

Experimental Studies on Chaotic Dynamics of Periodically Driven Rayleigh-Duffing Oscillator

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ABSTRACT

Chaotic dynamics of periodically driven Rayleigh Duffing oscillator has been studied experimentally with the help of an OPAMP based circuit diagram. Nonlinear equation of RDO oscillator has been designed by using a series of OPAMP. With the help a oscilloscope phase plane plot of oscillation has been plotted and with the help of a spectrum analyzer frequency spectrums of different oscillations have been checked. Experimental observation

Keywords : Duffing Oscillator, Rayleigh Duffing Oscillator, Chaos

I. INTRODUCTION

In recent years, a good amount of research articles have been published on the nonlinear dynamics of mechanical oscillators and electronic oscillators. These studies shows that, there are possibility of occurrence of phenomena, like, bifurcations, chaos, multi-stability etc. in conventional oscillators [1-3]. Also several new fields of application of broad band chaotic signals have been proposed with considerable advantages over narrow band signal based signals [4-6]. In the microwave frequency band, Gunn oscillators (GOs) are very popular low noise signal generators having applications in low noise mixers, demodulators frequency synthesizers etc.[7-9]. However, a GO is an inherently nonlinear system and as such its nonlinear dynamics has been studied in the literature. Dynamics of a GO can be explained by considering the nonlinear system equation of RDO. When a RDO operates in normal active region of biasing, bifurcation and chaos can be induced into it by injecting external signals into it. Some studies

considered presence of high frequency signal and observed spatiotemporal chaos in the system [10-13]. Besides a single RDO, a system of coupled RDOs has a rich nonlinear dynamics which demands thorough investigation. The present chapter aims to discuss different aspects of nonlinear dynamics of a RDO experimentally.

RDO is a very popular mathematical model, which is used to describe the nonlinear properties of different physical, chemical and biological systems. This oscillator consists of linear and cubic type restoring and velocity damping forces. Due to presence of different linear and nonlinear damping forces, behaviour of RDO is highly affected by the presence of any periodic force. It has been observed that, in presence of any periodic force if the coefficient of linear velocity damping term is varied from negative to positive value through zero, then for certain range of damping coefficient, RDO shows chaotic behaviour. In non oscillatory state, an external periodic force of proper strength and frequency can

make the oscillator chaotic. In oscillatory state, when RDO is forced by any periodic force then it oscillates chaotically or with more than two frequencies, which are quasiperiodic in nature. From the analysis of frequency spectrum, we have observed when a transverse intersection of unstable phase trajectory with the stable one occurs then, system shows

chaotic behaviour. The rest of the article has been organised as follows.

In section 2, experimental circuit diagram representing RDO has been shown and discussed. Experimental observation and results have been discussed in section-3. Finally, in section-4 some conclusions are given.

II. Experimental verification of the predicted chaotic dynamics for RDO

Analytically and numerically predicted chaotic dynamics of RDO has been properly verified experimentally by designing an op-amp based circuit to simulate system equations of RDO in presence of an external sync signal.

General system equation of RDO is,

$$\frac{d^2q}{dt^2} = aq - bq^3 + c\left(\frac{dq}{dt}\right) - d\left(\frac{dq}{dt}\right)^3 \quad (1)$$

The circuit equation (1) of RDO has been designed by using some electronic filters, OPAMPs and resistors. The simplified circuit diagram of the circuit has been shown in Fig.1. The experiment has been performed in low frequency range around operating frequency 2 KHz.

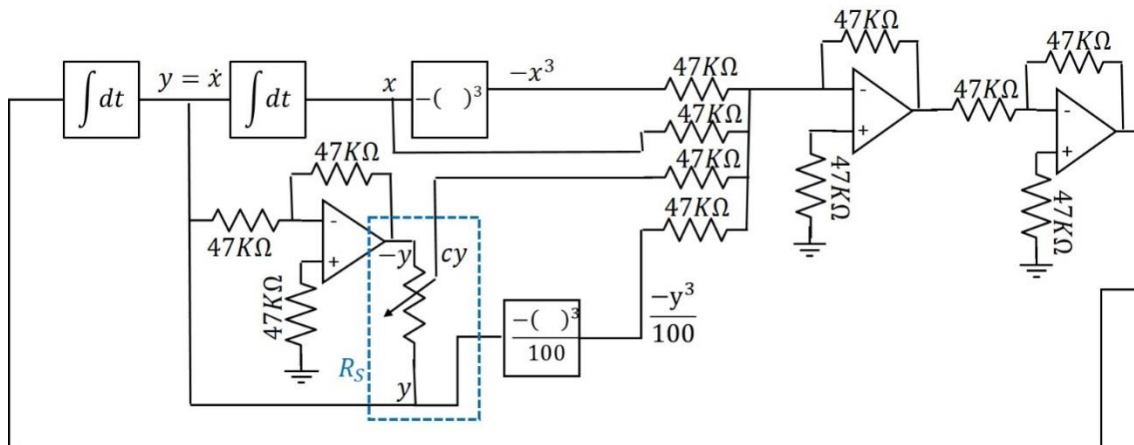


Fig.1. Simplified circuit diagram of OPAMP based Rayleigh Duffing Oscillator. All the Op-Amps used are TL 082 and nonlinearities are generated using AD-633JN. The circuit is biased with dc voltage of $\pm 15V$.

