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Two-Step Weight Iterative Methods for Solving Nonlinear Equations

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ABSTRACT

In this paper, a study on two-step weight iterative methods for solving nonlinear equations and also its cubic convergence. Nonlinear equation is the fundamentals inengineering, computer science and other many related fields. Many procedures have been developed during the past few decades. Yet there is need of procedures which is less time consuming in nature and more accurate when considering less iteration. We proposed a new method to give better result (minimum number of iterations) compared to other existing methods. Proposed method can be considered as a significant improvementof the Newton method and its variant forms.

Keywords: Nonlinear equation, Order of convergence, Taylor series expansion, asymptotic convergence.

I. INTRODUCTION

We consider a single variable nonlinear equation

$$f(p) = 0 \qquad (1)$$

Finding zeros of a single variable nonlinear equation (1) efficiently, is an interesting very old problem in numerical analysis and has many applications in applied sciences. In recent years, researchers have developed many iterative methods for solving equations (1). These methods can be classified as one-step, two-step, three-step, see [1-4]. These methods have been proposed using Taylor's series, decomposition techniques and quadrature rules, etc. Soheili [1], Noor [2], Bahgat [3], Mir et al [4] and John [5] have proposed many two-step methods.

In this paper we proposed some two-stepweight iterative methods based on series expansion of nonlinear equation. We have proved that the methods are of order three convergences. The methods and their algorithms are in section 2. Improved two-step methods and their algorithms are in section 3. The convergence analysis of the methods is discussed in section 4. In section 5, we tested the performance of the algorithms on some selected functions. Comparison of the methods performance was also made with performance of some existing methods and in many cases gives better results.

II. IMPLEMENTOF METHOD

Consider the nonlinear equation of the type (1).If r is a simple root and q is an initial guess close to r. Using the Taylor's series expansion of the function $f(p)$,we have

$f(p) = f(q) + (p-q)f'(q) + \frac{1}{2}(p-q)^2 f''(q)$ (2)

From which have $p = q - \frac{2f(q)}{2f'(q)+(p-q)f''(q)}$

(3)

This formulation allows us to suggest the following iterative methods for solving the nonlinear equations.

Algorithm 2.1

For a given initial choice p_k, find the approximate solution p_{k+1} by iterative schemes:

$$p_{k+1} = p_k - \frac{2f(p_k)}{2f'(p_k)+(p_{k+1}-p_k)f''(p_k)} \quad (4)$$

Equation (4) is an implicit method, since p_{k+1} is on the both side of the equation, which itself a difficult problem. Note that if $f''(p_0) = 0$, the algorithm 2.1 will reduce to the famous Newton's Method given by

$$p_{k+1} = p_k - \frac{f(p_k)}{f'(p_k)} \quad (5)$$

From Algorithm 2.1, we suggest the following iterative methods.

III. IMPROVED TWO-STEP ITERATIVE METHODSFOR SOLVING NONLINEAR EQUATIONS.

In this section, to present an improvement of two-step iterative methods for solving nonlinear equations. We are carried out and reformulate (rearrange) the regular method it is improved version of two-step iterative method.

Properties:

Let α is the possible of acceptance region to take more than 60% of probability.Here we take $\alpha = \frac{2}{3} = 66\%$.

Algorithm 3.1

For a given initial choice p_k, find the approximate solution p_{k+1} by the iterative schemes:

$$t_k = p_k - \frac{f(p_k)}{f'(p_k)} \quad (6)$$

$$p_{k+1} = p_k - \frac{2f(p_k)}{2f'(p_k)+(t_k-p_k)f''(p_k)}, k = 0, 1, 2, 3, \ldots \quad (7)$$

If we use Taylor's series to approximate $f''(p_k)$ in (7) with $\alpha\frac{f'(t_k)-f'(p_k)}{t_k-p_k}$, where t_k is as defined in (5). This enables us to suggest an iterative scheme which does not involve the second derivatives.

