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# Intuitionistic Fuzzy Orienteering Problem and Its Work-Depth Analysis

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## ABSTRACT

Orienteering is an NP-hard problem that originated from a water sport where a player is required to visit a set of control points connecting the source and the destination, collect the maximum possible rewards or scores associated with the control points and arrive at the destination within the time bound. It finds its application in the tourism industry, telecommunication networks and other computational problems where things like human behaviour and hesitancy of the decision maker must be considered. To tackle the uncertainty involved in the parameters we represent them using trapezoidal intuitionistic fuzzy numbers (TIFN) resulting in intuitionistic fuzzy orienteering problem (IFOP). A technique based on max-min formulation is presented to deal with IFOP using a new method for ranking TIFNs. Also, a work-depth analysis for the parallel version of IFOP is presented to show that IFOP is work-preserving and can be implemented on a multiprocessor model like PRAM to obtain the solution for large instances efficiently.

**Keywords:** Centroid of Centroids, Fuzzy Optimization, Intuitionistic Fuzzy Orienteering Problem, Orienteering Problem, Trapezoidal Intuitionistic Fuzzy Number.

## I. INTRODUCTION

The orienteering problem (OP), which is a mixture of the two well-known problems of combinatorial optimization i.e. the travelling salesman problem (TSP) and the knapsack problem (KP), is NP-Hard. This problem originated from a game where the player has to visit a set of control points connecting the source and the destination within a limited time budget and collect the maximum rewards possible. A lot of real life situations and applications from several fields like logistics, home delivery systems, tourism, building telecommunication networks etc. can be depicted in the form of OP. Several types of OP have been discussed in the literature which includes the team orienteering problem and the simple orienteering problem and a variation of both with time windows [1].

As stated before, the two important parameters associated with OP are time and score, both of which are imprecise in nature and cannot be determined exactly. The best way to tackle the prevailing vagueness is to represent the parameters using fuzzy numbers. Here we prefer intuitionistic fuzzy numbers (IFN) over fuzzy numbers because in OP the endeavor is to obtain a path that helps in achieving the maximum rewards or scores within the specified time bound but in practice, the areas where this problem finds its application, considers the human behavior, knowledge etc. like in tourism industry which leads to uncertainty in determining the values of the two parameters i.e. score and time. The most desirable method of handling these circumstances of insufficient information and lack of precision and certainty is to model the parameters using IFN. In this paper, we model the two parameters (score and time) using trapezoidal intuitionistic fuzzy numbers

(TIFN). TIFNs have been successfully used in several decision-making, applied engineering and scientific problems [2].

A number of heuristics have been proposed to solve the crisp OP. Also, a few approximation algorithms have been stated in the literature that considers this problem. The first heuristic for OP was suggested by Tsiligirides in 1984 [3]. Since then, a lot of heuristics were proposed by Golden et al, Ramesh and Brown, Wang et al and Chao et al [4]-[7]. The other approaches presented to solve OP include the genetic algorithm, tabu search and ant colony optimization suggested by Tasgetiren, Gendreau et al and Liang et al respectively [8]-[10]. Fischetti et al stated a branch and cut heuristic for OP in 1998 and in 2009, two techniques called the pareto ant colony optimization algorithm and the variable neighborhood search algorithm were proposed by Schilde et al for the multi-objective variant of OP [11]-[12]. A Greedy Randomized Adaptive Search Procedure for solving OP was proposed by Campos et al in 2012 [13]. Blum et al suggested a constant factor approximation for the rooted version of OP [14] and Johnson et al proposed an approximation algorithm for the un-rooted version of OP [15].Another approximation algorithm for the time dependent variant of OP was presented by Fomin et al in 2002 [16].

In section II, some necessary definitions are stated and section III, presents a brief description about fuzzy optimization. The mathematical representation of IFOP and the steps of IFOP algorithm is described in section IV and section V respectively. An illustrative example is presented in section VI. In section VII, a work-depth analysis of IFOP is explained. Finally, the paper is concluded in section VIII.

