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Adomian Decomposition Method for Multiphase Miscible Flow the Porous Media

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ABSTRACT

Multifluid Miscible Fluid flow in porous media givens some problem solve by nonlinear partial differential equations considering Adomian decomposition method. We generate the being of accurate solutions for those problem. The mathematical results show the capability and correctness of this method.

Keywords : Adomian decomposition method, nonlinear partial differential equations, Miscible Fluid.

I. INTRODUCTION

Many type of scientific and engineering applies Nonlinear partial differential equations. Many important mathematical models can be expressed in terms of nonlinear partial differential equations. The most general form of nonlinear partial differential equation is given by:

$$F\left(u, u_t, u_x, u_y, x, y, t\right) = 0 \qquad \text{(1a)}$$

With initial and boundary conditions

$$u(x, y, 0) = \emptyset(x, y), \forall x, y \in \Omega, \Omega \in R^2 \qquad \text{(1b)}$$

$$u(x, y, t) = f(x, y, t), \forall x, y \in \partial \qquad \text{(1c)}$$

Where Ω is the solution region and $\partial\Omega$ is the boundary of Ω.

In recent years, much research has been focused on the numerical solution of nonlinear partial equations by using numerical methods and developing these methods (Al-Saif, 2007; Leveque, 2006; Rossler & Husner, 1997; Wescot & Rizwan-Uddin, 2001). In the numerical methods, which are commonly used for solving these kind of equations large size or difficult of computations is appeared and usually the round-off error causes the loss of accuracy.

The Adomian decomposition method which needs less computation was employed to solve many problems (Celik et al., 2006; Javidi & Golbabai, 2007). Therefore, we applied the Adomian decomposition method to solve some models of nonlinear partial equation, this study reveals that the Adomian decomposition method is very efficient for nonlinear models, and it results give evidence that high accuracy can be achieved.

II. Mathematical Methodology

The principle of the Adomian decomposition method (ADM) when applied to a general nonlinear equation is in the following form(Celik et al., 2006; Seng et al., 1996):

$$\mathrm{Lu} + \mathrm{Ru} + \mathrm{Nu} = \mathrm{g} \qquad \text{(2)}$$

The linear terms decomposed into Lu+Ru, while the nonlinear terms are represented by Nu , where L is an easily invertible linear operator, R is the remaining linear part. By inverse operator L, with $L^{-1}(\cdot) = \int_0^t (\cdot)dt$. Equation (2) can be hence as;

$$u = L^{-1}(g) - L^{-1}(Ru) - L^{-1}(Nu) \qquad \text{(3)}$$

The decomposition method represents the solution of equation (3) as the following infinite series:

$$u = \sum_{n=0}^{\infty} u_n \quad (4)$$

The nonlinear operator $Nu = \Psi(u)$ is decomposed as:

$$Nu = \sum_{n=0}^{\infty} A_n \quad (5)$$

Where A_n are Adomian's polynomials, which are defined as (Seng et al., 1996):

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}[\psi(\sum_{i=1}^{n} \lambda^i u_i)]_{\lambda=0} \quad n = 0,1,2,\ldots \quad (6)$$

Substituting equations (4) and (5) into equation (3), we have

$$u = \sum_{n=0}^{\infty} u_n = u_0 - L^{-1}(R(\sum_{n=0}^{\infty} u_n)) - L^{-1}(\sum_{n=0}^{\infty} A_n) \quad (7)$$

Consequently, it can be written as:

$$\left.\begin{aligned}
u_0 &= \varphi + L^{-1}(g)\\
u_1 &= -L^{-1}(R(u_0)) - L^{-1}(A_0)\\
u_2 &= -L^{-1}(R(u_1)) - L^{-1}(A_1)\\
&\;\;.\\
&\;\;.\\
&\;\;.\\
u_n &= -L^{-1}(R(u_{n-1})) - L^{-1}(A_{n-1})
\end{aligned}\right\} \quad (8)$$

where φ is the initial condition,

Hence all the terms of u are calculated and the general solution obtained according to ADM as $u = \sum_{n=0}^{\infty} u_n$. The convergent of this series has been proved in (Seng et al., 1996). n=0

However, for some problems (Celik et al., 2006) this series can't be determined, so we use an approximation of the solution from truncated series

$$U_M = \sum_{n=0}^{M} u_n \; with \; \lim_{M\to\infty} U_M = u$$

Statement of the Problem:

Many Important problems in water resources engineering involve the mass-transport of a miscible fluid in a flow.

A fluid is considered to be a continuous material and hence in addition to the velocity of a fluid element, the molecules in this element have random motion. As a result of the random motion, molecules of a certain material in high concentration at one point will spread with time. So the velocity considered here is time dependent. The net molecular motion from a point of higher concentration to one of lower concentration is called molecular diffusion.