Algorithm3.2

For a given initial choice p_k, find the approximate solution p_{k+1} by the iterative schemes

$$t_k = p_k - \frac{f(p_k)}{f'(p_k)} \quad (8)$$

$$p_{k+1} = p_k - \frac{3f(p_k)}{2f'(p_k)+f'(t_{k)})}, k = 0, 1, 2, 3, \ldots \quad (9)$$

If we approximates t_k with Halley's method, we suggest the following iterative scheme.

Algorithm 3.3

For a given initial choice p_k, find the approximate solution p_{k+1} by iterative schemes:

$$t_k = p_k - \frac{2f(p_k)f'(p_k)}{2(f'(p))^2 - f(p_k)f''(p_k)} \quad (10)$$

$$p_{k+1} = p_k - \frac{3f(p_k)}{2f'(p_k)+f'(t_k)}, k = 0,1,2,3, \ldots \quad (11)$$

IV. CONVERGENCE ANALYSIS

Theorem:4.1

Let $r \in I$ be a simple zero of sufficiently differentiable function $f: I \subset R \rightarrow R$ for an open interval I. If q is sufficiently close to r then the iterative method defined byAlgorithm 3.1 has third-order convergence.

Proof:

Let r be a simple zero of $f$and $e_k = p_k - r$. Using Taylor expansion around$x = r$ and taking into account $f(r) = 0$, we get

$f(p_k) = f'(r)[e_k + c_2 e_k^2 + c_3 e_k^3 + c_4 e_k^4 + \cdots]$ (12)

$f'(p_k) = f'(r)[1 + 2c_2 e_k + 3c_3 e_k^2 + 4c_4 e_k^3 + 5c_5 e_k^4 + \cdots]$ (13)

$f''(p_k) = f'(r)[2c_2 + 6c_3 e_k + 12c_4 e_k^2 + 20c_5 e_k^3 + \cdots]$ (14)

Where $c_n = \frac{1}{n!}\frac{f^n(r)}{f'r)}$, $n = 1, 2, 3, \ldots$ and $e_k = p_k - r$

Combining (12 and (13) in (6) we have

$t_k = r + c_2 e_k^2 - 2(c_2^2 - c_3)e_k^3 - (7c_2 c_3 + 4c_2^3 - 3c_4)e_k^4 + \cdots$ (15)

Where$(t_k - p_k) = -e_k + c_2 e_k^2 - 2(c_2^2 - c_3)e_k^3 - (7c_2 c_3 + 4c_2^3 - 3c_4)e_k^4 + \cdots$ (16)

$(t_k - p_k)f''(p_k) = f'(r)[-2c_2 e_k + (2c_2^2 - 6c_3)e_k^2 + (10c_2 c_3 - 4c_2^3 - 12c_4)e_k^3 + (12c_3^2 - 8c_2^4 + 18c_2 c_4 - 26c_2^2 c_3 - 20c_5)e_k^4 + \cdots]$(17)

$2f'(p_k) = f'(r)[2 + 4c_2 e_k + 6c_3 e_k^2 + 8c_4 e_k^3 + 10c_5 e_k^4 + \cdots]$ (18)

$2f(p_k) = f'(r)[2e_k + 2c_2 e_k^2 + 2c_3 e_k^3 + 2c_4 e_k^4 + \cdots]$ (19)

$2f'(p_k) + (t_k - p_k)f''(p_k) = f'(r)[2 + 2c_2 e_k + 2c_2^2 e_k^2 + (10c_2 c_3 - 4c_2^3 - 4c_4)e_k^3 + (12c_3^2 - 8c_2^4 + 18c_2 c_4 - 26c_2^2 c_3 - 10c_5)e_k^4 + \cdots]$(20)

Thus, $\frac{2f(p_k)}{2f'(p_k)+(t_k-p_k)f''(p_k)} = [e_k + (c_3 - c_2^2)e_k^3 + o(e_k^4)]$ (21)

This shows that Algorithm 3.1 has third-order convergence.

Theorem: 4.2

Let $r \in I$ be simple zero of sufficiently differentiable function $f: I \subset R \rightarrow R$ for an open interval I. If q is sufficiently close to r then the iterative method defined by Algorithm 3.2 has third-order convergence.

