure of the device/mechanical system/mechanical-chain-group–in fact, temperature of components, e.g. hardmetal or cermets, constitutes also an important factor-and stress of materials also. Since materials testing apparatus have became more sophisticated and at the same time more accurate, the testing-process economical cost, therefore, has increased in recent times –we refer to them as the so-called tribotest in general [16]. Tribotests could be based on almost realistic simulations for all the components of the mechanical system, some of them, or a reduced number of them [16] –simplified-tests or single-component tests. As a result of the optimal variable-magnitude determinations with the mathematical model, it is imperative to link this objective data to perform, subsequently, experimental testing at lab. Then figure out a definite evaluation, in order to choose the optimal material usually for coatings or other structures [1,3]. Tribotests for biomedical wear and corrosion involve different and more uncommon/sophisticated conditions since the human physiology and biomechanics comprise different and rather more complicated parameters in several circumstances compared to classical mechanical systems [ref].

This contribution deals with an up-to-date modeling-presentation of tribology/biotribology wear,erosion, corrosion, and erosion-corrosion mathematical models, both from an objective and critical point of view. Complementary, in this article, we explained basic/functional nonlinear/linear optimization techniques to make an optimal choice of erosion and corrosion models, in order to minimize materials/machinery/device damage. The results and conclusions comprise a group of modern series of data, applicable in materials selection optimization, both for further research, and engineering design in the energy field. In general is a continuation of previous modeling contributions but complemented and developed towards a biomedical and biotribology scope [ref].
The simulations that are presented comprise both mechanical sytems modeling for tribology and biomedical modeling also. Optimization algorithms and computational examples are also shown with detailed and sharp-learning explanations. A group of highlights and important key points following all the article development from theory to computational practice are gathered at final sections to summarize the results of this contribution.

II. CLASSIFICATION OF MECHANICAL MATHEMATICAL MODELS FOR EROSION AND CORROSION

2.1 General Classification

Erosion and corrosion concepts imply the interaction between/among physical structures that could be in any physical state, namely, solid, have been developed several classifications for erosion and corrosion mathematical models.

However, at present and for future research, we do not try to emulate the already published classifications [17,10]. Instead, it is liquid, gas, metastates, or varieties of them. The interaction complexity is rather high, (Table 2). In the literature [17,10], there possible to simplify the classification(s) on the basis that, given the rather large number of models, it is guessed that the extensive complexity of E/C causes the necessity to design particular models almost for every type of interaction. In other words, the lack of existence of widely-applicable general models for E/C, constitutes the main reason for such kind of mathematical models variety. A brief of conditional factors is included in Table 2.

It was intended to set a common classification frame both for erosion and corrosion, in terms of simplification and fast practical use/selection of models in each particular materials choice (Tables 3, -4) – proposal of authors to be improved in further research. The predominant criterion of the classification is the practical engineering selection, that is, *for what is used every model*, and its advantages and limitations.The frame of classification is just the same for erosion and corrosion. Erosion-corrosion models can be included at anyone.

TABLE II

MATERIALS INTERACTION CONDITIONS FOR MECHANICAL TRIBOLOGY AND BIOMEDICAL TRIBOLOGY [Improved from [50]]

Conditional Factor	**Variables/Parameters**
State	solid (cristallographyc variety),liquid,gas,metaestates
Physical Magnitude	particles velocity,kinetic energy,materials particle temperature
Geometry	rather difficult in most cases,particle impact angle(s),interaction angle(s), interaction surface(s)
Material Compositio n	chemical,molecular,nano-quantum composition
Material Structure	physical-chemical and nanomaterial complexity
Material Origin	natural (unpredictable), artificial
Environmen t	temperature, humidity, thermical insulation, adiabatic and/or isothermical conditions
Residual Stress and Fatigue	influence in erosion and corrosion rates and surface cracks
Mutual Interaction	any possible interaction among/between all the former factors
Biomechani cal Conditions	For Biotribology and biomedical devices very important rather essential in engineering design
Physiologic al-chemical-composition of plasme, blood flow, and surrounding tissue composition	Very important for the tribocorrosion conditions and durability of the implant. Acid and base ions of plasma and surrounding chemical pH parameter constitutes a corrosion factor for metal/composites/plastic surfaces
Associated diseases in the patient subject of biomechanic al implant	Any concomitant disease of the patient that is subject of biomedical device implant surgery is a factor interacting with the implant materials and biomechanics, e.g., ostheoporosis, diabetes, clots, metasthasis, tumoral physical growth/pressure, etc

MATERIALS INTERACTION CONDITIONS FOR MECHANICAL TRIBOLOGY AND BIOMEDICAL TRIBOLOGY [Improved from [50]]
Conditional Factor | Variables/Parameters

In this line, according to the variety of physical states of the materials performing E/C at any kind of interaction, whether wear, corrosion or erosion, and their applications, it is defined,

Type 1 (T1) Mathematical E/C Models.-Those ones that can be implemented for several applications/material-interactions.Degree of usage is from 1 (lowest application range) -4 (highest application range).

Type 2 (T2) Mathematical E/C Models.-Those ones that can be implemented, and are designed/optimized for a specific or super-specific physical application. Degree of usage 1.

2.2 Mathematical Methods/Modelling Techniques for E/C

It is up to the researcher to include them in classification, or take the methods as a reference to characterize any Type 1 or Type 2 model.The criterion actually is the inclusion within the classification to clarify any model analysis precisely. In Table 4 a brief of the models presented in this paper is gathered with advantages, degree of usage, classification, and specific parameters for each one.

TABLE III

TRIBOLOGY AND BIOTRIBOLOGY MATHEMATICAL MODELS CLASSIFICATION WITH DETAILS [50]		
Group/Brand	**Model Type**	**Definition/Examples**
TYPE 1 (T1)	Models with several applications	Models for several E/C interactions in different conditions
TYPE 2 (T2)	Specific, and superspecific models with one application	Precise or extremely-accurate design for a unique materials physical interaction
Mathematical Methods	Mathematical And Optimization Techniques applicable to characterize Type 1 and Type 2, linked to any model	Heuristic (H) Empirical (E) Random (Monte Carlo) (R) Deterministic (D) Mixed (M) Finite Element (FE) Dynamic Model (DM) Others (O) Degree of Usage (1-4)

It is convenient/obliged to discuss a few concepts about the extensively used methods for E/C, usually characterized as heuristic and/or empirical. Additionally, to remark the essentials/significance of Finite Elements Method, which is a formal mathematical theory instead a simple method. Given the complexity of E/C, all models can be considered heuristic. Heuristic means, *grosso modo*, an approximate solution for a problem, non-perfect but functional in practice. The engineering heuristic method [18,19] comprises a pre-evaluation, an evaluation, the discussion of the evaluation, and finally the usability discussion.

E/C models are considered heuristic in our criteria. Empirical, [20], means knowledge based on whether experience, evidences, facts, experimental, or whether combinations of these factors. Formally, empirism asserts that the knowledge comes from perceptual representation systems and perceptual states [20].

In E/C modeling, the empirical ones are not necessarily bad/limited, and can be considered in some cases as the initial stage for a more theoretical model, e. g., the classical Finnie model [4,21].

Besides, it is extensively denominated in the literature FEM as a modelling/exclusive modelling method, and we respectfully differ from this interpretation. FE is a mathematical theory, [17,10], that can be widely applied from numerical methods, differential/partial-differential equations, physical applications of differential equations, (e. g., Boltzmann diffusion equation in radiotherapy), to mechanical systems, E/C modeling, and many others.

Therefore, in Classification of Table 3, FEM is used as a reference to develop/characterize/improve an equational model, but not as a model itself.

III. MATHEMATICAL FORMULATION OF EROSION MODELS WITH ADVANTAGES AND LIMITATIONS

In This section deals with a bibliographic description of E/C models, setting advantages, inconvenients, and prospective considerations. However, the citations/mentions are brief, and more extensive mathematical development is oncoming in next contributions.

3.1 Finnie Model (T1) This simple model, [21], was one of the first model invented for quantification of eroded material magnitude. This model (T1, ductile materials) is a cutting model and sets a rigid-plane surface. Finnie model is the base for further developments of other models, and remains today as a formal reference. Its basic formulation reads,

$$W = c \times \frac{M V^2}{\psi p K} \times \quad f(\alpha); \quad \text{with,} \qquad (1)$$

$$f(\alpha) = \sin(2\alpha) - \frac{6}{K}\sin^2(\alpha), \quad \alpha \le \operatorname{ar}\tan(\frac{K}{6});$$

$$f(\alpha) = \frac{K\cos^2(\alpha)}{6} \quad , \quad \alpha \rangle \operatorname{ar}\tan(\frac{K}{6});$$

where,

K= geometrical ratio between vertical to horizontal forces,V, particle speed, p, material flow stress, W, material volume remove, c is a correction factor for impact failure/mutual-particle-impact. Ψ is the ratio of depths, contact to cut. Note the factor MV^2 that corresponds to a kinetic energy magnitud inserted implicitlly within the formula.

3.2 Bitter Model (T1) This model sums erosion for plastic deformation (W_d) and cutting erosion (W_c). Its formulation derives from Finnie T1. Main equations for both removals are,

$$\text{Deformation Wear Erosion,}$$
$$W_D = \frac{M\left[V\sin(\alpha) - V_{el}\right]^2}{2\varepsilon_b} \text{ for } V\sin(\alpha) \rangle V_{el} \text{ and}$$
$$\text{null if } \langle\ V_{el};$$
$$\text{and subsequent, cutting wear erosion,}$$
$$W_C = \frac{2M\,C'\left[V\sin(\alpha) - V_{el}\right]^2}{2\varepsilon_b} \times$$
$$2MC'\left[V\cos(\alpha) - \frac{C' \times \left[V\sin(\alpha) - V_{el}\right]^2}{\sqrt{V\sin(\alpha)}}\ \phi_C\right]$$
$$\text{if } \ \alpha \le \alpha_0 \quad \text{or}$$
$$M\left[V\cos(\alpha) - \frac{M\left[V^2\cos^2(\alpha) - K_1\left[V\sin(\alpha) - V_{el}\right]^{3/2}\right]}{2\phi_C}\right]$$
$$\text{for } \alpha \rangle \alpha_0 \quad ; \tag{2}$$

This Bitter Model has many parameters, detailed in [18,21], and the most important ones that are in (1), namely, Alpha is the attack angle, ε_b is the deformation wear factor (obtained experimentally), and V_{el} is the threshold velocity (velocity at collision at which the elastic limit of the workpiece material is just reached). V_{el} can be calculated from the Hertzian contact theory. V_{el} depends on several factors, and some approximations were carried out. Parameter Φc is a material dependant wear factor obtained experimentally and C', K1 are constants [21].

3.3 Bitter Model Simplified (Neilson and Gilchrist's Model, T1) Neilson and Gilchrist's, simplified the Bitter model combined to express a ductile erosion model and using this Bitter model for brittle erosion, as follows,

$$W = \frac{MV^2\cos^2(\alpha)^2}{2\phi_C} + \frac{M\ \left[V\sin(\alpha) - V_{el}\right]^2}{2\varepsilon_b};$$
$$\alpha \ge \alpha_0 \quad ; \text{and,} \tag{3}$$
$$W = \frac{MV^2\cos^2(\alpha)^2\sin(n\alpha)}{2\phi_C} +$$
$$+ \frac{M\ \left[V\sin(\alpha) - V_{el}\right]^2}{2\varepsilon_b}; \alpha \le \alpha_0;$$

The details of parameters are rather extensive and correspond to the previous equations, [21,22,23]. However, this simplification does not save the experimental work required to determine the erosion constants.