## II. PRE-REQUISITES

### A. Trapezoidal Intuitionistic fuzzy numbers (TIFN)

In 1986 [17], Atanassov introduced the concept of intuitionistic fuzzy set which is an extension of the Zadeh's fuzzy set where two values are associated with every element of the set, one depicting the degree of belongingness and the other being the degree of non-belongingness. Both these values lie within the real unit interval [0, 1] [2].

An intuitionistic fuzzy set X in U where U is the universe of discourse can be represented as [2]

$$X = \{\langle x, \mu_X(x), \nu_X(x)\rangle : x \in U\}$$

such that

$$0 \le \mu_X(x) + \nu_X(x) \le 1 \ \forall x \in U \qquad (1)$$

Another term that can be linked with every element x in the set X is called the hesitancy degree of x to X and can be defined in the following way:

$$\pi_X(x) = 1 - \mu_X(x) - \nu_X(x)$$

such that

$$0 \le \pi_X(x) \le 1 \, , x \in U$$

In this paper, we use an IFN for which the real line is the universe of discourse i.e. $U = \mathcal{R}$ and can be stated as $X = \{\langle x, \mu_X(x), \nu_X(x)\rangle : x \in \mathcal{R}\}$. An IFN X possess the following properties [18]:

a. The membership function and the non-membership function is fuzzy convex and fuzzy concave respectively (if-convex).
b. There exists at least two points $x_1$ and $x_2$ in U such that $\mu_X(x_1) = 1$ and $\nu_X(x_2) = 1$
c. The membership function ($\mu_X$) and the non-membership function ($\nu_X$) is upper semicontinuous and lower semicontinuous respectively.

According to the above definition, TIFN A can be represented using eight numbers $A = \langle (a, b, c, d), (e, f, g, h)\rangle$ where $a, b, c, d, e, f, g, h \in \mathcal{R}$ such that $e \le a \le f \le b \le c \le g \le d \le h$ and the functions $L_A, M_A, N_A, K_A : \mathcal{R} \to [0,1]$. The definition of the membership function and a non-membership function of A is as follows [2]:

$$\mu_A(x) = \begin{cases} 0 & \text{if } x < a \\ L_A(x) & \text{if } a \le x < b \\ 1 & \text{if } b \le x \le c \\ M_A(x) & \text{if } c < x \le d \\ 0 & \text{if } d < x \end{cases} \qquad (2.a)$$

$$\nu_A(x) = \begin{cases} 1 & \text{if } x < e \\ N_A(x) & \text{if } e \le x < f \\ 0 & \text{if } f \le x \le g \\ K_A(x) & \text{if } g < x \le h \\ 1 & \text{if } h < x \end{cases} \qquad (2.b)$$

Where $L_A(x) = \frac{x-a}{b-a}$, $M_A(x) = \frac{x-d}{c-d}$, $N_A(x) = \frac{x-f}{e-f}$, $K_A(x) = \frac{x-g}{h-g}$. If $b = c$ and $f = g$ then the trapezoidal intuitionistic fuzzy number is reduced to a triangular intuitionistic fuzzy number.

## B. Addition of TIFN

Two TIFN $A_1 = \langle (a_1, b_1, c_1, d_1), (e_1, f_1, g_1, h_1) \rangle$ and $A_2 = \langle (a_2, b_2, c_2, d_2), (e_2, f_2, g_2, h_2) \rangle$ can be added using the following formula [2]:

$$A_1 + A_2 = \langle (a_1, b_1, c_1, d_1), (e_1, f_1, g_1, h_1) \rangle + \langle (a_2, b_2, c_2, d_2), (e_2, f_2, g_2, h_2) \rangle = \left\langle \begin{matrix} (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2), \\ (e_1 + e_2, f_1 + f_2, g_1 + g_2, h_1 + h_2) \end{matrix} \right\rangle \qquad (3)$$

## C. Expected Value of TIFN

To determine the degree up to which the constraints of the problem are satisfied by the TIFN (representing either the total score or the total time taken), we need to determine the expected value of TIFN using the below stated formula [2]:

For a given TIFN $A = \langle (a, b, c, d), (e, f, g, h) \rangle$

$$EV(A) = \frac{1}{8}(a + b + c + d + e + f + g + h)$$
(4)

