

# Implementation of a Native Regularization Scheme Named As Fantope for Similarity Search

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## ABSTRACT

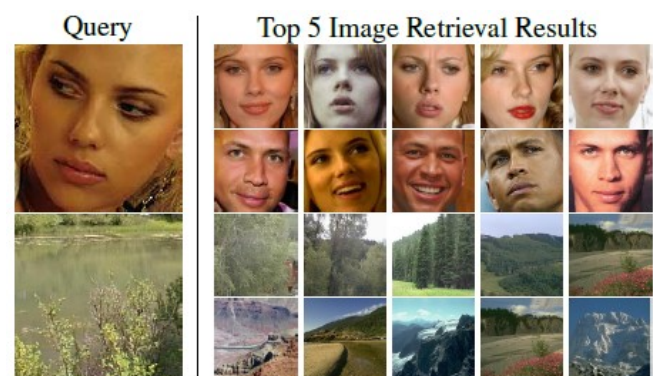
This paper introduces a regularization method to explicitly control the rank of a learned symmetric positive semidefinite distance matrix in distance metric learning. To this end, we propose to incorporate in the objective function a linear regularization term that minimizes the  $k$  smallest eigenvalues of the distance matrix. It is equivalent to minimizing the trace of the product of the distance matrix with a matrix in the convex hull of rank- $k$  projection matrices, called a Fantope. Based on this new regularization method, we derive an optimization scheme to efficiently learn the distance matrix. We demonstrate the effectiveness of the method on synthetic and challenging real datasets of face verification and image classification with relative attributes, on which our method outperforms state-of-the-art metric learning algorithms.

**Keywords:** Distance Metric, Mahalanobis Distance Metric, Metric Learning Fantope Regularization, Ky Fan's Theorem

## I. INTRODUCTION

Distance metric learning is useful for many Computer Vision tasks, such as image classification, retrieval or face verification. It emerges as a promising learning paradigm, in particular because of its ability to learn with attributes. Metric learning algorithms produce a linear transformation of data which is optimized to fit semantical relationships between training samples. Different aspects of the learning procedure have recently been investigated: how the dataset is annotated and used in the learning process, e.g. using pairs, triplets or quadruplets of samples; design choices for the distance parameterization; extensions to large scale context, etc. Surprisingly, few attempts have been made for deriving a proper regularization scheme, especially in the Computer Vision literature. Regularization in metric learning is however a critical issue, as it often limits model complexity, the number of independent parameters to learn, and thus overfitting. Models learned with regularization usually better exploit correlations between features and often have improved predictive accuracy. In this thesis, we propose a novel regularization approach for metric

learning that explicitly controls the rank of the learned distance matrix.



**Figure 1.** The relevance of our approach

Above figure illustrates the relevance of our approach. We present retrieval results after metric learning with the proposed method, and provide an illustrative comparison with LMNN algorithms.

## II. Brief Literature Survey

Image representation for classification has been deeply investigated in recent years [3, 7]. The traditional Bag-of-Words representation [10] has been extended for the coding step [14] as well as for the pooling [1], or with bioinspired models [9, 11]. Nonetheless, similarity

metrics are also crucial to compare, classify and retrieve images. We focus in this work on supervised distance metric learning methods. Some of them consider sets of similar and dissimilar pairs of images for training [4, 6, 13]. They learn a distance metric that preserves distance relations among the training data. Other methods consider triplets [2, 5, 8, 12] of images, which are easy to generate in classification. For instance, LMNN [12] learns a distance metric for k-Nearest Neighbors (k-NN) approach using those triplet-wise training sets. In this thesis, we will consider the widely used Mahalanobis distance metric  $DM$  that is parameterized by the PSD matrix  $M$  and investigate a new optimization scheme with a regularization term that explicitly controls the rank of  $M$ . Such a scheme allows to avoid overfitting without any trick such as early stopping.

The main contributions of this thesis will be:

- We introduce a new regularization strategy based on the convex hull of rank- $k$  projection matrices, called Fantope, which allows to explicitly control the rank of distance matrices.
- We propose an efficient algorithm to solve the new optimization scheme.
- Our framework outperforms state-of-the-art metric learning methods on synthetic and challenging real Computer Vision datasets.