3.4 Hutchings Model (T1) This model and its derivations were a primary ones [23]. It was designed for erosive wear by plastic deformation, without deformation factors. The angle of impact is 90 degrees, that is, normal incidence. The result is a summatory of impacts, with an erosion rate, E, as follows,

$$E = \frac{K\rho U^2}{2H} \quad ; \tag{4}$$

where, [23], ρ is the density of the material being eroded, U is the initial particle velocity and H is the target surface hardness. K represents the fraction of material removed from the indentation as wear debris and is also known as the wear coefficient.

The value of K can be thought of as a measure of the efficiency of the material removal process. Derivations of this model inserting the impact angle have been developed and constitute an specific variety [21,23]. This model was used to make an optimization example with software-subroutine in Section VI.

3.5 Hashish modified model for erosion (T2) This model is based on Finnie one and includes the velocity term and the conditions of the particle shape [21]. Basic formulation is as follows,

$$W = \frac{7}{\pi} \times \frac{M}{\rho_p} \times \left(\frac{V}{C_K}\right)^{5/2} \sin(2\alpha) \times \left(\sin(\alpha)\right)^{1/2} \quad ;$$
$$\text{and,} \tag{5}$$
$$C_K = \sqrt{\frac{3\ \sigma_f\ R_f^{3/5}}{\rho_p}} \quad ;$$

where R_f is the particle roundness factor. This model does not require any experimental constants. It is uniquely based on the ductile properties of the eroded material, and therefore useful/focused for shallow impact angles for ductile materials, T2.

3.6 Computational Fluid Dynamics Models (T1) This method is used for solid particle erosion inside pipe geometries, rather T2 but since it could be applied on several kinds of materials, T1. Its weakness is that this technique is complicated and time consuming and as such is most appropriate for complex, non-standard geometries.

Additional difficulties are, [23], the determination of percentage of particle on a fixed surface, their impacting angle, and specific/individual velocity. An example of formulation for this type of modelling is,

$$E = A\, F_S\, V_0^n\, f(\theta) \;; \qquad (6)$$

where, E is the erosion rate, V_0 is the particle impingement velocity, A is a material dependent coefficient, F_s is a particle shape coefficient, n is an empirical constant, 1.73, and $f(\theta)$ is a particle impact angle dependent function.

3.7 Micro Scale Dynamic Model (MEDM,T1) This model, [23,24], is designed to be implemented with FE method and is useful for erosion-corrosion. It is based on fundamental physical forces equations, such as,

$$\vec{F} = m \times \frac{d^2\vec{r}}{dt^2} \;; \qquad (7)$$

The MEDM approach is applied to modelling an abrasion process compared to plastic-elastic mechanical elements, such as wheels or similar mechanical components. This tribotesting method is widely used to rank wear-resistant materials under low stress condition. Abrasive particles pass through the opened-gap between the mechanical sample and the specimen. As a result the specimen surface is eroded/abraded. The mass loss of a tested material is dependent on the mechanical properties of the tested material and the abrasive particles as well as the wear conditions. All this is carried out with 2D modeling and the resulting equations have a physical mechanical frame and do not present important complications.

3.8 A series of models with corresponding approximations Nepomnyashchy, [4], asserted that erosive wear of metals is caused by low-cycle fatigue or microcutting, and depends on the impact angle. Abramov [4] applied Hooke's law for metal erosive deformation, and supposed breakings are linked to maximum shear stress magnitude.

Beckmann and Gotzmann [4] derived an analytical expression for the erosion of metals from the hypothesis that, in abrasive and erosive wear, the volume removed is proportional to the work of shear forces in the surface layer. The basic model was formulated from the study of deformation caused by a single spherical particle.

Peter [4] in his model used Beckmann and Gotzmann`s erosion theory after the replacement of the equations for computing the indentation depth of the particle and the specific shear energy density.

3.9 Finite Element (FEM) and Monte-Carlo/Quasi Monte-Carlo Models Broadly, [10], FEM is a mathematical method, and not an specific model. Therefore, what is included here is the FEM that has been applied on specific model equations to obtain practical results for erosion determination.The same consideration holds for Monte-Carlo, that is, Monte Carlo is a mathematical method that was used for erosion modeling, e.g.,thermal barrier coatings or physical vapor deposition.

Monte Carlo simulation techniques uses continuous software random loops to reach an optimal value for particle size, properties, the material surface condition and the local dynamic impact condition.

IV. MATHEMATICAL FORMULATION OF CORROSION MODELS WITH ADVANTAGES AND WEAKNESSES

In this section corrosion models are explained with their main formulation. One difference between erosion modeling compared to corrosion is the rather complexity of the chemical process of corrosion equations. In the following a series of corrosion models are presented. In corrosion, depending of the imperative condition of every chemical compounds of the materials, T2 models are found very frequently in the literature [23]. Usually the most frequent is that erosion causes corrosion, and less common that corrosion causes erosion –oxidative-corrosion is an important engineering question in seawater technology and marine engineering [24]. Corrosion in power plants is caused principally by oxidation, [5], whose general chemical equation reads,

$$\begin{aligned}&aM + \frac{b}{2} O_2 \leftrightarrow M_aO_b \;\; ; \\ &\text{feasible corrosion, if,} \\ &\Delta G \langle 0 \;\; , \\ &G : \text{Gibbs Free Energy} \;\; ;\end{aligned} \tag{8}$$

Apart from that, recently, [26], corrosion combined with wear/erosion, i. e., wear plus abrasion, has become a promising and applicable new investigation line.

Energy Plants, such as nuclear or other kinds of steam-turbines type got significant improvements from these recent advances. In the following we pass on the corrosion models direct description [26].

4.1 Chemo-hygro-thermo-mechanical model for concrete (T2) This model, [8], is developed in FEM and is used for reinforcement concrete at any kind of special construction. It comprises chemical and mechanical characteristics. It can be considered an specific model of T2, and with features of corrosion-erosion duality.

4.2 Pipe Corrosion Models based on Neural-Network Theory (T2) This model works in pipes, based on Neural-networks mathematical methods. It si applicable in Power Plants since pipes constitute an important structure in energy systems -corrosion in oil-gas pipelines. The internal corrosion of pipeline is a multivariable nonlinear system, and Genetic Algorithms (GA), such as Neural Network analysis, are used in its optimization. The computational development of this model follows the usual steps of the GA programming –it can be considered specific T2 model.

4.3 Stress Corrosion Model (T1) Stress corrosion, in combination with environmental agents, causes cracks in a number of mechanical structures [27]. the environment component diffuses within the cracks and causes a positive feedback for the cracking-mechanical process.

The modeling is rather complex, and some approaches were done [27]. The role of the geometry of the cracks, added to fracture mechanics principles constitute additional factors to increase the difficulties. Some equations [27] for this kind of stress are published in the literature, as follows, for an hyperbolic notch,

$$\begin{aligned}&\sigma_X = \frac{K}{(2\pi r)^{1/2}} \times \\ &\times\left[\cos\left(\frac{\theta}{2}\right)\times\left(1-\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right)-\frac{\rho}{2r}\times\cos\left(\frac{3\theta}{2}\right)\right]; \\ &\sigma_Y = \frac{K}{(2\pi r)^{1/2}} \times \\ &\times\left[\cos\left(\frac{\theta}{2}\right)\times\left(1-\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right)-\frac{\rho}{2r}\times\cos\left(\frac{3\theta}{2}\right)\right];\end{aligned} \tag{9}$$

where θ is the polar angle of r, K a geometrical constant, and ρ the curvature parameter. The study and modeling of the interrelation among cracks (mechanical) and corrosion (chemical) is a complex mathematical-geometrical challenge.

4.4 Three-Dimensional geometric models of corroded steel bars (T2) This geometrical model, (T2), is based on the experimental fact that corrosion pit can be given with a hyperbola. The effects/physical-consequences of geometric parameters for a hyperbola on mechanical properties of corroded steel bars are applied –there is a link, therefore, with any kind of energy plant. It is a rather empirical model based on simple hyperbolic geometry of pits and steel bars. Stress and strain parameters are fundamental in the implementation of this model.

4.5 Wagner Model and derived equations for oxidative corrosion This equation is basic for the mathematical analysis of the kinetic process of oxidation-corrosion rates. Oxidative corrosion rate usually has two stages, the initial stage (formation of superficial layer) and the main stage (the growth of the thickness of oxidative layer and formation of the multilayer of oxide), with an intermediate stage between both [5]. The Wagner primary equation is useful to derive practical formulas for high-temperature corrosion and low-temperature corrosion, and a series of intermediate approximations. Wagner's Differential Equation reads,

$$J = -CB\left[\frac{d\varphi}{dx} + ze\frac{dE}{dx}\right];$$
$$J \text{ is, Rate of Number of particles through oxide layer;} \quad (10)$$

were C is particle concentration, B is the particle velocity for unit of force applied, φ is the chemical potential (we refer to Nerst fundamental equation), z is the valence of the particle, e is the electron charge and x is the thickness of the oxide layer. From this Wagner equation a series of models for different oxidative stages have been developed in the literature [4, 5, 28, 29], mainly in a exponential differential equation frame or integral equation [5]. This model is a milestone for power plant functionality Survival Time Function R(t) in Reliability determination of the plant.

In classic contributions, Ots, [5], developed corrosion models both in metal in general at low,high, discrete or continuous temperature, and metal pipes with the same variations, but under the effect of oil shale combustion [5].

The series of equations/approximations is rather large, nevertheless it is possible to refer some fundamental formulas that could be modified according to specific metal material or geometry of basic plant components.

For general metals corrosion, at high temperature, the following equation holds,

$$W = K_{02}\, e^{\left(\frac{-Ez}{RT}\right)} \times t^{n} \; ;$$
$$W \text{ is, Amount of oxidized metal;} \quad (11)$$

where t is time, T absolute temperature, K_{02} is derived from a temperature-dependent coefficient, and n is a corrosion rate factor. Variations of these formulas are extensive and detailed [5], i.e., specific for diffusion-controlled region of the oxide layer, particular for the kinetic region of the layer, etc.

4.6 Models of Corrosive-Erosive Wear of Heat-Transfer Tubes (T1) In the literature, Ots, [5], developed also in his contributions a series of equations/approximations for Erosion-Corrosion Models prefereably/more-specific for oil shale combustion. In order to refer/show a basic equation with differential-frame of a function of several variables which is W, the specific mass of corroded material, in function of, namely, P, force acting on the layer, K, corrosive activity of the deposit (e.g., a boiler), and t, the time. That formulation reads,

$$dW(P,K,t) = \left(\frac{\partial W(P,K,t)}{\partial t}\right)_{P,K} + \left(\frac{\partial W(P,K,t)}{\partial P}\right)_{t,K} + \left(\frac{\partial W(P,K,t)}{\partial K}\right)_{P,t} ;$$
$$W(P,K,t) \text{ is, Specific mass of corroded metal;} \quad (12)$$

The similar mathematical observation is applicable, this and those formulas in general, [5], could be modified according to specific metal material or geometry of basic plant components [7, 30].