## D. Ranking of TIFN

To rank the TIFN, we introduce a technique called Centroid of Centroids (CoC). The centroid of a fuzzy number signifies its geometric centre and is denoted using the formula: $\int_{-\infty}^{\infty} xf(x)dx / \int_{-\infty}^{\infty} f(x)dx$. A trapezoid can be divided into three figures (two triangles and a rectangle) and finding out the centroid of each and joining them forms a triangle. The centroid of this resultant triangle can be considered to be a balancing point and a better point of reference. As the task here is to rank a trapezoidal intuitionistic fuzzy number, we evaluate the centroid for both the trapezoids using the following formula and the figure shown below:

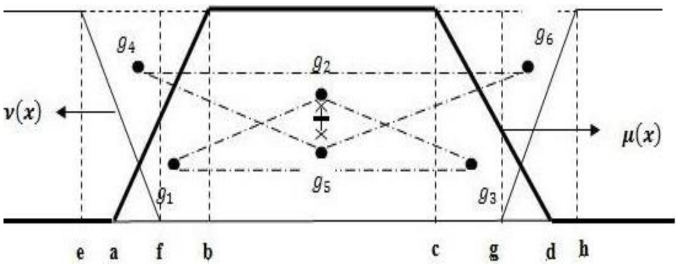



**Figure 1.** The point of reference used for ranking a TIFN

The centroid of the triangle $g_1g_2g_3(C_1)$

$$C_1 = (x_1, y_1) = \left[\frac{(2a+b+7c+2d)}{18}, \frac{7}{18}\right] \qquad (5)$$

The centroid of the triangle $g_4g_5g_6(C_2)$

$$C_2 = (x_2, y_2) = \left[\frac{(2e+f+2h+7g)}{18}, \frac{11}{18}\right] \qquad (6)$$

Then the rank of the TIFN can be calculated using the following formula:

$$\text{Rank (R)} = \sqrt{\left(\frac{x_1+x_2}{2}\right)^2 + \left(\frac{y_1+y_2}{2}\right)^2} \qquad (7)$$

## E. Fuzzy Decision Set ($Z$)

The fuzzy version of the problem under consideration, when formulated as an integer programming problem, can have several goals each of which can be depicted as a membership function and a fuzzy set ($F_i$) consisting of the elements along with their membership values [19]. The set comprising of the feasible solution elements is called a fuzzy decision set which is as follows:

$Z = F_1 \cap F_2 \cap \ldots \ldots \ldots \cap F_i$

$$\text{i.e. } \mu_Z(x) = \mu_{F_1}(x) * \mu_{F_2}(x) * \ldots \ldots \ldots \ldots . * \mu_{F_i}(x) \quad (8)$$

Here, $*$ is a $t-$norm denoting any operation like algebraic product, minimum etc. and for the stated problem $*$ represents the minimum operation. The element in the fuzzy decision set Z with the highest membership value is the most desirable solution represented by the set $Z^*$ as shown below:

$$\mu_{Z^*}(x^*) = \max[\mu_Z(x)] \quad (9)$$

## III. FUZZY OPTIMIZATION

In most of the problems from the field of engineering design and decision making, it is difficult to conclude with the most optimal solution from a set of feasible solutions. The most appropriate method to deal with this kind of a situation is to tackle the uncertainty in the variables that lead to the optimal solution. The randomness that comes into existence due to natural variations and fluctuations can be handled using the probabilistic concepts but to take care of the uncertainty that is due to the vague nature of the objective, linguistic statements of the decision maker showing his willingness (like acceptable solution or satisfactory solution etc.), qualitative statements etc., we introduce the concept of fuzzy optimization where the optimization problems are solved using fuzzy logic. In the crisp optimization problems, there is an objective function which is to be maximized or minimized and at the same time the stated constraints should be satisfied, if not the solution is unacceptable. However, in fuzzy optimization we induce a certain amount of relaxation to this restriction of satisfying each and every constraint completely. In case of fuzzy optimization, the solution is a matter of degree i.e. we define degree of acceptability or degree of satisfaction which can be expressed using membership functions. So, the objective function and the constraints can be represented as fuzzy goals using membership functions and we intend to come out with a solution which is called the “best compromise solution” that helps in achieving these goals. Along with fuzzy goals, crisp constraints may also be required to state the physical conditions, technological feasibility etc. that should be present in a solution. The technique of fuzzy optimization provides flexibility to the objective function and latitude to the constraints as a result of which we can obtain more than one solution for a particular problem but each one may have a different degree of acceptability and depending upon the willingness and requirement, the decision maker can select the most appropriate solution. Also, fuzzy optimization helps in obtaining a solution which is not a 0-1 type solution by quantifying the preferences of the decision maker and tackling the uncertainty in the decision making problems, that comes into existence due to imprecision ,vagueness etc. through membership functions [20]-[22].