### III. Problem Formulation

Metric learning Fantope regularization (Objective function): a metric learning algorithm aims at determining  $M$  such that the metric satisfies most of the constraints defined by the training information. It is generally formulated as an optimization problem of the form:

$$\min_M \mu R(M) + l(M, A)$$

where  $l(M, A)$  is a loss function that penalizes constraints that are not satisfied,  $R(M)$  is a regularization term on the parameter  $M$  of the metric, and  $\mu \geq 0$  is the regularization parameter.  $l(M, A)$  measures the ability of the matrix  $M$  to satisfy some distance constraints given in the training set. The type of constraints depends on the way relationships between training samples are provided, e.g. relations between pairs, triplets, quadruplets [13] etc. In thesis,

we focus on defining an effective regularization term  $R(M)$ .

## IV. METHODOLOGY

We evaluate the proposed metric learning regularization method in two different Computer Vision applications. The first experiment is a face verification task, for which the similarity constraints come from relations between pairs of face images that are either similar or dissimilar. In the second experiment, we evaluate recognition performance on image classification with relative attributes. In this context, we work with features defined in attribute space. Algorithm 1 shows the metric learning with Fantope Regularization.

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**Algorithm 1** Metric Learning with Fantope Regularization

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**input** : Training constraints  $\mathcal{A}$ , hyper-parameter  $\mu$  and step size  $\eta > 0$ .

**output** :  $M \in \mathbb{S}_+^d$

Initialize  $M \in \mathbb{S}_+^d$ ,  $W \leftarrow V_M \text{Diag}(w) V_M^T$  (Eq. (7))

**repeat**

  Compute  $\nabla_M$  (Eq. (12))

$M \leftarrow \Pi_{\mathbb{S}_+^d}(M - \eta \nabla_M)$

$W \leftarrow V_M \text{Diag}(w) V_M^T$  (Eq. (7))

**until** stopping criterion (e.g. convergence)

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## V. RESULTS AND DISCUSSION

We now provide a quantitative evaluation of our method in the described setup. The target rank  $e$  of our regularization term is fixed to  $e = 40$ , as in [18].

**Impact of regularization:** we compare here the impact of Fantope regularization over trace regularization. Table 1 shows classification accuracies when solving Eq. with both regularization methods. Fantope regularization outperforms trace regularization by a large margin (82.3% vs. 77.6%). This illustrates the importance of having an explicit control on the rank of the distance matrix.

TABLE 2 : ACCURACIES OBTAINED ON LFW IN THE “RESTRICTED” SETUP WITH OUR LEARNING FRAMEWORK IN DIFFERENT REGULARIZATION SETTINGS.

Regularization Method	Accuracy (in %)
Trace-norm Regularization	77.6 ± 0.7
Fantope Regularization	82.3 ± 0.5

**State-of-the-art results:** we now compare Fantope Regularization to other popular metric learning algorithms. Table 3 shows performances of ITML [6], LDML [10] and PCCA [18] reported in [10] and [18] in the linear metric learning setup. These methods are the most popular metric learning methods when the task is to decide whether a pair is similar or dissimilar. Fantope regularization, which reaches  $82.3 \pm 0.5\%$  accuracy, outperforms ITML and LDML and is comparable to PCCA on LFW in this setup. We explain in the following how our method can reach  $83.5 \pm 0.5\%$ .

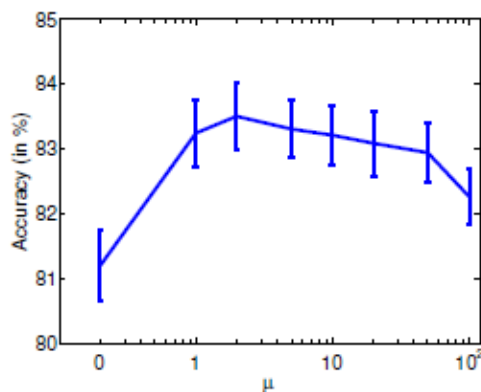
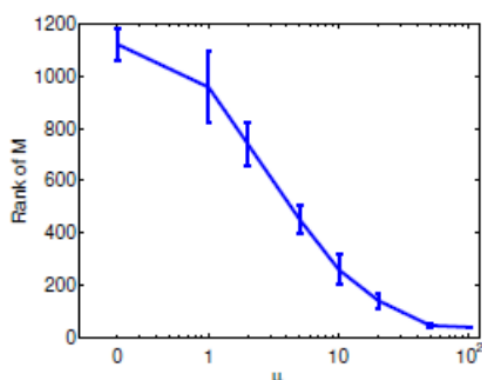
TABLE 3 : RESULTS (MEAN AND STANDARD ERROR) ON LFW IN THE “RESTRICTED” SETUP OF STATE-OF-THE-ART LINEAR METRIC LEARNING ALGORITHMS AND OF OUR METHOD WITH EARLY STOPPING.