4.7 Todinov Synergic Model of Erosion and Corrosion A model for Erosion and Corrosion for powdered materials coatings was developed by Todinov [29]. Synergism between erosion and corrosion reads,

$$T=E+C+S \quad (13)$$

where T is the total mass loss rate from [29], (i.e., the erosion–corrosion rate), E is the pure erosion rate, C is the pure corrosion rate, and S is the mass loss rate due to the synergistic effect between erosion and corrosion –the object of interest of this equation. This synergistic term may be separated into two terms:

$$S=S_{EC}+S_{CE}; \quad (14)$$

where, [29], S_{EC} represents the erosion-induced corrosion rate (i.e., increase in corrosion rate due to erosion) and S_{CE} represents corrosion-induced erosion rate (increase in erosion rate due to corrosion). This is the synergism modelling base, and further developments and approximations can be found in the literature, [29].

By way of explanation, it is sharply different a corrosion process over a previously eroded powdered material surface, S_{EC}, from an erosion with loss of material in a previous corroded area, $S_{CE.}$

V. MATHEMATICAL MODELS FOR BIOMEDICAL TRIBOLOGY AND TRIBOCORROSION, AND INTRODUCTION

This section is mainly focused on hip prostheses models for femur acetabular joint replacement. The reasons are multiple, from the high prevalence /incidence of hip articulation degradation/fracture or similar surgical/medical pathologies to traumatological or genetic malformations that involve severe biomechanical problems in hip articulation system-which in fact, hip gait constitutes a fundamental part for walk, run and general mobility of the whole human anatomy. In other words, hip is the biomechanical mesh between the trunk and the legs walking muscular-articular system. No matter whether legs are functional or not, a mobility default in the hip causes such a complicated biomechanical consequences that all the inferior member of the body claudicates completely [ref]. Load magnitudes on knee, taking into account tendons and ligaments forces during walk, are around 2000 N, and similar values can be expected in hip, bath natural articulation or implant. This number gives an idea of the severe constraints/difficulties, both biomechanical and material characteristics (stress, strain, hardness, etc) when designing the prostheses.

A classical model for wear in hip arthroplasty is,

$$W = K \bullet (L\,X)/H \qquad (14.1)$$

where K is a wear parameter/constant, L is biomechanical load, X is sliding distance, and H is hardness of implant. This equation is optimized in computational section. Lubrication modeling in hip prostheses, [53], constitute also a base for development of mathematical formulation, and as an example we refer to Rabinowitsch model that reads,

$$\eta = \eta_\infty + \frac{\eta_0 - \eta_\infty}{1+\left(\frac{\tau}{G}\right)^2}; \qquad (14.2)$$

where mu is viscosity, and tau is shear-stress, the other parameters are constants determined by regression, [53]. And also the Carreau's and Cross model,

$$\eta = \eta_\infty + \frac{\eta_0 - \eta_\infty}{1+\left(1+\left[\frac{\dot{\gamma}}{\dot{\gamma}_c}\right]^2\right)^{m/2}}; \qquad (14.3)$$

where gamma is the notation for the shear rate. Actually hip arthroplasty constitutes an important branch of medical devices industry with several super-specialization branches for the extensive area of investigation.

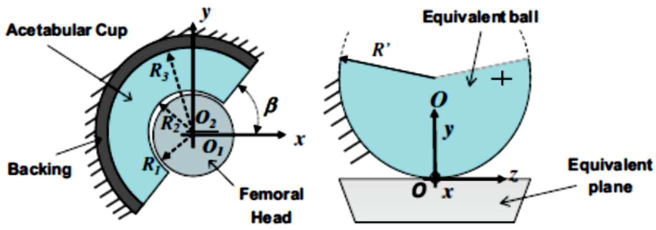

Figure 1. From reference [53], excellent biomechanical sketck of L. Mattei, F.DiPuccio, B.Piccigallo, E.Ciulli showing biomechanical coordinates and ball and socket modeling of hip implant with sharpness.

The wear of a hip prosthesis is a complicated phenomenon, which generally depends on the contact status between the ball and the cup (i.e., friction regime), characteristics of the tribocouple, physiological conditions [1], production quality of the prostheses [2], lubricants [3], etc. For example, despite a low friction torque, the polymer-on-metal configurations exhibit higher wear, than metal-on-metal or ceramic-on-ceramic ones [4] due to the boundary lubrication regime between the wearing surfaces [4,5]. For the same reason, small-size metal-on-metal hip joints perform worse, than large-size ones [4]. Properly designed and manufactured metal-on-metal hip joint prosthesis work, *vice-a-versa*, under mixed lubrication regime [5], and ceramic-on-ceramic hip joints function even under hydrodynamic lubrication conditions [4], what provides extremely low friction.

The three principal wear mechanisms in hip joints were found to be adhesive wear, abrasive wear and fatigue wear [55], accompanied by tribocorrosion in the case of metal-on-metal configurations [60]. With time, one mechanism may change to another [55]. For polymer-on-ceramic hip joints, adhesive wear of polymer with the subsequent formation of the tribolayer on the ceramic surface is characteristic [55]. For polymer-on-metal configurations, both adhesive and abrasive wear mechanisms were reported, whereas the last was found to be more probable [55]. Surface fatigue in combination with three-body abrasion and tribochemical reactions was found to cause wear in the case of metal-on-metal tribocouples [60]. Despite the absence of clear literature data, for ceramic-on-ceramic configurations, surface fatigue and abrasion may be named as the most probable wear mechanisms.

For simulation of wear of a hip prostheses the Archard's wear law is usually applied [55,61,62]. Its is more convenient to present the integral equation from this model once obtained from the finite elements method mathematical development. According to it, the wear volume V (mm^3), vanished from the contact surface, may be determined as

$$W = \int_{\Gamma_{t0}}^{\Gamma_t} \int_{S_{t0}}^{S_t} K_W \, \sigma \, dS \, dA \quad ;$$

(14.4)

where Γ is the contact surface, mm^2;S_t is the sliding distance, m; k_w is the wear coefficient; k_w = (0.18–0.80)×10^{-6} mm^3/Nm for the ultra-high-molecular-weight polyethylene (UHMWPE) in tribocouple with the stainless steel [55,61,63], and k_w = 0.10–0.31 × 10^{-6} mm^3/Nm for UHMWPE in tribocouple with alumina (Al_2O_3) [1];σ is the normal contact stresses (Hertz contact stresses), N/mm^2, which may be calculated by the corresponding formulas.The maximum normal force (F_N) may be taken as F_N = 3500 N [55,61] and the swing angle of foot is 23 degrees in the forward and backward directions [55].

For the real simulations, the volumetric wear rate (mm^3/year) is usually calculated. By the literature data, it is in the range of 5–50 mm^3/year.

The difference between the modeling of hip and knee is given mainly by the methods used. In knee implants, because of the extreme loads that are acting over a rather small bone surface, the usual method is Finite Elements modeling, with precise distribution of stress and strain magnitudes [47,51]. However, substitution of tibial parts are also made with metallic implants, e. g., titanium plasma spray coatings [47,51].

Spinal biomechanics modeling is also usually focused on Finite Elements Modeling, [Casesnoves, ASME, ref 7]. In spinal reconstruction, a large number of prostheses types are used given the complicated and risky system of the vertebral biomechanics. Finite elements are combined with other biomechanical constraints in order to obtain precision and functionality.

All in all, in Tables IV, a succinct brief of biotriboly are presented with advantages and inconvenients. The extension of the optimization/simulations of Appendix 1 will give in following publications additional algorithmic data for this important field of the Biotribiology. Next section deals yet focused on strictly mathematical contents for usually large-scale or multiobjective optimization

VI. BRIEF OF OPTIMIZATION METHODS AND ALGORITHMS

In this section we pass on the subject of optimization techniques to optimize the modelling of E/C. There are two main subsections, namely, the first is the general optimization method that is used/selected for E/C, which is Multiobjective, in general –the condition is not exclusive, since there are other types of applicable optimization methods [30]. The second is a series of modern/classical algorithms that can be used for this kind of Multiobjective Optimization, namely, from evolutionary algorithms to, e. g., Monte-Carlo formulation.

All in all, we refer in short to the most important methods/algorithms, and explain advantages, limitations, and their best area of use/applications [7,8,30]. In the last subsection, the important probabilistic link between the group of E/C models and the statistical models/concepts of Engineering reliability for Power Plants is analyzed concisely with formulation and mathematical lemmas.

4.1 Generic Multiobjective Optimization (MO)

Multiobjective Optimization [7,30] has developed in recent years large-scale optimization methods to determine a series, or combination of series, of optimal parameters for a number of variables into the Objective Function. MO with Least Squares L_2 Norm is the most frequent technique used actually in literature for large scale computational problems [30,31,32]. MO Objective Function with L_1 Norm or so-called Chevyschew Multiobjective, is also useful although not so frequent.

4.2 Specific Optimization Algorithms

Optimization specific methods/algorithms can be divided into Deterministic and Stochastic ones.

Deterministic methods (DM), [30,31,32], are steepest descent; conjugate gradient, linear programming, maximum likelihood, dynamically penalized likelihood, quasi Newton methods, Broyden Fletcher method, Davidson Fletcher method and others. Random techniques (RT) are principally Monte Carlo methods, Quasi-Monte Carlo, simulated annealing and genetic algorithms. All of them show advantages and weaknesses, and the aim of the section is concerned with the most useful and practical of all these –given in short.

4.2.1 Interior-Reflective Newton Method This (DM) method is an evolution of the classical good Newton/Newton-Raphson Method. We obtained acceptable results in multiobjective optimization with several variables [8] in CAD modeling, and large-data/variables polynomial fits with results of high Determination Coefficients previously. It can be considered also according to literature suitable for mathematical models for E/C.

4.2.2 Levenberg-Marquardt (DM) It is a method whose objective function is sum of squares of nonlinear functions. Levendberg-Marquard algorithm is considered, in general, as an acceptable multiobjective method, and it also has been used efficiently, i. e., in several of our contributions [31,32].

4.2.3 Conjugate Gradient (CG) Algorithms (DM) and variations/refinements This group of CG methods derive from the original Steepest Descent (SD) method with important mathematical improvements. SD can be considered useful but obsolete and with approximate solutions, and CG methods are still useful although cannot be considered as extraordinary [30]. CG running time is acceptable also.

4.2.4 Genetic/Evolutionary algorithms (RT) These methods are extensively used today in many varieties and extensive branches of science, economy, and statistics. Evolutionary algorithms intend to resemble the nature selection process through continuous random generations of solutions (so-called chromosomes), at every program loop. The process is continuously repeated at any step, conditioned by a settled tolerance. The method is considered good, although not extraordinary [7,30].

4.2.5 Simulated Annealing (SA) SA is a global optimisation method (RT). The algorithm, [30], searches for new values for input parameters in three ways: grid search, linear interpolation and discrete values. SA is a simple method that in principle should converge to a global optimal solution but parameters settings could be a problem and time required could be too long for time critical applications –SA is considered in our criterion useful for a search of a primary useful approximation towards global minimum, and because of its proper random algorithm, the obtained optimal value does not match *necessarily* the global minimum of the objective function.
For optimization of a large number of parameters it is especially difficult to obtain optimal results. Inconvenients, as said, are that the search point could be trapped in a deep concavity of the OF and the program call would give it as global minimum. Therefore, SA is useful for an accurate initial search.