## IV. PROBLEM DEFINITION

The OP can be presented in the form of a graph $G(V, E)$ where V and E denote the set of vertices and the set of edges respectively. This graph is a weighted undirected completely connected graph. The weight assigned to every vertex $v_i \in V$ and every edge $e_{ij} \in E$ denotes the parameter score ($S_i$) and the time taken to traverse each edge $(t_{ij})$ respectively. The task in OP is to obtain a path P that connects the source vertex ($v_1$) and the destination vertex ($v_n$) and also includes any subset ofV such that the total collected score is maximized within the specified time budget $T_{max}$ [1].

Here, we introduce the intuitionistic fuzzy orienteering problem (IFOP) where the parameters (score and time) are represented using TIFN. In IFOP, the strict requirements of the crisp formulation which include the maximization or minimization of the objective function, satisfying each and every constraint and giving equal importance to all the constraints are relaxed to some extent by using fuzzy logic with the aim to provide a more accurate and realistic modeling of the real world. In the fuzzy formulation, we consider the willingness of the decision maker, his aspiration levels and the degree up to which a solution is acceptable or its degree of satisfaction and using intuitionistic fuzzy numbers this modeling can be made more apt as the extra information stating the degree of non-belongingness along with the degree of belongingness is the best way to tackle the vagueness [23].

The fuzzy version of OP provides latitude to the solution by relaxing the constraints to some extent and representing the objective function of maximizing the score and the constraint of satisfying the time bound as fuzzy goals using linear membership functions. The remaining constraints are crisp as shown below:

$$\sum_{i=1}^{N-1}\sum_{j=2}^{N} \widetilde{S_i}\, x_{ij} \gtrsim S_{min} \quad (10)$$

$$\sum_{j=2}^{N} x_{1j} = 1 \quad , \quad \sum_{i=1}^{N-1} x_{iN} = 1 \quad (11)$$

$$\sum_{i=1}^{N-1} x_{ik} \leq 1 \quad \forall\, k = 2, \dots\dots, N-1 \quad (12)$$

$$\sum_{j=2}^{N} x_{kj} \leq 1 \quad \forall\, k = 2, \dots\dots, N-1 \quad (13)$$

$$\sum_{i=1}^{N-1}\sum_{j=2}^{N} \widetilde{t_{ij}}\, x_{ij} \lesssim T_{max} \quad (14)$$

$$2 \leq u_i \leq N \quad \forall\, i = 2, \dots\dots, N \quad (15)$$

$$u_i - u_j + 1 \leq (N-1)\left(1 - x_{ij}\right) \quad \forall\, i, j = 2, \dots\dots\dots, N \quad (16)$$

$$x_{ij} \in \{0,1\} \quad \forall\, i, j = 1, \dots\dots, N \quad (17)$$

The variables with a tilde denote a fuzzy parameter and here we use a TIFN. In the above stated equations, the position of vertex $v_i$ is denoted by the variable $u_i$ and if vertex $v_j$ is explored after $v_i$ then $x_{ij}$ = 1else it is 0.The restriction that for every path the beginning and the end point should be $v_1$ and $v_N$, every path remains connected without any vertex being visited more than once and the necessity of removing sub-tours is implemented by the crisp constraints (11), (12)-(13) and (15)-(16) respectively [1]. The two fuzzy goals (10) and (14) represent the necessary condition of maximizing the total reward or score collected and of the total time taken for traversing a path being within the specified upper limit respectively. In the fuzzy formulation, the symbols '≥' and '≤' indicating the 'greater than or equal to' and 'less than or equal to' of the crisp case are replaced by the symbols '≳' signifying the 'fuzzy greater than or equal to' and '≲'signifying the 'fuzzy less than or equal to' relation respectively. These symbols suggest that there is no strict boundaries for the constraints and the violation of the constraint to some extent is also acceptable but with differing degrees [24].