Method	Accuracy (in %)
ITML [10]	$76.2 \pm 0.5$
LDML [10]	$77.5 \pm 0.5$
PCCA [18]	$82.2 \pm 0.4$
Proposed Method	$83.5 \pm 0.5$

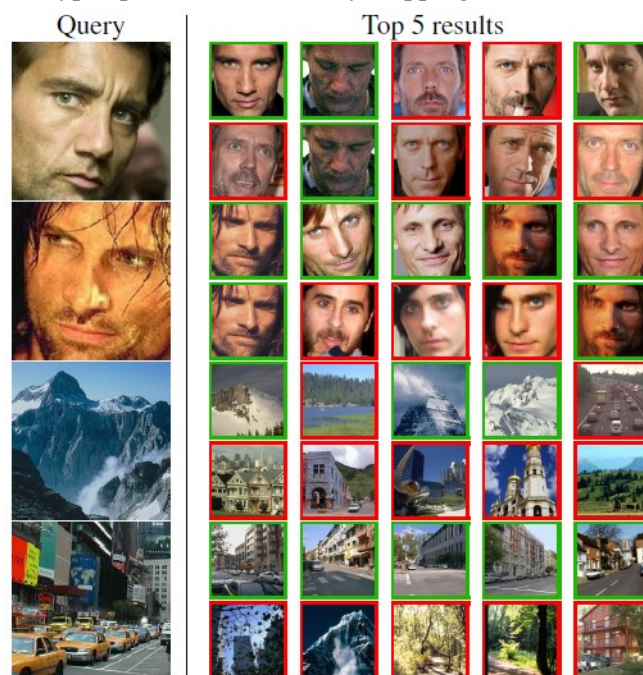
TABLE 4 : ACCURACY OF MIGNON’S CODE [18] ON LFW AS A FUNCTION OF THE NUMBER OF ITERATIONS OF GRADIENT DESCENT. THE PERFORMANCE OF PCCA [18] GREATLY DEPENDS UPON THE EARLY STOPPING CRITERION

Number of iterations	10	100	1000	$10^4$
Accuracy (in %)	$79.2 \pm 0.5$	$79.3 \pm 0.5$	$75.8 \pm 0.5$	$63.2 \pm 0.5$

**Impact of the hyper-parameter:** Fig. 2 illustrates the impact of the Fantope regularization:



**Figure 2.** (left) rank and (right) accuracy of the learned metric on LFW in the “restricted” setup as a function of the hyper-parameter with early stopping.



**Figure 3.** Some results of similarity search on the PubFig and OSR datasets. We show for each query the 5 nearest neighbors returned by our method (first row) and by LMNN (second row).

Results in green correspond to images in the same class as the query whereas results in red are images from different classes.

## VI. CONCLUSION

We proposed a new regularization scheme for metric learning that explicitly controls the rank of the learned distance matrix. Our method generalizes the trace regularization, and can be applied to various optimization frameworks to impose a meaningful structure on the learned PSD matrix. We also derived an efficient metric learning algorithm that combines the

regularization term with a loss function that can incorporate constraints between pairs or triplets of images. We also demonstrate that regularization greatly improves recognition on both controlled and real datasets, showing the relevance of this new regularization to limit overfitting. Future work includes the learning of a better designed ADMM formulation scheme that takes into account the fact that the objective function is not convex.