4.2.6 Stochastic Optimization (SO) Group This group comprise random methods in general, and SO denomination is used generically in the literature to characterize the applied method as a reference of its group of origin [7,30,31,32]. Markov chain models are also considered stochastic and their variants are widely used to date.

4.2.7 Monte-Carlo Methods (RT) Generically, [32], Monte Carlo (MC) is considered a random/stochastic method applicable in a large number of science specializations and mathematical statistics science. Basically, MC uses a continuous group of computational loops with a fixed/input closing-tolerance value for all the variables to be determined. Each loop is generating random values that can stop its circle when the tolerance value is accomplished for that/those variables. It is quite similar to evolutionary algorithms but not the same.

In the past, computers running time was rather slow compared to nowadays, so MC was a method only implemented for particular calculations with powerful computers. Today, with microelectronics advances in microprocessors and operating systems mainly, standard programs such as GEANT and many evolutions/variants of this type, e.g., GEANT-FLUKA [31,32], are computationally able to bring results after a reasonable running time – GEANT is an example among a large variety. MC is used in physics extensively, e.g. radiation therapy, numerical methods (numerical integral calculations, for example), and several branches of science and mathematical statistics – in statistics, for instance, to select/optimize randomly samples, sorting the tedious task of collecting a large bulk of empirical data. MC methods are used also, for example, to determine the random reliability of a mechanical chain linked to probability calculations of the system or plant under certain conditions.

4.2.8 Finite Element Modelling Optimization (DM,RT) Commonly, FEM is a mathematical method, and not an specific model. FEM is a method based on a geometrical discretization/geometrical-discretization of parameters. Therefore, what is included here is the FEM that has been applied on specific model equations to obtain practical results for erosion determination. It is convenient remark, finally, that this overview is a simplification/selection of the current nonlinear optimization methods available today. Since FEM is used in a large number of science, technology, and engineering branches, the amount of varieties and applications range, it is a must to carry out an specific/detailed selection of the type of FEM that is optimal for a defined numerical development. And prefereably with objective parameters to check the final accuracy, errors and other fitting parameters [33,17].

4.2.9 Graphical Optimization This method is used nowadays to facilitate the applications of the imaging tools over intervals of objective functions. The visual identification of local or global minima or maxima is sharp, although the condition mandatory to obtain an accurate 3D or 2D image of the objective function is strictly necessary. This approach is explained clearly in following sections with a number of consequent 3D and 2D images.

4.3 Erosion, corrosion, and wear in Reliability Engineering Modelling There is a close conceptual and mathematical link among this group of E/C models,their physical phenomena, and, in general, a Power Plant Functional Operation. Namely, E/C and Wear Modeling with Reliability Engineering (RE) Mathematical Modelling -usually statistical models and/ro distribution, from Gaussian to t-student,chi-square, binomial and others [28,29]. The objective definition of RE is given in brief in [28], as follows,
Mechanical Reliability is the probability that a component, device, or system will perform its prescribed duty without failure for a time interval when operated correctly in a specified technical environment

That probability is formally implemented in mathematical statistics, according to Probability Theory, as function of F=F(t), wich is the Probability of Failure, and the function R=R(t), which is the Probability of Survival Time. Both concepts set formally the above definition without any misinterpretation/confussion. F(t) is usually expressed by a Gaussian distribution. The quotient F(t)/R(t) is used, [28,29] to define a third concept, namely, the Hazard Rate Function (HRF), that reads,

$$\begin{aligned}
&\text{For Probability Theory},\\
&F(t)+R(t)=1,\\
&HRF = Z(t), \text{ and},\\
&z(t)\,dt = \frac{F(t+dt)-F(t)}{R(t)}; \text{ or},\\
&z(t)\,dt = \frac{f(t)\,dt}{R(t)}, \text{and finally},\\
&z(t) = \frac{f(t)}{R(t)},\\
&\text{if and only if HRF is constant},\\
&\text{HRF Model},\\
&R(t) = e^{-\lambda t}
\end{aligned} \tag{15}$$

According to Eqs (10), it is defined for Plant Operation,

Lemma 1.-The Hazard Rate Function, (HRF),in magnitude, is the numerical quotient that defines the operational state of a power plant as follows,

$$\begin{aligned}
&HRF = \frac{f(t)\,dt}{R(t)},\\
&\frac{f(t)\,dt}{R(t)} = \begin{bmatrix} \rangle\rangle 1, \text{difficult plant operation;} \\ = 1, \text{neutral state;} \\ \langle\langle 1, \text{normal operativity / functionality} \end{bmatrix};
\end{aligned} \tag{16}$$

Proof:

Maximum probabilistic values of F(T) and R(T) are 1. Their variations, i.e., R(T), probability of survival time, high value, and F(T), probability of failure, low value, result in a quotient
‹‹ 1, that is an acceptable operational state. Same condition is applicable for the other inequalities with different quotiens/magnitudes.

TABLES IV

A REVIEW OF MAIN TYPE 1 AND TYPE 2 MODELS FOR E/C IN A QUICK-CAPTION SETTING [IMPROVED FROM [50]]						
EROSION MODELS/CLASSIFICATION						
NAME	**TYPE**	**SETTING VARIABLES**	**ADVANTAGES**	**WEAKNESSES**	**USAGE GRADE**	**COMMENTS**
Finnie	T1	Impact angle (ia),Velocity	Simplicity	Obsolete accuracy	2	Milestone classic model
Bitter	T1	Impact angle (ia),Velocity	Improved Finnie for ductile and brittle erosion	Useful today	2	More specific than Finnie
Hutchings Model	T1	Velocity,density,hardness,	Simplicity/applications diversity ductile metals	Too simple accuracy	2	Derived models are more improved
Hashish modified model for erosion	T1	Velocity,density,particle roundness,experimental constants	no many experimental constants required	Experimental validation weak	2	Not extensively proven but initially acceptable
Computational Fluid Dynamics Models	T1	Velocity,fluid flown,geometric factors	Accurate adaptation on pipe geometry	Determination of impact spot,time-consuming	2	It had better for simple geometries.
Micro Scale Dynamic Model (MEDM)	T1	Angle,fixed particle geometry,forces physical parameters	Physical accuracy,ductile and brittle materials	Particle diverse geometry	2	A model based on Newton's laws mainly
Finite Element and Monte-Carlo/Quasi Monte-Carlo Models.	T2 On Previous equations	Variables depending on the selected specific equation	Accuracy with selected equations and Monte Carlo precision	Monte-Carlo requires specific software and running time	2	Monte Carlo is accurate, FEM is joint to any suitable equation
CORROSION MODELS/CLASSIFICATION (sometimes applicable on Biotribology)						
NAME	**TYPE**	**SETTING VARIABLES**	**ADVANTAGES**	**WEAKNESSES**	**USAGE GRADE**	**COMMENTS**
Chemo-hygro-thermo-mechanical model for concrete	T1	Physical and electrochemical variables	For concrete in general with chemo-chloride corrosion, applicable in FE	Further numerical/exp data necessary	1	Simulation processes higher than 6
Pipe Corrosion Models based on Neural-Network Theory	T2	Genetic algorithms software design, temperature, CO_2 partial pressure,chemical constants	Specific accuracy,global optimization	For pipes, oil-gas and large calculations	1	Computational-Optimization background required for implementation
Oil Shale Energy Plants, Ots models series	T1	Partial pressure O_2 , chemical constants, temperature, thickness of oxide layer,	Applicability to some other energy plants in general, large number of specific equations	Mostly chemical oxidation	3	Useful for oil shale plants, boilers,tubes, and pipes
Three-Dimensional geometric models of corroded steel bars	T1	Geometrical and angles in 2D and 3D,polar coordinates and crack geometry shape	Prediction of erosion-corrosion after cracks and stress distribution	Rather complicated only for elliptical and hyperbolic gometries	1	Geometrical coordinates well-selected required
Seawater-water corrosion models	T2	Chemical parameters (concentrations),pH, temperature,salinity	Applicability to some other energy plants in general, low temperature corrosion for water steam	Too specific fro marine corrosion	1	Extensive usage and types in marine technology
Wagner Law,Equation and Corrosion Model	T1	Several chemical constants,Oxigen partial pressure,thickness of layer (variable)	Applicability extensive general for dense corrosion layer (oxide)	Differential equation rather complicated for fitting	2	Fundamental Law of corrosion with many derivations
CORROSION-EROSION MODELS/CLASSIFICATION (sometimes applicable on Biotribology)						
Corrosive-Erosive Wear of Heat-Transfer Tubes	T1	Corrosive activity,time, and force acting on the oxide layer	Both corrosion and erosion determination Extensive applicability	Specific for tubes and boilers	2	Largely developed by Olts with series of equations
Corrosion-Erosion Synergic Model of Todinov	T1	Synergic model for erosion and corrosion	Applicability on powdered materials coatings mainly	Not very specific parameters defined	2	Connected with statistical and probability theory for Reliability in Engineering Modelling

Tables IV. Brief of Mechanical and Biomedical Tribological Models

BRIEF OF MEDICAL-BIOTRIBOLOGICAL MODELS						
BIOTRIBOLOGICAL MODELS/ALGORITHMS FOR HIP ARTHROPLASTY						
NAME AUTHOR, [REF]	**TYPE**	**SETTING VARIABLES**	**ADVANTAGES**	**WEAKNESSES**	**USAGE GRADE**	**COMMENTS**
Classical General Model Jin and others [51]	T1	Hardness, Load, Rotation Velocity	Simplicity	Accuracy to be improved and specified	2	classic model for further developments
Rabinowitsch Model for lubrication [53]	T2	Viscosity constants and shear-stress	Specific for lubrication and minimize wear	Required precision in constants	2	useful
Carreau's Model [53]	T2	Viscosity constants and shear, shear rate	Evolution of previous model with shear rate	Not useful totally for synovial fluid	2	Derived models are more improved
BIOTRIBOLOGICAL MODELS/ALGORITHMS FOR KNEE ARTHROPLASTY (THOSE MODELS ARE USUALLY DEVELOPED WITH SPECIFIC FINITE ELEMENTS GENERIC METHOD)						
NAME	**TYPE**	**SETTING VARIABLES**	**ADVANTAGES**	**WEAKNESSES**	**USAGE GRADE**	**COMMENTS**
Finite Elements Modelling	T1	Physical variables	Extensive and multifunctional applications	Errors at implementation	2	Simulation processes feasible
Corrosion Models applied on solid metallic implants for knee [52]	T2	Chemical parameters	Specific accuracy	For durability of metal-coated knee implants	1	useful
BIOTRIBOLOGICAL MODELS/ALGORITHMS FOR SPINAL ARTIFICIAL IMPLANTS						
Finite Elements Modeling	T1	Corrosive activity,time, and force acting on the oxide layer	Both corrosion and erosion determination Extensive applicability	Specific for tubes and boilers	2	Largely developed by Olts with series of equations
Dynamics of deformable Compliant Artificial Intervertebral-Lumbar Disks [Casesnoves 2017]	T1-2	THE NUMERICAL REULEAUX METHOD APPLIED ON ARTIFICIAL DISKS [Casesnoves, 2007, [34,64]] Synergic model for deformation-biomechanical-stress of artificial disks	Applicability on deformable solids dynamics/kinematics	Computational algorithms And framework necessary	2	Connected with Deformable solid theory and General Numerical Reuleaux Method (Casesnoves, 2007) Modelling

VII. MECHANICAL-COMPUTATIONAL NUMERICAL EXAMPLE

In this section a computational example with a selected classical empirical (T1) model, Hutching, is shown with the software algorithm. The software/formulation in this instance was developed originally from numerical methods by authors with Freemat4.1. The model is easy and the program also, it is intended for sharp understanding to be applied more largely/ improved in other types of modeling—This introductory example is taken from previous publications since the programming technique is basically explained [ref]. Then we get the [Eq 4] ,

$$E = \frac{K\rho U^2}{2H} \quad ;$$

The simulation will be for several experimental erosion rates (4 measurements of E, around 10^{-2} mm/Kg, Fe alloy) to determine both the wear coefficient and the optimal particle velocity contained in the gas. This simulation is an example, and it is intended to show to show the computational-numerical method, not the model fitting accuracy. That is, 2 optimal variables to be found and a L^2 Norm Least-Squares Objective Function (OF) without constraints,

$$OF = \sum_{i=1}^{i=4} W_i \times \left\| E_i - \frac{K \times V^2 \times \rho}{2H} \right\|_2^2 \quad ;$$

with W_i as the notation for the weights, (17)

so, we get, in concise notation,

$$x = K,\ y = V,\ M = \frac{\rho}{2H};\ \text{then,}$$

$$OF = \sum_{i=1}^{i=4} W_i \times \left\| E_i - xy^2 \times M \right\|_2^2 \quad ;$$

Setting units, just to interpret right numerical values. For E (L/M, mm/Kg),V(LT^{-1} m/s), ρ (ML^{-3} Kg/mm^3), K (L/M mm/Kg), H (N/mm^2) . Values taken from [ref], are included in Table 4. The software selected was Freemat 4.2, (Samit Basu General Public License).