Fluid flows in nature are usually turbulent, but we have considered the porous medium through which the fluid flows, to be homogeneous and for this reason, in the direction of flow, we assume laminar flow in which miscible fluids mix.

In moving through the random passages of the medium, two fluid elements adjacent to each other at one time will separate, as they may take different routes. The geometrical dispersion is coupled with molecular diffusion and dispersion due to no uniformity of the velocity across the cross-section of the passages. By considering the passage as randomly connected tubes, de Jong (1) and safman (2) have shown that the dispersion in an isotropic medium can be described with a coefficient D for longitudinal dispersion in the direction of seepage velocity.

Mathematical Modeling:

The equation of continuity for the mixture is given by

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0 \tag{9}$$

Where ρ is the density for mixture and $\vec{v}$ is the pore velocity (vector).

The equation of diffusion for a fluid flow through homogeneous porous medium with no addition or subtraction of the dispersing material, is given by

$$\frac{\partial c}{\partial t} + \nabla \cdot (c\vec{v}) = \nabla \cdot [\,\rho \bar{D} \nabla \left(\frac{c}{\rho}\right)] \tag{10}$$

Where c is the concentration of the fluid A in the other host fluid B (i.e. c is the mass of A per unit volume of the mixture), $\bar{D}$ is the tensor co-efficient of dispersion with nine components D_{ij}.

In a laminar foe through homogeneous porous medium at constant temperature, ρ may be considered to be constant. The equation (9) gives

$$\nabla \cdot \vec{v} = 0$$

And equation (10) becomes

$$\frac{\partial c}{\partial t} + \nabla \cdot (c\vec{v}) = \nabla \cdot [\,\bar{D} \nabla c] \tag{11}$$

When the seepage velocity $\vec{v}$ is along the X-axis, the non-zero components are

$D_{11} = D_L$ and $D_{22} = D_T$ (coefficient of transverse dispersion), and other D_{ij} are zero.

Solution of the Problem:

Consider the Problem

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} = D_L \frac{\partial^2 c}{\partial x^2}$$

with the initial Condition

$$c(x,0) = \gamma_0, \quad 0 \le x \le L$$
$$c(0,t) = \gamma_1$$
$$c(L,t) = \gamma_2$$

Solution: In this problem, we have

$N(c) = \Psi(c) = -u\frac{\partial c}{\partial x}$

$g(\xi, t) = D_L \frac{\partial^2 c}{\partial x^2}$

$R\theta = 0$

$L(c) = \frac{\partial c}{\partial t}$

And $\phi = c(x,0) = \gamma_0$

By using equation(6) Adomain's polynomials can be derived as follows:

$$\left.\begin{aligned}
A_0 &= -u_0 \frac{\partial c_0}{\partial x} \\
A_1 &= -u_1 \frac{\partial c_0}{\partial x} - u_0 \frac{\partial c_1}{\partial x} \\
A_2 &= -u_2 \frac{\partial c_0}{\partial x} - u_1 \frac{\partial c_1}{\partial x} - u_0 \frac{\partial c_2}{\partial x} \\
A_3 &= -u_3 \frac{\partial c_0}{\partial x} - u_2 \frac{\partial c_1}{\partial x} - u_1 \frac{\partial c_2}{\partial x} - u_0 \frac{\partial c_3}{\partial x} \\
&\vdots
\end{aligned}\right\} \tag{12}$$

And so on. The rest of the polynomials can be constructed in similar manner.

By using Equation (8), we have

$c_0 = \gamma_0$

$c_1 = -\,u_0 \frac{\partial \gamma_0}{\partial x}\, t$

$c_2 = -\,u_1 \frac{\partial \gamma_0}{\partial x}\, t + u_0\, u_1 \frac{\partial \gamma_0}{\partial x}\frac{t^2}{2} - {u_0}^2 \frac{\partial^2 \gamma_0}{\partial x^2}\frac{t^2}{2}$

$c_3 =$
$- u_3 \frac{\partial \gamma_0}{\partial x} \frac{t^2}{2} + {u_1}^2 \frac{\partial \gamma_0}{\partial x} \frac{t^2}{2} + u_0 \frac{\partial^2 \gamma_0}{\partial x^2} \frac{t^2}{2} - u_0\, u_2 \frac{\partial \gamma_0}{\partial x} \frac{t^2}{2} + u_0\, u_1 \frac{\partial^2 \gamma_0}{\partial x^2} \frac{t^2}{2} -$ $u_0\, {u_1}^2 \frac{\partial \gamma_0}{\partial x} \frac{t^3}{6} -$
${u_0}^2 u_2 \frac{\partial \gamma_0}{\partial x} \frac{t^3}{6} + {u_0}^2 u_1 \frac{\partial^2 \gamma_0}{\partial x^2} \frac{t^3}{6} - {u_0}^3 \frac{\partial^3 \gamma_0}{\partial x^3} \frac{t^3}{6}$