Proof:

Let r be a simple zero of $f$and $e_k = p_k - r$. Using Taylor expansion around $x = r$ and taking into account $f(r) = 0$, we get

$f'(t_k) = f'(r)[1 - c_2 e_k + (3c_2^2 - 6c_3)e_k^2 + (15c_2 c_3 - 6c_2^3 - 14c_4)e_k^3 + o(e_k^4)]$ (22)

Next adding the equation (18) and (22), we get,

$2f'(p_k) + f'(t_k) = f'(r)[3 + 3c_2 e_k + 3c_2^2 e_k^2 + (15c_2 c_3 - 6c_2^3 - 6c_4)e_k^3 + \cdots]$ (23)

$3f(p_k) = 3f'(r)[e_k + c_2 e_k^2 + c_3 e_k^3 + c_4 e_k^4 + \cdots]$ (24)

And dividing (23) and (24),we get,

$\frac{3f(p_k)}{2f'(p_k)+f'(t_k)} = e_k + (c_3 - c_2^2)e_k^3 + o(e_k^4)$ (25)

Equation (25) establishes that Algorithm 3.2 has order of convergence equal to three. This completes the proof of the theorem.

Theorem: 4.3

Let $r \in I$ be simple zero of sufficiently differentiable function $f: I \subset R \rightarrow R$ for an open interval I. If q is sufficiently close to r then the iterative method defined by Algorithm 3.3 has third-order convergence.

Proof:

Let r be a simple zero of f and $e_k = p_k - r$. Using Taylor expansion around $x = r$ and taking into account$f(r) = 0$, we get

$$t_k = r + (c_2^2 - c_3)e_k^3 + o(e_k^4) \quad (26)$$

$$f'(t_k) = f'(r)[1 - c_2 e_k - 6c_3 e_k^2 + (3c_2^3 - 3c_2 c_3 - 14c_4 e_k^3 + \cdots] \quad (27)$$

$$2f'(p_k) + f'(t_k) = f'(r)[3 + 3c_2 e_k + (3c_2^3 - 3c_2 c_3 - 6c_4)e_k^3 + \cdots] \quad (28)$$

Dividing (24) and (28), we get

$$\frac{3f(p_k)}{2f'(p_k) + f'(t_k)} = e_k + c_3 e_k^3 + o(e_k^4). \quad (29)$$

V. EXPERIMENTAL RESULT

In this section, we present some examples to illustrate the efficiency of our proposed methods which are given by the Algorithm 3.1, 3.2 and 3.3 We compare the performance of these Algorithms with that of Newton Method (NM), the method of Soheili (SM) and Bahgat (BM), John method (JM). Displayed in Table 1are the number of iterations (NT) required to achieve the desired approximate rootα_n. The following stopping criteria were used.

$$(i).\ |\alpha_{n+1} - \alpha_n| < \varepsilon \qquad (ii).\ f(\alpha_{n+1}) < \varepsilon \quad (30)$$

Where $\varepsilon = 10^{-15}$.

We used the following, some of which are same as in [1-3]

$$\begin{cases} f_1(x) = e^{x^2 - 7x - 30} - 1 \\ f_2(x) = x^3 - 10 \\ f_3(x) = x^2 - e^x - 3x + 2 \\ f_4(x) = x^{10} - 1 \\ f_5(x) = 11x^{11} - 1 \end{cases}$$

Table 1. Comparison between improved version of two-step iterative method and other methods.

Functions	x_0	**NI**				
		NM	**SM**	**BM**	**JM**	**Proposed Method**
f_1	4	20	13	11	11	10
f_2	1.5	7	5	4	3	3
f_3	2	6	4	4	4	3
f_4	1.5	10	7	5	5	4
f_5	0.7	8	6	5	3	3

VI. CONCLUSION AND SUGGESTION

In this paper, to established the two-step weight iterative methods and it is applied for solving the nonlinear equations. The comparison and experimental results (in the table 18) shows that our proposed new method is very effective and provide highly accurate results in a less number of iterations as compared with Newton method, Soheili method , Bahgat method and John method. Our proposed method can be considered as a significant improvement of the Newton method and its variant forms. In future research we are trying to develop this method to tuning up based on accurate result, time and number of iteration is to be reduced.

VII. REFERENCES

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