Equivalence with commercial Matlab is complete in handle functions [31,32], loops, and programming sentences –any programmer can design this computational algorithm. Newton-Raphson (NR) is used in several variables with Jacobian Matrix and a tolerance of at least 10^{-4}. NR cannot be considered excellent optimization method, rather its developments in further algorithms in Least-Squares. However for illustration is sufficient currently. The matrix that was programmed in the software, for n variables, reads,

$$A = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \cdots & \cdots & \frac{\partial f_m}{\partial x_n} \end{pmatrix} ; \qquad (18)$$

This example of optimization has not been programmed with smooth-gradient factor within the objective function and the L_2 Norm is the most extensively used for these algorithms although L_1 Chevyshev Multiobjective could also fit for this type of nonlinear programming.

TABLE V

NUMERICAL OPTIMIZATION SIMULATION VALUES SELECTED				
TYPE	**SIMULATION VARIABLE VALUES**			
E_i /with W_i	0.01 /0.5	0.03/0.25	0.05/0.175	0.04/0.175
ρ	Fe Alloy 7900 Kg/m^3			
H	200 Hv Vicker equivalent to 640 N/mm^2			
Optimization Parameters	**K , wear coefficient**		**V , Optimal Particle Speed**	

Results and Errors are shown in Table 5 and numerical values of the objective function after programming are almost null. That means that the algorithm at least has been correctly programmed¸because the average error for minimization of objective function is about E-3 and in some complementary runnings was found around E-5-6. The interpretation of results of this, we recall simple optimization example, is that the process of programming optimal fitting for erosion and corrosion models can be carried out with modern computational programs, e.g., Freemat (Samit Basu, General Public License), Matlab, or similar available software.In Table 6, the average iterations mean the number of times that the program is trying to obtain the desired precision that was set, namely, the tolerance set, in this case, 10^{-8}. The average error significance is the absolute value of the objective function with the inset optimal values got by the program. In other words, to make null the objective function (average error narrowly null) implies that the optimization process is acceptable.

TABLE VI

NUMERICAL OPTIMIZATION RESULTS	
PARAMETER	**SIMULATIONS RESULTS**
K	**75E-6**
V	**17.100**
Average iterations	**6**
Average tolerance	**10E-8**
Average error of Objective Function Minimization	**10E-4**

VIII. COMPUTATIONAL SIMULATIONS OF MECHANICAL MODELS WITH PROGRAMMING RECIPES

Simulation Series 1.-This example of 2 variables simulation is done with the Menguturk and Sverdrup (1979) model, [ref], developed as an empirical erosion model for carbon steel material eroded by coal dust. The model shows that erosion is largely a function of particle impact velocity and angle. Selection of this algorithm is justified for primary new 3D simulations with surfaces in an attempt to demonstrate the practical materials engineering/bioengineering usage of this kind of 3D representation—in other words, the cursor of the software can give the numerical desired values for lab or experimental of any type.The model for small and large particle impact angles is given as follows,

$$E = v^{2.5} \times \left[\left(1.63e-6 \bullet (\cos\alpha)^{2.5} \right) + \left(4.68e-7 \bullet (\sin\alpha)^{2.5} \right) \right];$$

This is the simplest equation valid for particle impact angles ≥ 22.7°. For angles < 22.7°, the model formulation reads [ref],

$$E = v^{2.5} \times \left[\left(\left(1.63e-6 \bullet (\cos\alpha)^{2.5} \bullet \sin\left(\frac{180}{45.4} \alpha \right) \right) \right) + \left(4.68e-7 \bullet (\sin\alpha)^{2.5} \right) \right];$$

(19)

where E is the erosion rate in mm3 g−1 , and impact velocity and angle α, measured in m s−1 and radians,respectively. The volumetric erosion rate (mm3 g−1). That is, 2 variables. This simple equation illustrates the following series of computational simulations, because the implementation of programming matrices algebra-operations is fast, although the application of the matrix-algebra concepts in programming requires special calculations to obtain accurate/realistic/precise results.

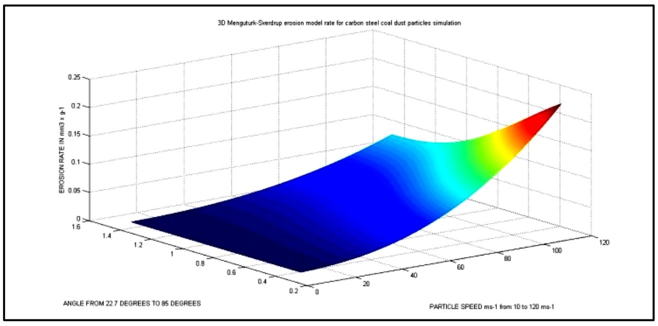

Fig 1.1- Pictured, a numerical Matlab-2009-10, jpg format, surface-matrix-simulation for a velocity range from 10-120 ms^{-1} of Menguturk and Sverdrup (1979) model and matrices 1000x1000, quite large numerical imaging programming. The choice of the imaging perspective is intended to show the smooth surface growth towards the maximum speed and angle optimal value that gives the maximum erosion magnitudes for the model. Note the cosine and sine variations and exponentials low values in the model formulation, according to changes within the angles range.

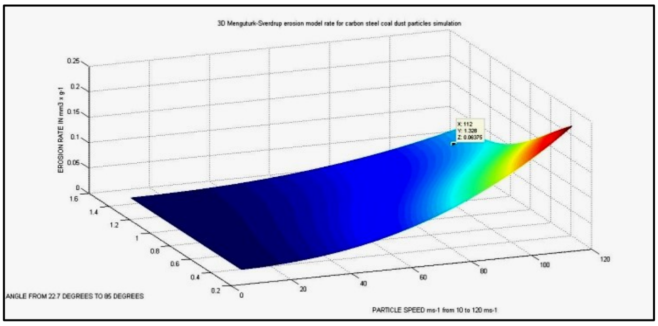

Fig 2.- Pictured with cursor-magnitude-inset, the narrowly-same numerical Matlab-2009-10, jpg format, surface-matrix-simulation for a velocity range from 10-120 ms^{-1} of Menguturk and Sverdrup (1979) model and matrices 1000x1000, quite large numerical imaging programming. Cursor indicates speed 112 ms^{-1} , angle of particle 1.326 radians, and erosion rate about 0.084 $mm^3\ g^{-1}$. The choice of the imaging perspective is intended to show the smooth surface growth towards the maximum speed and angle optimal value that gives the maximum erosion magnitudes for the model. Note the the practical utility of the cursor to search optimal experimental-theoretical values for modeling research both for simulation and nonlinear optimization.

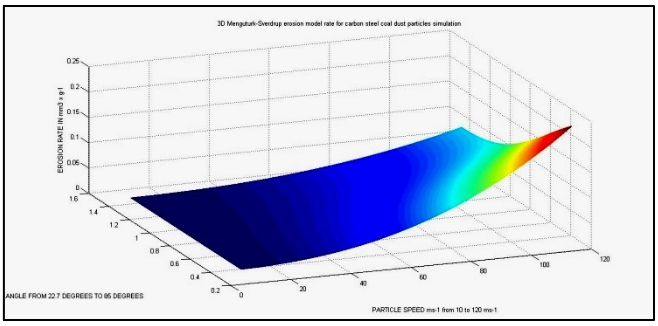

Fig 3.- Simulation of a different perspective to show the surface extension, jpg format, a matrix-simulation for a velocity range from 10-120 ms^{-1} of the model and matrices 1000x1000, quite large numerical imaging programming—running time around 4 seconds, perspective-imaging change time about 10 seconds, taking into account the large matrices. The choice of the imaging perspective is intended to show better the smooth surface growth towards the maximum-peak speed and angle optimal value that gives the maximum erosion magnitudes for the model, and also the surface part for minimum values.

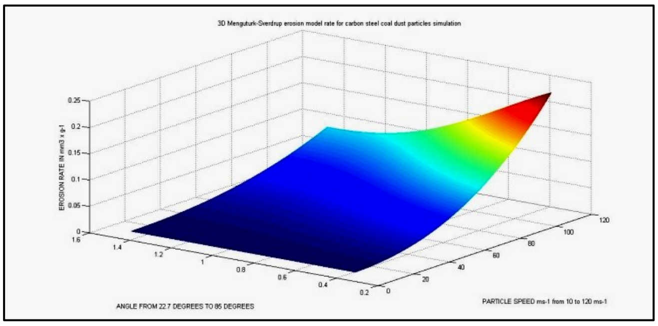

Fig 4.- Rather the same simulation of the previous figure in different angles, to show the surface extension, jpg format, a matrix-simulation for a velocity range from 10-120 ms^{-1} of the model and matrices 1000x1000, quite large numerical imaging programming—running time around 4 seconds, perspective-imaging change time about 10 seconds, taking into account the large matrices. The choice of the imaging perspective is intended to show better the smooth surface growth towards the maximum-peak speed and angle optimal value, with the surface-sheet totally pictured, that gives the maximum erosion magnitudes for the model, and also the surface part for minimum values.

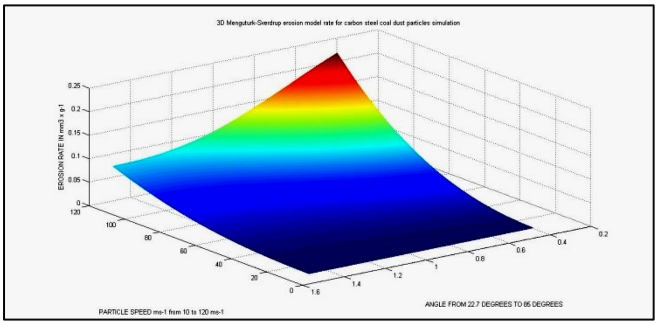

Fig 5.- Inverted-angle simulation with a different perspective to show the surface extension, jpg format, a matrix-simulation for a velocity range from 10-120 ms^{-1} of the model and matrices 1000x1000, quite large numerical imaging programming—running time around 4 seconds, perspective-imaging change time about 10 seconds, taking into account the large matrices. The choice of the imaging perspective is intended to show better the smooth surface growth towards the maximum-peak speed and angle optimal value that gives the maximum erosion magnitudes for the model, and also the surface part for minimum values.