In this paper, we symbolize the two fuzzy goals of minimizing the total time taken by each path and maximizing the total score collected by each path by membership functions as shown in Figure 2 and Figure 1 of [26] respectively. As can be observed from the diagram below, the total score collected for a path should be either equal to $S_{min}$ or greater than $S_{min}$ in the ideal case which gives the most desirable solution but to consider the practical situations, we also accept solutions which have their total score within the range of $S_{min}$ and $S_{min} - P$, each having a different degree of satisfaction. Similarly, for the constraint of satisfying the time budget we specify the limit $T_{max}$ but also accept solutions up to $T_{max} + L$ with different degrees of acceptability. The fuzzy decision set Z and $Z^*$ and the "best compromise solution" derived from the max-min formulation is shown in Figure 3 of [26].

## V. IFOP ALGORITHM

Following are the steps to determine the most appropriate path for a given graph $G(V, E)$ with N nodes. The steps are explained in the next section with the help of an illustrative example:

**Step1**: Compute all the paths ($P_m$) that connect the source node ($v_1$) and the destination node ($v_N$) and fulfil the condition stated by (11), (12), (13), (15), (16), (17).

**Step 2**: The following values are calculated for each of the possible paths computed in Step1:

(A) The total collected reward or score and the total time taken (using Definition II(B) and (3)).

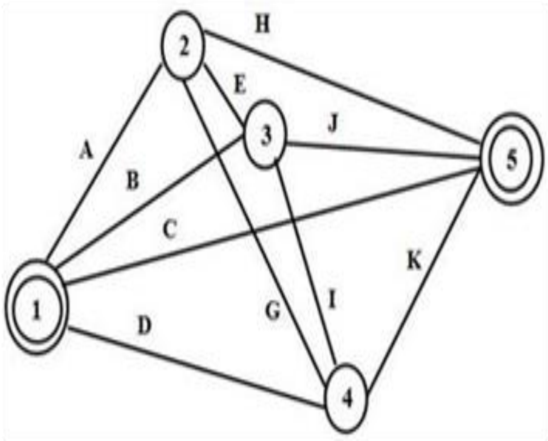
(B) The expected value for the total time taken and the total collected score (using Definition II(C) and (4)).

(C) The membership value for the total time taken and the total collected score represented by the fuzzy set $F_1$and $F_2$respectively (using (14), Fig. 2 of [26] and (10), Fig. 1 of [26] ).

**Step 3**: Compute the set of feasible paths depicted by the fuzzy decision set Z (using Definition II (E) and (8)).

**Step 4**: The final solution representing the most desirable path is denoted by the fuzzy decision set $Z^*$ (obtained using Definition II (E) and (9)). If the set $Z^*$ contains more than one path then to conclude with the path that maximizes the total collected score, the paths in $Z^*$ are ranked according to their total collected score (using (5), (6), (7)).

## VI. LUSTRATIVE EXAMPLE




| Node | Label | Intuitionistic Fuzzy Score Values $\langle(\mu(x)),(\nu(x))\rangle$ |
|---|---|---|
| $v_1$ | 1 | $\langle(1,2,8,9),(0,2,8,10)\rangle$ |
| $v_2$ | 2 | $\langle(8,9,11,12),(6,8,12,14)\rangle$ |
| $v_3$ | 3 | $\langle(3,5,9,11),(1,4,10,13)\rangle$ |
| $v_4$ | 4 | $\langle(17,20,24,27),(14,18,26,30)\rangle$ |
| $v_5$ | 5 | $\langle(1,2,4,5),(0,1,5,6)\rangle$ |
| | $S_{min}=25, P=13$ | |