## VII. REFERENCES

- [1] S. Avila, N. Thome, M. Cord, E. Valle, and A. de A. Arajo. Pooling in image representation: The visual codeword point of view. *Computer Vision and Image Understanding (CVIU)*, 117(5):453 – 465, 2013. 2
- [2] S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein. Distributed optimization and statistical learning via the alternating direction method of multipliers. *Foundations and Trends in Machine Learning*, 3(1):1–122, 2011. 4
- [3] G. Chechik, V. Sharma, U. Shalit, and S. Bengio. Large scale online learning of image similarity through ranking. *JMLR*, 11:1109–1135, 2010. 1, 2
- [4] M. Cord and P. Cunningham. *Machine learning techniques for multimedia*. Springer, 2008. 2
- [5] J. Dattorro. *Convex optimization and Euclidean distance geometry*. Meboo Publishing USA, 2005. 3
- [6] J. V. Davis, B. Kulis, P. Jain, S. Sra, and I. S. Dhillon. Information-theoretic metric learning. In *ICML*, 2007. 2, 3, 6
- [7] K. Fan. On a theorem of weyl concerning eigenvalues of linear transformations. *Proceedings of the National Academy of Sciences of the United States of America*, 35(11):652, 1949. 3
- [8] A. Frome, Y. Singer, F. Sha, and J. Malik. Learning globally-consistent local distance functions for shape-based image retrieval and classification. In *ICCV*, 2007. 1, 2
- [9] H. Goh, N. Thome, M. Cord, and J. Lim. Unsupervised and supervised visual codes with restricted boltzmann machines. In *ECCV*, 2012. 2
- [10] M. Guillaumin, J. Verbeek, and C. Schmid. Is that you? metric learning approaches for face identification. In *ICCV*, 2009. 1, 6
- [11] G. B. Huang, M. Ramesh, T. Berg, and E. Learned-Miller. Labeled faces in the wild: A database for studying face recognition in unconstrained environments. Technical Report 07-49, University of Massachusetts, Amherst, 2007. 6
- [12] N. Kumar, A. Berg, P. Belhumeur, and S. Nayar. Attribute and simile classifiers for face verification. In *ICCV*, 2009. 7
- [13] M. T. Law, N. Thome, and M. Cord. Quadruplet-wise image similarity learning. In *ICCV*, 2013. 1, 2, 3
- [14] D. Lim, B. McFee, and G. Lanckriet. Robust structural metric learning. In *ICML*, 2013. 1, 2, 4
- [15] D. Lowe. Distinctive image features from scale-invariant keypoints. *IJCV*, 60(2):91–110, 2004. 6
- [16] B. McFee and G. Lanckriet. Metric learning to rank. In *ICML*, 2010. 2
- [17] T. Mensink, J. Verbeek, F. Perronnin, and G. Csurka. Distance-based image classification: Generalizing to new classes at near-zero cost. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 35(11):2624–2637, 2013. 1, 2, 5
- [18] A. Mignon and F. Jurie. Pcca: A new approach for distance learning from sparse pairwise constraints. In *CVPR*, 2012. 1, 2, 3, 5, 6
- [19] A. Oliva and A. Torralba. Modeling the shape of the scene: A holistic representation of the spatial envelope. *IJCV*, 42(3):145–175, 2001. 2, 7
- [20] D. Parikh and K. Grauman. Relative attributes. In *ICCV*, 2011. 1, 5, 7, 8
- [21] M. Schultz and T. Joachims. Learning a distance metric from relative comparisons. In *NIPS*, 2003. 1, 2
- [22] T. Serre, L. Wolf, S. Bileschi, M. Riesenhuber, and T. Poggio. Robust object recognition with cortex-like mechanisms. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 29(3):411–426, 2007. 2
- [23] C. Shen, J. Kim, L. Wang, and A. van den Hengel. Positive semidefinite metric learning with boosting. In *NIPS*, 2009. 2
- [24] J. Sivic and A. Zisserman. Video google: A text retrieval approach to object matching in videos. In *ICCV*, 2003. 2
- [25] C. Theriault, N. Thome, and M. Cord. Extended coding and pooling in the hmax model. *IEEE Transactions on Image Processing*, 22(2):764–777, 2013.
- [26] K. Weinberger and L. Saul. Distance metric learning for large margin nearest neighbor classification. *JMLR*, 10:207–244, 2009. 1, 2, 7
- [27] E. Xing, A. Ng, M. Jordan, and S. Russell. Distance metric learning, with application to clustering with side-information. In *NIPS*, 2002.
- [28] J. Yang, K. Yu, Y. Gong, and T. Huang. Linear spatial pyramid matching using sparse coding for image classification. In *CVPR*, 2009.