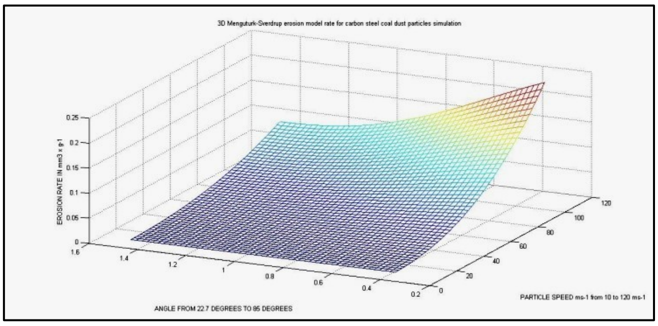

Fig 6.- Totally different simulation of the previous figure in also different angles, to show the surface extension, jpg format, a matrix-simulation for a velocity range from 10-120 ms^{-1} of the model and matrices 50x50, rather simple numerical imaging programming—running time around 2 seconds, perspective-imaging change time about 1-3 seconds, taking into account in this case the small matrices. The choice of the imaging perspective is intended to show better the smooth surface growth towards the maximum-peak speed and angle optimal value, with the surface-sheet totally pictured, that gives the maximum-medium-minimum erosion magnitudes for the model, and also the surface part for minimum values.

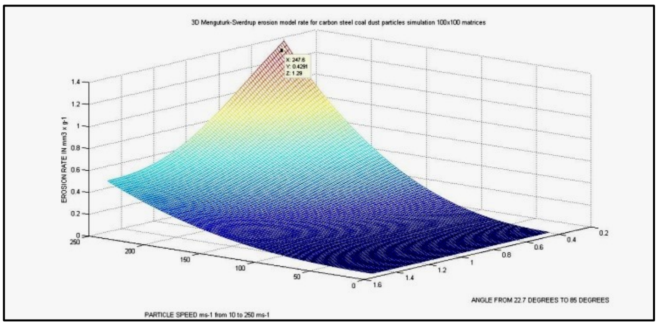

Fig 7. (enhanced in Appendix 1)- This simulation shows maximum-model cursor-values of speed about 247 ms^{-1} , angle of particle 0.4291 radians, and erosion rate about 1.29 $mm^3\ g^{-1}$ .So pictured with inset-cursor it is a different simulation of the previous figure in also different angles, to show the surface extension, jpg format, a matrix-simulation for a velocity range from 10-250 ms^{-1} of the model and matrices 100x100, rather simple numerical imaging programming—running time around 2 seconds, perspective-imaging change time about 1-3 seconds, taking into account in this case the small matrices. Cursor in at peak of The choice of the imaging perspective is intended to show better the smooth surface growth towards the maximum-peak speed and angle optimal value, with the surface-sheet

totally pictured, that gives the maximum-medium-minimum erosion magnitudes for the model, and also the surface part for minimum values.

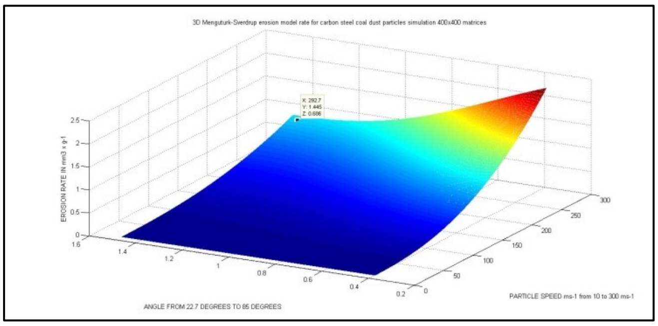

Fig 8.- In this program the speed range was increased from 10-300 ms^{-1} .The simulation with inset cursor is showing speed 292 ms^{-1} , angle of particle 1.445 radians, and erosion rate about 0.686 mm^3 g^{-1} . It was selected a cursor-point with maximum particle angle and maximum velocity. Speed range is 10-300 ms^{-1} , velocity tha could be physically reached in power-energy plants systems pipes or tubes, and is narrowly near to the sound velocity. The matrices of imaging programming are not too large, 400x400. The choice of the imaging shoot is varied to show the smooth surface growth towards the maximum/minimum speed and angle value that gives the maximum erosion magnitudes for the model.Note the cosine and sine variations and exponentials low values in the model according to changes within the angles range.

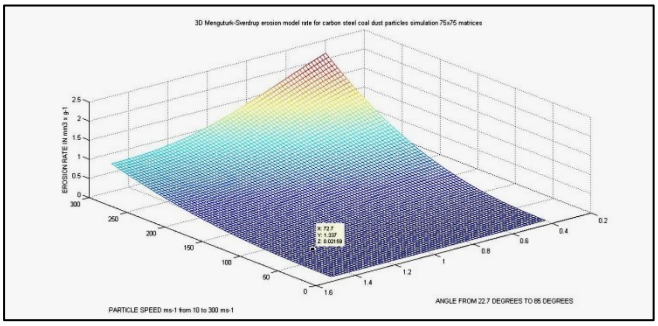

Fig 9.-Pictured with cursor inset, a minimum value for a speed range from 10-300 ms^{-1} and matrices of imaging programming 75x75. The choice of the imaging shoot is varied to show the smooth surface growth towards the maximum speed and angle value that gives the maximum erosion magnitudes for the model.Note the cosine and sine variations and exponentials low values in the model according to changes within the angles range.

Simulation Series 2.-This example of 2 variables simulation is developed with the same model running similar speed and angles ranges but selecting one variable. It is simpler programming and can be easily executed both in Freemat or Matlab as it is here presented.

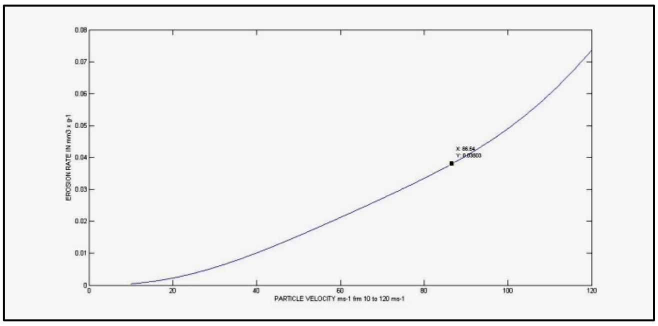

Fig 10.-In this 2D-simulation-program the speed range was from 10-120 ms^{-1} .The simulation with inset cursor is showing speed about 67 ms^{-1} , and erosion rate about 0.036 mm^3 g^{-1} . It was selected a cursor-point with middle particle velocity.The matrices of imaging programming are 1000x1000.Note the cosine and sine variations and exponentials low values in the model according to changes within the velocity/angles range.

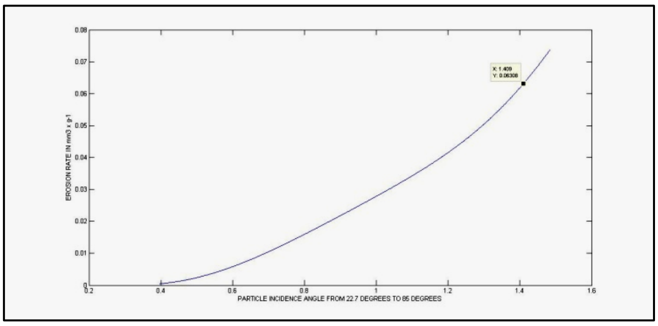

Fig 11.-In this 2D-simulation-program the speed range was from 10-120 ms^{-1} . This program was designed for erosion rate versus angle range of particle incident. The simulation with inset cursor is showing angle of 1.409 radians and erosion rate about 0.063 mm^3 g^{-1} . It was selected a cursor-point with rather extreme incident-angle value. The matrices of imaging programming are 1000x1000.Note the cosine and sine variations and exponentials low values in the model according to changes within the velocity/angles range.

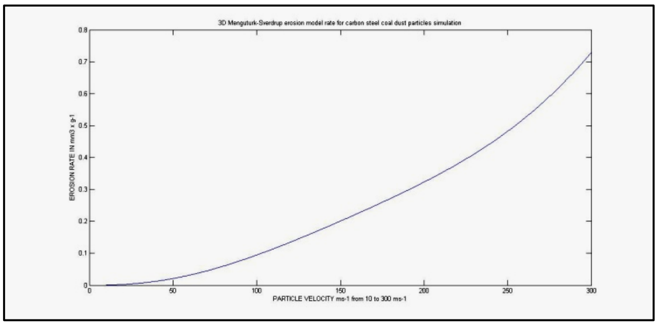

Fig 12.-In this 2D-simulation-program the speed range was from 10-300 ms^{-1} —that is, almost the sound velocity at upper bound of interval. This program was designed for erosion rate versus angle range of particle incident. The matrices of imaging programming are 1000x1000.Note the increment of erosion rate maximum within the wider velocity range.

Optimization Series 1.-In Table VII,pictured, a series of optimization trials in non-linear least squares for wear in biotribological hip prostheses and mechanical systems. Units according to references of biomedical tribology. The hip implants materials are selected as significantly different, with hardness interval from metal to the highest values of ceramic-ceramic implants [51]. An extensive discussion f these figures will be developed in further contributions.

TABLE VII (ENHANCED AT APPENDIX 1)

NON-LINEAR OPTIMIZATION NUMERICAL DATA FOR BIOMEDICAL/MECHANICAL TRIBOLOGY WEAR/EROSION MATHEMATICAL MODELS

MODEL TYPE AND SIMULATION #	NUMBER OF OPTIMIZATION VARIABLES	NUMBER OF RANDOM SIMULATED LAB MEASUREMENTS	SEARCH POINT	OPTIMAL SOLUTION	APPLIED DOUBLE PRECISION
Hutchings	2, k v	50	(0.2,10)	0.0003, 12.0	N
Hutchings	2, k v	100	x=(0.2 ,10)	0.0005 9.9919	N
Hutchings	2, k v	10000	x= (1, 14)	0.0003 13.8600	N
Menguturk and Sverdrup	2, v, angle 25 degrees	10000	X=5	138.4362	N
Menguturk and Sverdrup	2, v, angle 25 degrees	1000	X=5	139.0728	Y
Menguturk and Sverdrup	2, v, angle 25 degrees	10000	X=1	138.3685	N
Menguturk and Sverdrup	2, v, angle 25 degrees	10000	X=4	138.2630	N
hip metal implant	1, k, Hardness 350 MPa	10000	X=5	5.5802e-04	Y
hip metal implant	1, k, Hardness 350 MPa	10000	X=2	5.6087e-04	Y
hip metal-coated implant $C_OC_RM_O$	1, k, Hardness 884 MPa	10000	X=5	0.0014	N
ceramic implant 2300	1, k, Hardness 2300 MPa	10000	X=4	0.0037	N
ceramic implant 2300	1, k, Hardness 2300 MPa	10000	X=7	0.0037	N
ceramic implant 2300	1, k, Hardness 2300 MPa	10000 at interval [1,5]	X=7	0.0368	N
CONCLUSIONS: Initially acceptable optimization results conditioned to further improvements					

Simulation Series 3.-In this subsection global-local minima with a random simulated laboratory measurements are computed in order to obtain a 2D plot series of global minimum visual location. In the same way, a number of imaging optimization pictures are shown with additional comments.