| Edge | Label | Intuitionistic Fuzzy Time Values $\langle(\mu(x)),(\nu(x))\rangle$ |
|---|---|---|
| $e_{12}$ | A | $\langle(1,4,6,9),(0,2,8,10)\rangle$ |
| $e_{13}$ | B | $\langle(6,9,13,16),(5,8,14,17)\rangle$ |
| $e_{14}$ | D | $\langle(14,15,21,22),(12,14,22,24)\rangle$ |
| $e_{15}$ | C | $\langle(4,6,8,10),(2,4,10,12)\rangle$ |
| $e_{23}$ | E | $\langle(2,3,3,4),(0,3,3,6)\rangle$ |
| $e_{24}$ | G | $\langle(5,8,8,11),(4,8,8,12)\rangle$ |
| $e_{25}$ | H | $\langle(14,18,22,26),(12,16,24,28)\rangle$ |
| $e_{34}$ | I | $\langle(5,8,12,15),(3,6,14,17)\rangle$ |
| $e_{35}$ | J | $\langle(0,2,10,12),(0,1,11,12)\rangle$ |
| $e_{45}$ | K | $\langle(1,2,2,3),(0,2,2,4)\rangle$ |
| | $T_{max}=20, L=15$ | |

**Figure 2.** The input graph with $N = 5, v_1 = 1, v_N = 5$ and the time and score values associated with each edge and vertex respectively

$\mathbf{P_7}: 1-2-4-5$ ; $\mathbf{P_8}: 1-4-2-5$ ;
$\mathbf{P_9}: 1-3-4-5$ ; $\mathbf{P_{10}}: 1-4-3-5$ ;
$\mathbf{P_{11}}: 1-2-3-4-5$ ; $\mathbf{P_{12}}: 1-2-4-3-5$ ;
$\mathbf{P_{13}}: 1-3-4-2-5$ ; $\mathbf{P_{14}}: 1-4-3-2-5$ ;
$\mathbf{P_{15}}: 1-4-2-3-5$ ; $\mathbf{P_{16}}: 1-3-2-4-5$

**Step 2**: The actual values for the stated example obtained as a result of step 2 (A), (B), (C) of the IFOP algorithm are shown in Table II.

The fuzzy set $F_1$ and $F_2$ denoting the membership value for the total time taken and the total collected score respectively are as follows:

$F_1$
$= \{P_1/1, P_2/0.67, P_3/1, P_4/1, P_5/1, P_6/0.06, P_7/1, P_8/0, P_9/0.8, P_{10}/0.06, P_{11}/1, P_{12}/0.4, P_{13}/0, P_{14}/0, P_{15}/0, P_{16}/0.73\}$

$F_2 = \{P_1/0, P_2/0.23, P_3/0, P_4/1, P_5/0.77, P_6/0.77, P_7/1,$

$P_8/1, P_9/1, P_{10}/1, P_{11}/1, P_{12}/1, P_{13}/1, P_{14}/1, P_{15}/1, P_{16}/1\}$

**Step 3**: For the considered input, following is the fuzzy decision Z :

$Z = \{P_1/0, P_2/0.23, P_3/0, P_4/1, P_5/0.77, P_6/0.06, P_7/1, P_8/0, P_9/0.8, P_{10}/0.06, P_{11}/1, P_{12}/0.4, P_{13}/0\}$

**Step 4**: Following are the paths in $Z^*$ and their corresponding ranks obtained for the given network:

$Z^* = \{P_4/1, P_7/1, P_{11}/1\}$

**Table 1.** Ranks Of The Desirable Paths

| Path | Score | Rank |
|---|---|---|
| $P_4$ | $\langle(18,22,32,36),(14,20,34,40)\rangle$ | 3 |
| $P_7$ | $\langle(26,31,43,48),(20,28,46,54)\rangle$ | 2 |
| $\mathbf{P_{11}}$ | $\langle(\mathbf{29,36,52,59}),(\mathbf{21,32,56,67})\rangle$ | **1** |

The most desirable path is $P_{11}$ as it has the highest rank. To check the correctness of our result, we set the spreads of the TIFN (for both score and time) to zero in order to convert each input to its crisp equivalent and then perform exhaustive search. This gives the same solution as our algorithm.