$\vdots$

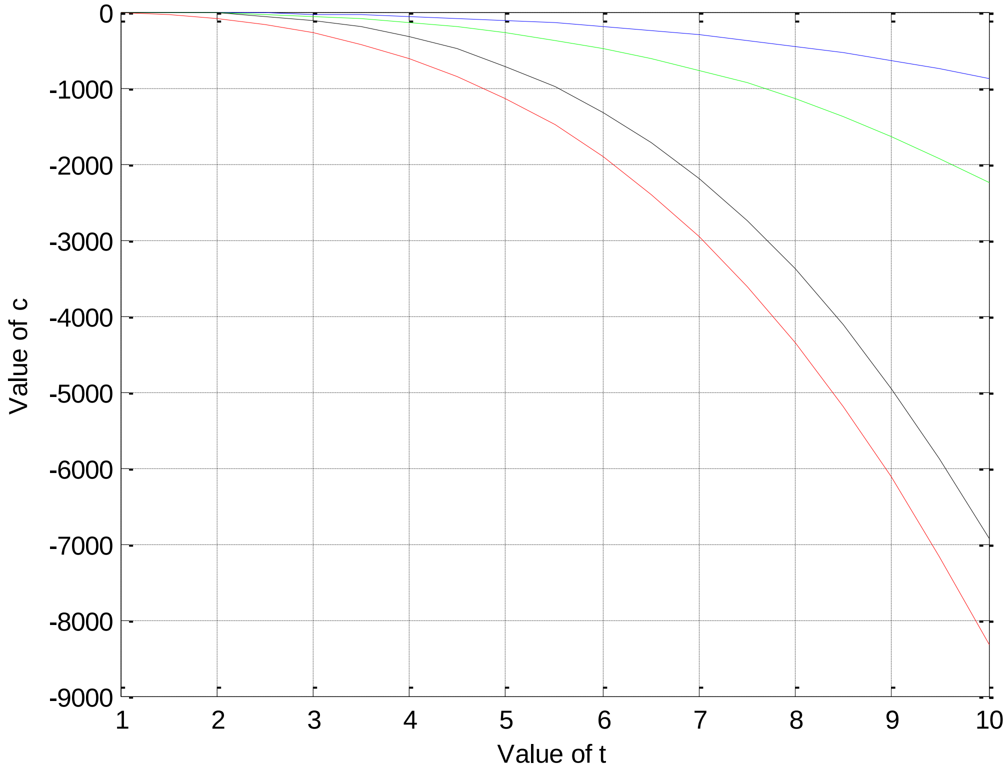
Substituting these individual terms in equation (4) obtain

$$c(x,t) = \gamma_0 - \left(u_0 \frac{\partial \gamma_0}{\partial x} + u_1 \frac{\partial \gamma_0}{\partial x}\right) t + \left(u_0\, u_1 \frac{\partial \gamma_0}{\partial x} - {u_0}^2 \frac{\partial^2 \gamma_0}{\partial x^2} - u_3 \frac{\partial \gamma_0}{\partial x} + {u_1}^2 \frac{\partial \gamma_0}{\partial x} + \quad {u_0}^2 \frac{\partial^2 \gamma_0}{\partial x^2} - u_0\, u_2 \frac{\partial \gamma_0}{\partial x} + u_0\, u_1 \frac{\partial^2 \gamma_0}{\partial x^2}\right) \frac{t^2}{2} - \left(u_0\, {u_1}^2 \frac{\partial \gamma_0}{\partial x} + {u_0}^2 u_2 \frac{\partial \gamma_0}{\partial x} - {u_0}^2 u_1 + {u_0}^3 \frac{\partial^3 \gamma_0}{\partial x^3}\right) \frac{t^3}{6} + \cdots$$

$$\boxed{\boldsymbol{\gamma_0 = 1, \quad \frac{\partial \gamma_0}{\partial x} = 2, \quad \frac{\partial^2 \gamma_0}{\partial x^2} = 1.5, \quad \frac{\partial^3 \gamma_0}{\partial x^3} = 3}}$$