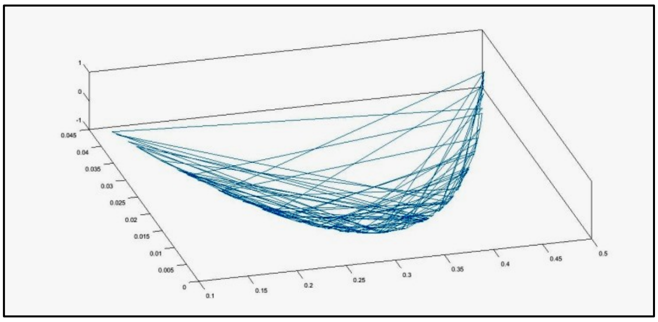

Fig 13.-In this 3D-simulation-program the Hutchings model search for the minimum with random simulations is shown. Since values of simulations are stochastic, the program is joining the curve model points in the neighbourhood of the minimum.

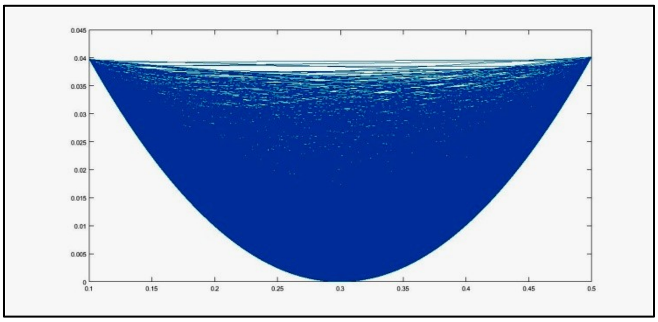

Fig 13.1. Pictured in 2D the Hutchings model with higher number of random points for simulation and in consequence the narrowly complete surface is fill with spiles joining curve points around the minimum.

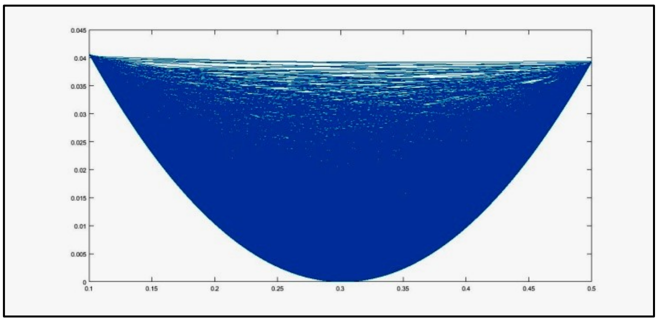

Fig 13.2.-The same technique of Fig 14 but in this case for Eq 14.1 corresponding to hip arthroplasty wear modelling. That is, a local 2D minimum plot of hip arthroplasty wear model [51, Equation 14.1], generated with random values of simulated lab measurements.

Optimization Series 2.-Introduction to Imaging Optimization with Constructive Approximation Graphical Constraints.-This section deals with new practical concepts in approximated/constructive optimization derived from the current available software facilities. It is not unfrequent that lab experimental requires fast calculations, roughly speaking approximated, to try tentative trials or get a quick view of maximum and minimum, usually local, optimal values of a model with/without constraints.

Given the significant improvements of the 3D/2D graphical software and the extensive choice of tools available in the graphics prompts, obtain the maximum of a function in a previously selected range with the simple program and parameters range takes a few seconds.

For example, in previous Fig 7 the approximated local maximum of the function is easily determined by the use of the cursor. Just the same approach for the approximated local minimum determination can be done.

Furthermore, it is possible to try a constructive approximation with straight lines, so settings constraints in 3D. That is, fixed a value for one variable, we drag the cursor in that direction to find the maximum or minimum of the z axis objective function obtaining at the same time the optimal local value of the other axis variable. In other words, we have set an upper graphical constraint for one parameter and set the search for both optimal values in the other parameter and the z-axis objective function.

In Fig 13 an even more evident instance is shown with a radiotherapy dose distribution of radiation dose distribution—from the author's previous publication that has very explicit imaging simulations examples [49]. The global maxima line of the radiation dose distribution is sharply found and just the same occurs for the global minima—minima and maxima are in a line since the distribution in 3D is symmetric. Therefore, to use this method when, for example, we are designing further programs of simulations/optimization and the need is to get a caption of approximate values is suitable and practical—and this happens usually in engineering fast experimental works/trials.

Advantages of this method are a quite series ones, provided that the program for simulation is precise and accurate—and this is a mandatory condition. Not all the laboratory staff are experts in programming and optimization, and instead, as skillfull ones in lab workshop, they can get sharp learning from this graphical method. In addition, it is not necessary to design more optimization codes, e. g., a multiobjective optimization program, to obtain a local minimum for any selected interval.

Other significant advantage is the fact, provided the accuracy of the simulation, that to use a searching-optimization program could yield wrong results from an inconsistent choice of the initial search, and a number of additional mathematical reasons. However, using the graph, if there are several concavities in the objective function surface, to locate the local minimum is visually fast/precise instead.

Definitely, the constant progress/improvements in software for simulations or optimization graphs justify this usage for practical engineering trials and experimental. Disadvantages of this method exist obviously, since it is an approximated method, and are the simplification/approxiamtion of data obtained, and the limitation of the function to 3D within a closed interval range of parameters. In conclusion, it is suggested this method for fast and visual optimization with simple computational programming and convenient tools at prompt.

To prove graphically-computationally these assertions about graphical optimization, some radiotherapy images developed but different from previous publications [49], are shown in the following to put forward the argument. That is, with Matlab software, for instance, the cursor can determine the optimal point of intervals, but with Freemat it is not possible, only available to get a general overview of the surface objective function and guess the regions of maxima and minima. Nevertheless, imaging both in Matlab and Freemat is overall acceptable and good for their respective subroutines.

In the following, a series of figures with 2D and 3D plots are shown and commented. No matter that it is a radiation therapy problem, what is meant is the usage advantages/possibilities of this type of simulations/optimization.

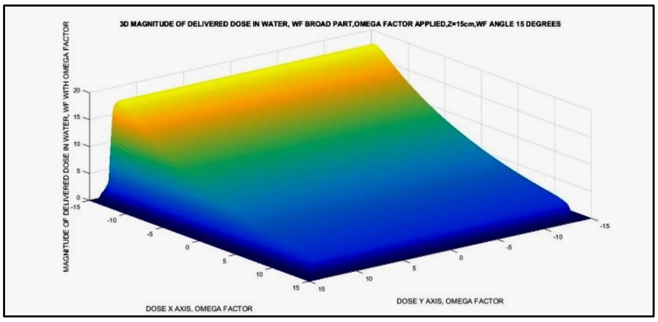

Fig 13.3.-A 3D objective symmetric-function radiotherapy photon-dose surface with clear determination of the straights marginal lines of global minima/maxima in radiation delivery dose distribution. This simulation is done with Matlab 2016 version and similar imaging computing-simulations were developed in previous publications. Cursor can reach and identify maxima, minima, and regions of interest, among other features.

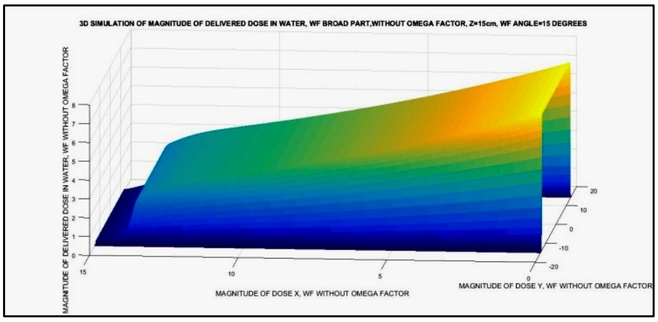

Fig 14.-A different algorithm and varied angle projection of 3D objective symmetric-function radiotherapy photon-dose surface with clear determination of the straights marginal lines of global minima/maxima in radiation delivery dose distribution. This simulation is done also with Matlab 2016 version—the most modern 2016, although highly-significant differences compared to previous ones are not strongly evident.

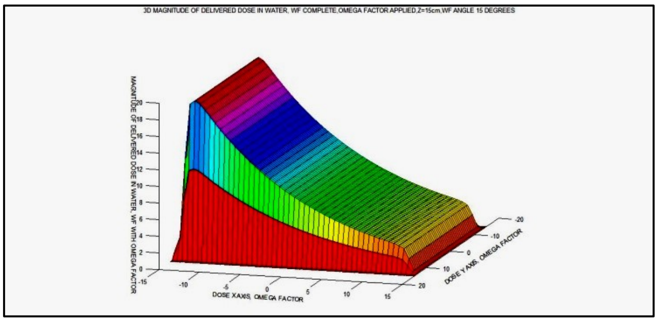

Fig 15.-A Freemat 4.1 (Samit Basu General Public License), 3D objective symmetric-function of radiotherapy photon-dose surface with clear determination of the straight marginal lines of global minima/maxima in radiation delivery dose distribution. Matrices are 50 x 50, and with Freemat 4.1 the running time is longer than Matlab—at the same time the matrices size for normal running-time is lower in standard microprocessors. This simulation is Freemat original based on previous computational contributions [49].For 50 x 50 matrices imaging view setting takes about 2 seconds, and spatial-changes of imaging-set about 4 seconds.

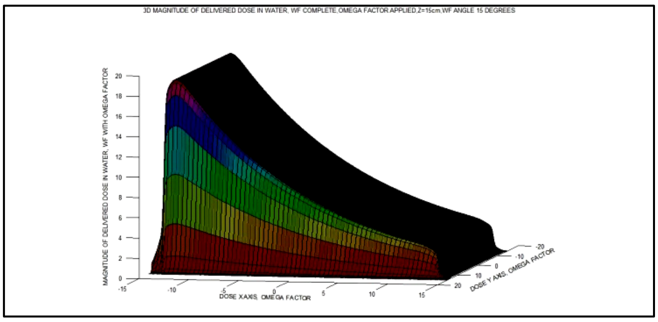

Fig 16.- Pictured, a Freemat 4.1 (Samit Basu General Public License), 3D objective symmetric-function of radiotherapy photon-dose surface with clear determination of the straight marginal lines of global minima/maxima in radiation delivery dose distribution. Matrices are in this case 350 x 350—running time about 5-6 seconds. This simulation is also Freemat4.1 original based on previous computational contributions [49].For 350 x 350 matrices imaging view setting takes about 6 seconds, and spatial-changes of imaging-set about 10 seconds. The sharpness and contrast are excellent with this subroutine.

Brief Notes of Functional Programming applications in Computational Simulations.-In this subsection some comments about F# programming advantages and inconvenients are detailed to put forward an even more wide software options available at present—according to a high-quality selection in this study, about 17 types of programs/languages are among the best today. Functional programming, [65], shows a sharp property, namely, the fast connection and implementation of large web load of data mainly through visual studio workstation. On the contrary, this could be a disadvantage given the current cyber-security frequent problems at net.

A large number of packages for complementary developments are linked to visual studio resources, but again it means that, when programming, the development of software is not kept totally safe from cyber-security events that could occur.