## VII. WORK-DEPTH ANALYSIS OF IFOP

We present a parallel formulation here, to achieve a better performance and solve the IFOP more efficiently for large instances. Parallel computers can be organized in various ways and several multiprocessor models are known but it is difficult to conclude with one model that is apt for all machines. The method to deal with this situation is to focus on algorithms than on machines. As stated in [25], work-depth model is a technique of presenting the parallelism of an algorithm. For any algorithm, the following terms can be calculated [25]:

**Work (W):** Total number of operations performed.
**Depth (D):** Longest chain of dependencies among its operations.
**Parallelism ($\mathcal{P}$):** The ratio $\frac{W}{D}$

Algorithms with efficient work-depth models can be converted into efficient multiprocessor models and then to actual parallel computers. Work-depth models can be represented in three possible ways:

(a) Circuit Model.
(b) Vector Machine Model.
(c) Language-based Model.

For the parallel formulation of IFOP we use the circuit model which is the most abstract one when compared to the other two models. A circuit has two important components: nodes and directed arcs. The directed arcs and the nodes denote the flow of values and the operations to be performed respectively. Fan-in and fan-out are two terms associated with each node signifying the number of incoming and outgoing arcs respectively. The input to the circuit is provided through input arcs which do not originate from any node. Similarly, the output arcs carry the result out of the circuit and do not have any destination node. The number of nodes denotes the work of the circuit, also called the size of the circuit. A circuit should not contain directed cycles and the count of the nodes on the longest directed path connecting the input and output arc specifies the depth of the circuit. If the parallelism computed for the work-depth model is at least as large as the number of processors then it is said to be work-preserving and can be translated into an efficient multiprocessor model like the PRAM model [25].

For the circuit of IFOP we assume two things:

(a) Number of processors

$= \text{Number of distinct paths between source } (v_1) \text{ and destination } (v_N)$

(b) Number of distinct paths (p) = $\sum_{i=1}^{N-1} \frac{(N-2)!}{[N-(i+1)]!}$ where N is the number of nodes in the given graph G.

The first step of the IFOP of determining all the possible distinct paths is performed sequentially as shown in Figure 3:

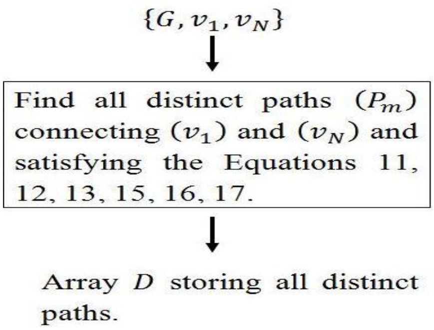



**Figure 3.** The sequential module executing Step 1 of IFOP that computes all the distinct paths in the given graph G.

The circuit of IFOP is shown in Fig. 4. The parallelism for IFOP is calculated below:

**Work (W) = $5p + 2$.**

**Depth (D) = 5.**

**Parallelism ($\mathcal{P}$)** = $\frac{W}{D} = \frac{5p+2}{5} = \left(p + \frac{2}{5}\right)$.

Therefore, IFOP is work-preserving because the parallelism ($\mathcal{P}$) is $\left(p + \frac{2}{5}\right)$ which is at least as large as the number of processors i.e. p.

## VIII. CONCLUSION

In this paper, we considered the orienteering problem which is a NP-Hard problem and formulated the intuitionistic fuzzy version of this problem accounting for uncertainty in the real life areas where this problem finds its application like the tourism industry, logistics etc. We state the problem as a fuzzy integer program with fuzzy goals and crisp constraints and present a fuzzy optimization technique for solving IFOP. The method suggested here considers the hesitancy, aspiration levels, degree of acceptability and satisfaction of the decision maker, thus providing latitude to the solution process. The generated solution is capable of tackling the uncertainty and vagueness involved in the two parameters score and time. To deal with larger instances efficiently, we presented the work-depth analysis of IFOP and showed that the algorithm is work-preserving and thus can be efficiently implemented on a multiprocessor model like the PRAM.

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