The different parameters a, b, c, d of the RDO can be selected by adjusting gain values, voltage division factors and variable resistors of the hardware circuit. In the circuit by using an op-amp based perfect integrator real time integration is performed. To generate cubic type nonlinearity analog multiplier ICs have been used and to change the parameter c from negative to positive a special arrangement is used. The experimental results have been discussed below:

3. Studies on chaotic dynamics with variation of c and strength of sync signal:

At first we select gain value and variable resistors used in the circuit to choose $a = 1, b = 1, d = 0.02$. By varying the variable resistor and bias voltage used in the circuit value of “ c ” can be changed from negative to positive by following the relation. Now by choosing some discrete value of the linear damping parameter

value “c” from negative to positive, minimum strength values of the external sync signal in which RDO becomes chaotic have been determined. To identify the regular and chaotic oscillation we have used phase plane plot and frequency spectrum. To observe phase plane plot frequency spectrum an oscilloscope and a spectrum analyser (specification?) have been used respectively.

We take three discrete values of bias voltage which gives three different values of the linear damping parameter “c”, first one is slight large negative, second is nearly zero and third is slight large positive. Phase plane plot and frequency spectrum for three bias voltages are shown in Fig.2, Fig.3 and Fig.4.

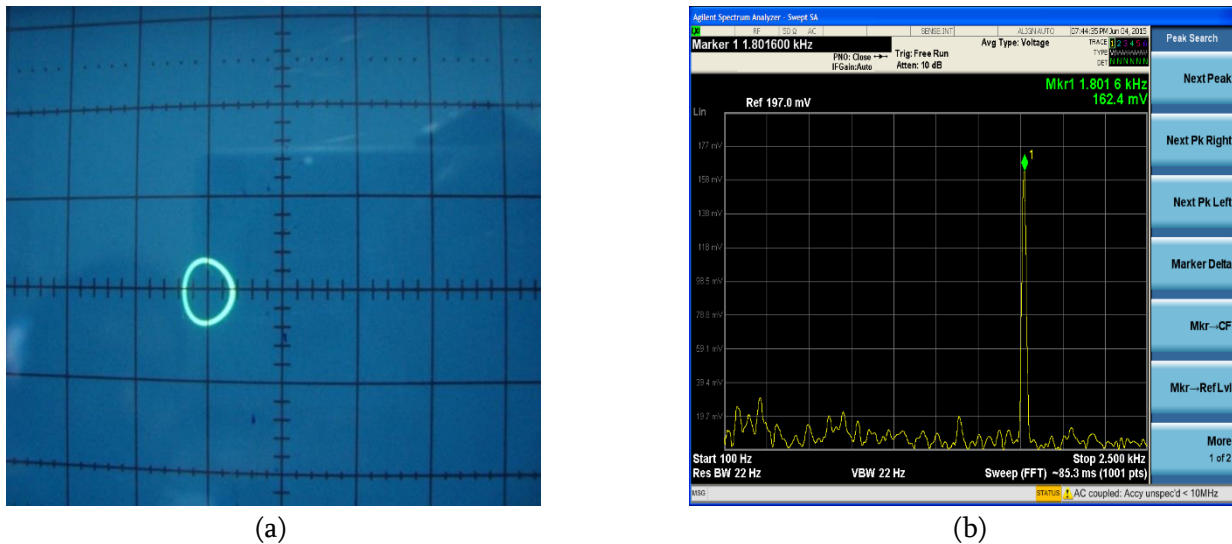


Fig.2. Experimental results showing (a) phase plane trajectory and (b) Frequency spectrum of RDO in presence of external sync signal for biasing voltage $V_s=0.12$ and strength of the sync signal $q_s = 0.1 V$. Other parameters are $a = 1, b = 1, d = 0.02$.

Here, in the Fig.2, phase trajectory is a single close curve and corresponding frequency spectrum consists of a finite peak, which ensures a stable periodic oscillation in RDO.

Now, when biasing voltage is increased slightly to 0.16 by keeping other parameters fixed, then phase trajectory is double scrolled in nature, consisting of many loops intersecting to each other and corresponding frequency spectrum is broad in nature indicating chaotic oscillation in RDO as shown in Fig.3.