Specific programming development in F# shows a reduction of complicated number of patterns, loops, and sentences compared to other computational languages, although definition borders and settings for this language are not as accurate as other classical languages, e. g., modern FORTRAN in numerical methods. As a casual comment of these contributors for our long experience in software programming during 27 years, fashion/marketing in computer science constitute a hurdle to be taken seriously into account, to ensure the high-quality standars of the scientific advances. In other words as a rule, , not all the new, more expensive, or nicely-sophisticated in programs/languages could be considered the best for a change of usage methods.

In the following, Figs 17 and 18, a brief code in F# for charting a simple simulation is shown, for a hip classical model, and a mathematical algorithm for prime numbers is executed. The use of the package is easy and fastly-implemented, although graphics are limited in several features. Definitely, according to [46], researchers are invited to judge objectively by themselves what they need and for what they want the computational tools/languages available.

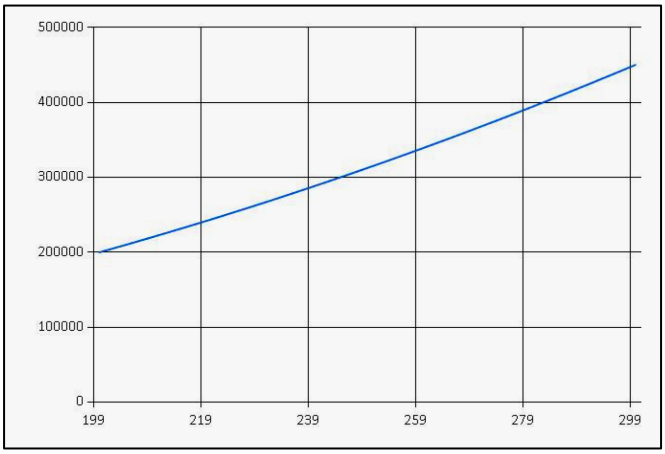

Fig 17.-A F# chart developed with functional programming software in visual studio for a Hutchings erosion model simulation.It is seen sharply the good image given by the compiler, although other types of programming software facilities could make better and faster plots without downloading chart-F# specific packages. This program was developed in F# by the authors originally [Casesnoves,2016].

```
//Program to generate prime numbers it is practical tu use the classical Euler algorithm for
prime numbers
//given in polynomial form, n**2+n+41=primes sequence, that was deetrmined by Euler as one of
the simplest and earliest
//algorithm for primes, w euse seq subroutine

let primes = seq { for i in  0 .. 15 -> (i,i*i+i+41) } ;;

seq {for (i,iPrimes) in primes -> (i,iPrimes) };;

//this Euler algorithm works right, that is, 43,47,53 are sequenced primes

//Program, we use this ( I studied several webs/books to develop and changing the information
for this exercises)

open System

let rec sieve = function
    | (p::xs) -> p :: sieve [ for x in xs do if x % p > 0 then yield x ]
    | []      -> []

let primes = sieve [2..100]
```

Fig 18.-A F# simple code to generate prime numbers with classical Euler algorithm. It was developed with functional programming software in visual studio for model simulation.It is seen sharply the good image given by the prompt interactive, although other types of programming software facilities could make better windows and faster debugs. This program was developed in F# by the authors originally [Casesnoves,2016].

IX. DISCUSSION AND CONCLUSIONS

A discussion of current wear, E/C models in Biomedical and Mechanical Tribology with a simplified classification has been presented -proposed by authors. This categorization is based on practical applications and functionality. The most important models have been explained, setting advantages, combinations, particular details, and weaknesses. Optimization methods to fit models and mathematical models in general have been explained succinctly. The essential probabilistic-statistical notions and formulation of Engineering Reliability for Mechanical Systems have been mathematically linked to the E/C conceptual models in concise lemmas and formulation.

A large series of computational simulations an d optimization with up-to-date computational software have been developed and presented extensively in 2D, 3D imaging systems and numerical tables. The illustration of all the first part of the paper with realistic values, optimization subroutines and imaging simulations in 2D is considered appropriate and acceptable. Examples with other different languages, such as functional programming, were also specified for sharp learning of options available today.

Taking a caption of the wide variety of Model-Formulation presented, it is in focus to guess how complicated is to obtain well-optimized and generalized models for Biomedical and Mechanical Tribology, wear and E/C. The reasons for this rather significant mathematical-empirical hurdle have been put forward along this contribution. However, what could be obtained is an improvement of the current models towards a continuous generalization process and get programming/experimentally more general models of Type1. Future implementation of this kind of techniques of optimization, as explained in the simple numerical example comprises the same application for other models with larger number of parameters, such angle, particle density, etc.

Finally and to summarize, it was presented a series computational-designed example with a selected models for erosion optimization –indicative computational illustration. The variables have been determined with software specially designed in a classical/acceptable Nonlinear Optimization subroutine, in Raphson method. It can be considered as an E/C modelling-programming instance, not given by automatic software subroutines, for future contributions to be developed/published.

X. ACKNOWLEDGEMENTS AND SCIENTIFIC ETHICS STANDARDS

TUT is gratefully acknowledged for all the facilities for research. This study was carried out, and their contents are done according to the European Union Technology and Science Ethics. Reference, 'European Textbook on Ethics in Research'. European Commission, Directorate-General for Research. Unit L3. Governance and Ethics. European Research Area. Science and Society. EUR 24452 EN. This study was completely done by the authors, the software, calculations, images, mathematical propositions and statements, reference citations, and text is original for the author. This article contains also unique numerical data and special new-improved images. The principal sketches were made originally, and the figures, tables, or data that corresponds/developed-from previous papers is properly clarified. When anything is taken from a source, it is adequately recognized [58].

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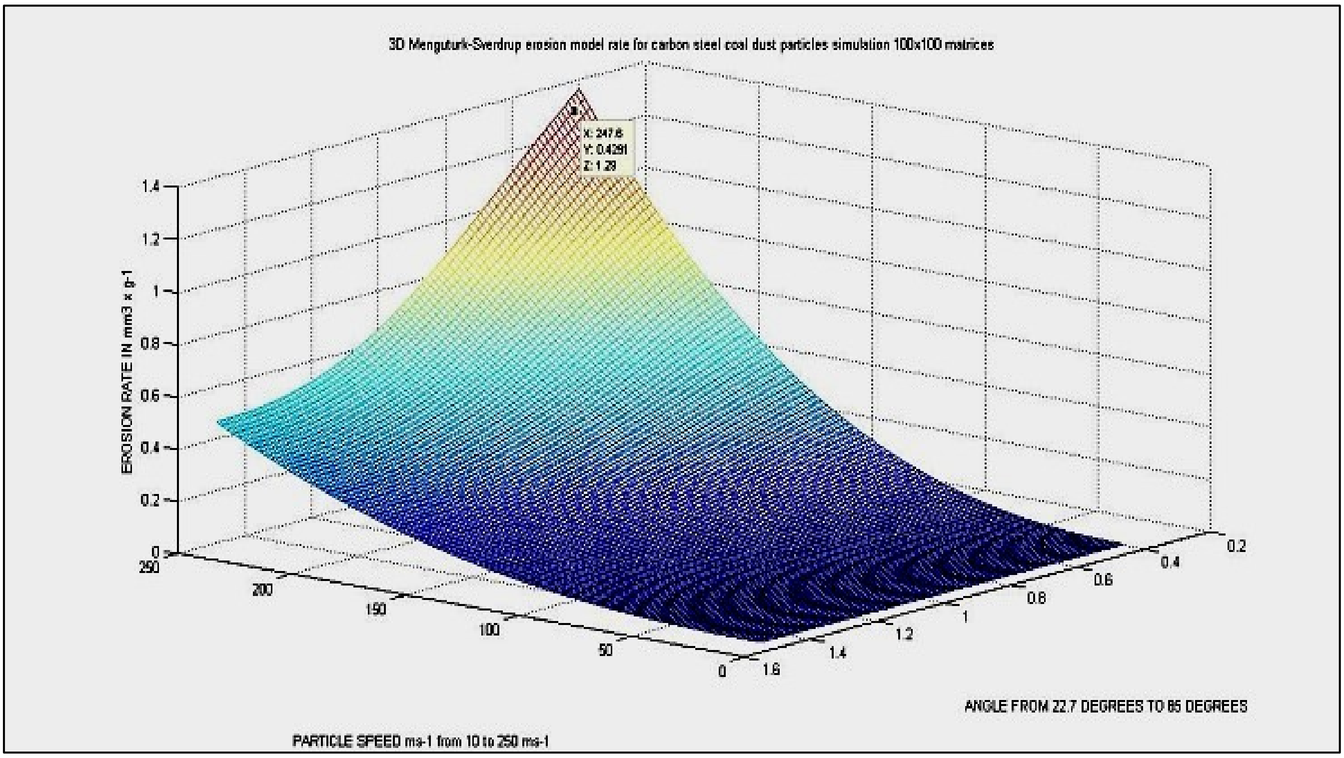
XII. APPENDICES

APPENDIX 1, ENHANCED IMAGES OF TABLE AND FIGURE VII

NON-LINEAR OPTIMIZATION NUMERICAL DATA FOR BIOMEDICAL/MECHANICAL TRIBOLOGY WEAR/EROSION MATHEMATICAL MODELS					
MODEL TYPE AND SIMULATION #	NUMBER OF OPTIMIZATION VARIABLES	NUMBER OF RANDOM SIMULATED LAB MEASUREMENTS	SEARCH POINT	OPTIMAL SOLUTION	APPLIED DOUBLE PRECISION
Hutchings	2, k v	50	(0.2,10)	0.0003, 12.0	N
Hutchings	2, k v	100	x=(0.2 ,10)	0.0005 9.9919	N
Hutchings	2, k v	10000	x= (1, 14)	0.0003 13.8600	N
Menguturk and Sverdrup	2, v, angle 25 degrees	10000	X=5	138.4362	N
Menguturk and Sverdrup	2, v, angle 25 degrees	1000	X=5	139.0728	Y
Menguturk and Sverdrup	2, v, angle 25 degrees	10000	X=1	138.3685	N
Menguturk and Sverdrup	2, v, angle 25 degrees	10000	X=4	138.2630	N
hip metal implant	1, k, Hardness 350 MPa	10000	X=5	5.5802e-04	Y
hip metal implant	1, k, Hardness 350 MPa	10000	X=2	5.6087e-04	Y
hip metal-coated implant $C_OC_RM_O$	1, k, Hardness 884 MPa	10000	X=5	0.0014	N
ceramic implant 2300	1, k, Hardness 2300 MPa	10000	X=4	0.0037	N
ceramic implant 2300	1, k, Hardness 2300 MPa	10000	X=7	0.0037	N
ceramic implant 2300	1, k, Hardness 2300 MPa	10000 at interval [1,5]	X=7	0.0368	N
CONCLUSIONS: Initially acceptable optimization results conditioned to further improvements					


3D Menguturk-Sverdrup erosion model rate for carbon steel coal dust particles simulation 100x100 matrices
X: 247.6
Y: 0.4291
Z: 1.28
EROSION RATE IN mm3 x g-1
1.4
1.2
1
0.8
0.6
0.4
0.2
0
250
200
150
100
50
0
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
ANGLE FROM 22.7 DEGREES TO 85 DEGREES
PARTICLE SPEED ms-1 from 10 to 250 ms-1