Table1

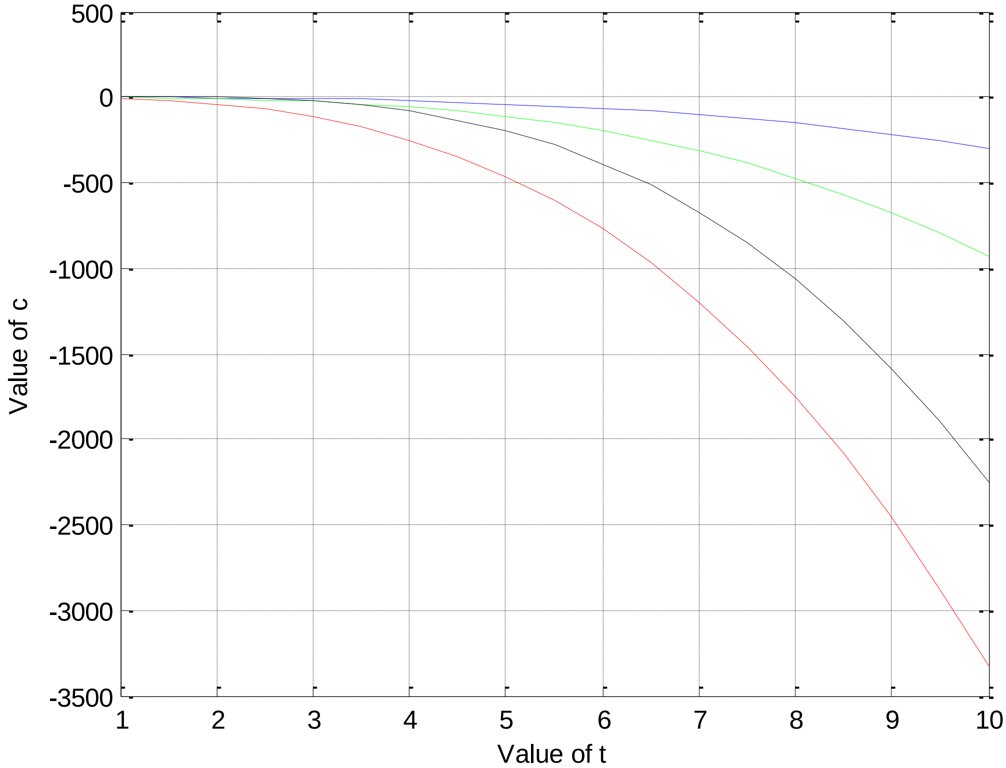
t		$c(x,t)$		$c(x,t)$		$c(x,t)$		$c(x,t)$
0.1	$u_0 = 1$, $u_1 = 1$, $u_2 = 1$	95.8333	$u_0 = 1$, $u_1 = 2$, $u_2 = 3$	0.0932	$u_0 = 2$, $u_1 = 1$, $u_2 = 3$	0.0803	$u_0 = 2$, $u_1 = 3$, $u_2 = 1$	0.0967
0.2		87.6667		0.0753		0.0027		0.0733
0.3		70.0000		0.0325		-0.1790		-0.0200
0.4		37.3333		-0.0493		-0.5107		-0.2333
0.5		-15.8333		-0.1842		-1.0383		-0.6167
0.6		-95.0000		-0.3860		-1.8080		-1.2200
0.7		-205.6667		-0.6688		-2.8657		-2.0933
0.8		-353.3333		-1.0467		-4.2573		-3.2867
0.9		-543.5000		-1.5335		-6.0290		-4.8500
1.0		-781.6667		-2.1433		-8.2267		-6.8333



$$\gamma_0 = 1, \quad \frac{\partial \gamma_0}{\partial x} = 1, \quad \frac{\partial^2 \gamma_0}{\partial x^2} = 1, \quad \frac{\partial^3 \gamma_0}{\partial x^3} = 1$$

Table 2

t		$c(x,t)$		$c(x,t)$		$c(x,t)$		$c(x,t)$
0.1		-0.8333		-2.0000		-0.0080		0.0010
0.2		-3.6667		-9.0000		-0.0410		-0.0010
0.3		-9.5000		-26.0000		-0.1160		-0.0230
0.4		-20.3333		-59.0000		-0.2510		-0.0830
0.5	$u_0 = 1$	-38.1667	$u_0 = 1$	-114.0000	$u_0 = 2$	-0.4640	$u_0 = 2$	-0.1990
0.6	$u_1 = 1$	-65.0000	$u_1 = 2$	-197.0000	$u_1 = 1$	-0.7730	$u_1 = 3$	-0.3890
0.7	$u_2 = 1$	-102.8333	$u_2 = 3$	-314.0000	$u_2 = 3$	-1.1960	$u_2 = 1$	-0.6710
0.8		-153.6667		-471.0000		-1.7510		-1.0630
0.9		-219.5000		-674.0000		-2.4560		-1.5830
1.0		-302.3333		-929.0000		-3.3290		-2.2490



III. CONCLUSION

Nonlinear partial equations is used to solve the problem and also applied the adomain decomposition method. This is method is presented quite efficient to get exact solution. The benefit of this method give the solution without a need for large size of computations. The numerical method indicate a high degree of accuracy.

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