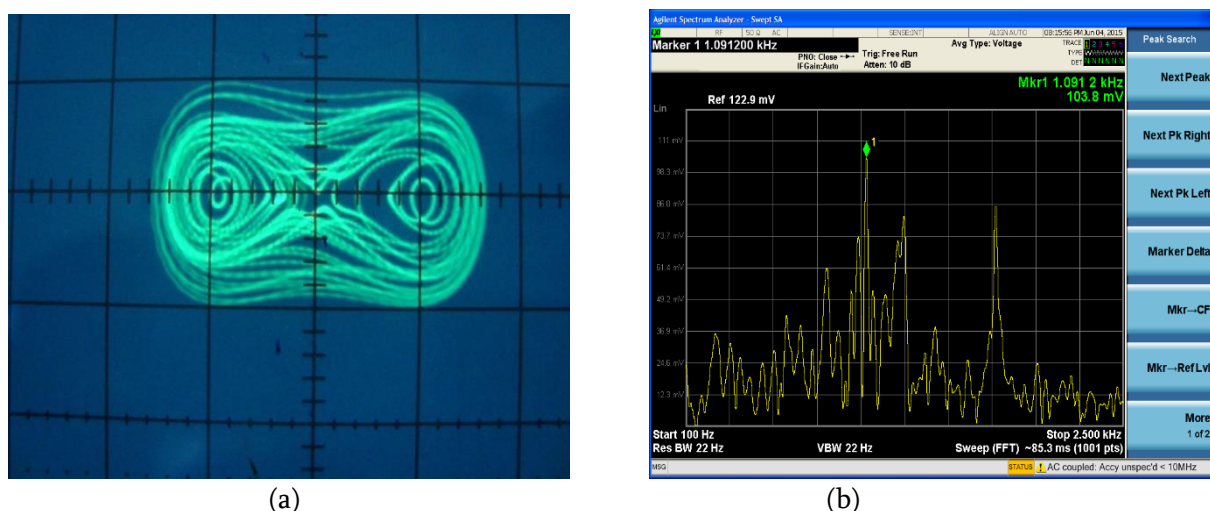


Fig.3. Experimental results showing (a) phase plane trajectory and (b) Frequency spectrum of RDO in presence of external sync signal for biasing voltage $V_s=0.16$ and strength of the sync signal $q_s = 0.1 V$. Other parameters are $a = 1, b = 1, d = 0.02$.

When amplitude of the external voltage is made to 0.27, then phase trajectory is again single loop and corresponding frequency spectrum represents periodic oscillation as shown in Fig.4

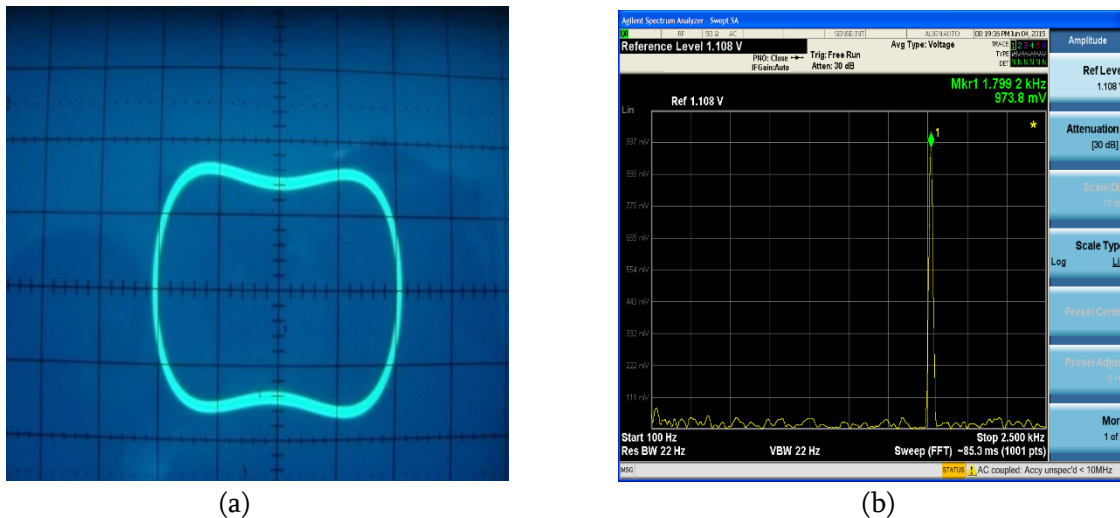


Fig.4. Experimental results showing (a) phase plane trajectory and (b) Frequency spectrum of RDO in presence of external sync signal for biasing voltage $V_s=0.27$ and strength of the sync signal $q_s = 0.1 V$. Other parameters are $a = 1, b = 1, d = 0.02$.

From the experimental results, it has been observed that for a particular values of bias voltage chaotic oscillation in RDO is observed but for low and high value of bias voltage only periodic oscillation occurs.

III. CONCLUSION

Dynamics of driven RDO is studied experimentally. Experimental evidence shows that when a free running RDO is driven by any external signal, then with proper value of amplitude and frequency of sync signal oscillation becomes chaotic. For large and small amplitude of sync signal RDO shows again periodic oscillation. Chaotic oscillation is observed for a limited range of amplitude of sync signal. There exists a range of frequency of sync signal for which chaotic oscillation of RDO is observed. This experimental results would be very helpful to explain the chaotic dynamic of RDO type oscillators such as microwave Gunn oscillator base chaos generators.

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