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Steady state analysis of M/M/1 Queue with Two-Phase, Server Time-Out and Breakdowns

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ABSTRACT

This paper analyzes M/M/1 vacation two-phase queueing model with server Start-up, Time-out and Breakdowns. Customers arrivals are assumed as Poisson process and be given batch mode in the first phase followed by individual models second phase. Arrivals during batch service are allowed to enter the batch without Gating. After providing the second phase of service to all customers individually, server returns to first phase to serve the existing customers followed by individual service. If no one presents, then server waits for a fixed time 'C' is called server Time-out. If units arrived during this fixed time, then the server starts the cycle again by providing them batch service followed by individual service, otherwise after expiration of fixed time he takes a vacation. The server comes back from vacation, after N-customers are accumulated. The server passes a random period as pre-service procedure after coming back from vacation. During individual service the server is susceptible to random failures. Various performance measures are evaluated in steady state. Cost function is established to define the threshold and sensitivity analysis is also presented through numerical examples.

Keywords: Vacation, Two-phase, Pre-service, Time-Out and Server Breakdown.

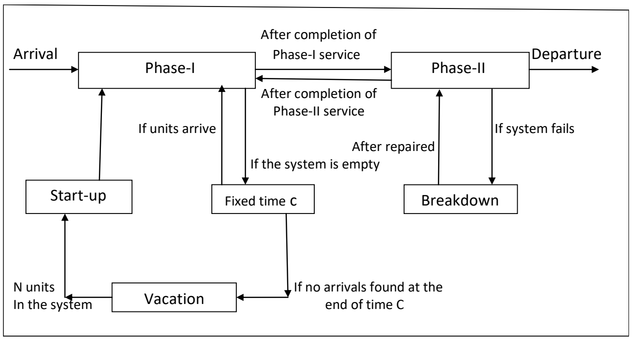
I. INTRODUCTION

We deal with the optimal analysis of M/M/1 two-phase vacation queueing system with server startup, timeout and breakdowns. The two-phase M/M/1 queueing model was first presented by Krishna and Lee(1990).Doshi (1990) studied the two-phase M/G/1 queueing system. Vacation queueing models by using the probability generating function technique was first introduced by Levy and Yechiali. Kim and Chae (1998) analyzed the two-phase queueing system with N-policy. Oliver C.Ibeet.al studied M/M/1 multiple vacation queueing system with differentiated vacation. Olive C.Ibe introduced the timeout concept and he derived mean waiting time of the vacation queue with timeout.

There are many papers in Two-Phase; this paper is an extension to improve server's utility with the concept of Timeout.

II.THE SYSTEM AND ASSUMPTIONS

Arrivals are assumed to follow Poisson process with mean arrival rate λ and join the first phase of batch service. The server delivers service to all the customers with mean service rate $1/\beta$. On completion of batch service, everyone of this batch receive individual service with mean rate of $1/\mu$. In the individual service phase, the server may fail with a failure rate α. It can be instantly repaired with a repair rate β, and resumes service immediately. After this server returns to first phase to serve all the customers if any and provide second phase then. If no one is waiting in batch queue then the server waits for a fixed time 'C' is called sever Timeout. If units arrived during this fixed time he does the service to that unit as batch service followed by individual service. If no units arrived during this fixed time, then it takes a vacation and after N customers accumulates in the batch queue and start pre-service work with mean $1/\theta$. Once the period of startup is ended, the server starts service cycle. This cycle is shown as follows:



III. ANALYSIS OF THE MODEL

Various steady state probabilities of the system are shown below:

$p_{0,i,0}$ (i=0,1,2,3,…) the server is on vacation.

$p_{1,i,0}$ (i=N,N+1,N+2,…) the server is on Start-up.

$p_{2,i,0}$ (i=0,1,2,3,…) the server is on Time-out.

$p_{3,i,0}$ (i=1,2,3,…) the server is in Batch service.

$p_{4,i,j}$ (i=0,1,2,3,… and j=1,2,3…) the server is in individual service.

$p_{5,i,j}$ (i=0,1,2,3,… and j=1,2,3,…) the server breakdown.

The following are the satisfied system size steady state equations:

$$\lambda p_{0,0,0} = Cp_{2,0,0} \quad (1)$$

$$\lambda p_{0,i,0} = \lambda p_{0,i-1,0}\,; \qquad 1 \le i \le N-1 \quad (2)$$

$$(\lambda+\theta)p_{1,N,0} = \lambda p_{0,N-1,0} \quad (3)$$

$$(\lambda+\theta)p_{1,i,0} = \lambda p_{1,i-1,0}; \quad i > N \quad (4)$$

$$(\lambda+C)p_{2,0,0} = \mu p_{4,0,1} \quad (5)$$

$$(\lambda+\beta)p_{3,1,0} = \lambda p_{2,0,0} + \mu p_{4,1,1} \quad (6)$$

$$(\lambda+\beta)p_{3,i,0} = \lambda p_{3,i-1,0} + \mu p_{4,i,1}; \quad 2 \le i \le N-1 \quad (7)$$

$$(\lambda+\beta)p_{3,i,0} = \lambda p_{3,i-1,0} + \mu p_{4,i,1} + \theta p_{1,i,0}; \quad i \ge N \quad (8)$$

$$(\lambda+\alpha+\mu)p_{4,0,j} = \mu p_{4,0,j+1} + \beta p_{3,j,0} + +\gamma p_{5,0,j}; \ \ j \ge 1 \quad (9)$$

$$(\lambda+\alpha+\mu)p_{4,i,j} = \mu p_{4,i,j+1} + \lambda p_{4,i-1,j} + +\gamma p_{5,i,j}; \ \ i,j \ge 1 \quad (10)$$

$$(\lambda+\gamma)p_{5,0,j} = \alpha p_{4,0,j}, \qquad j \geq 1 \tag{11}$$

$$(\lambda+\gamma)p_{5,i,j} = \alpha p_{4,i,j} + \lambda p_{5,i-1,j}; \qquad i,j \geq 1 \tag{12}$$

These equations can be solved with the following PGFs:

$G_0(z) = \sum_{i=0}^{N-1} p_{0,i,0}z^i$, $G_1(z) = \sum_{i=N}^{\infty} p_{1,i,0}z^i$, $G_2(z) = p_{2,0,0}$, $G_3(z) = \sum_{i=1}^{\infty} p_{3,i,0}z^i$,

$G_4(z,y) = \sum_{i=0}^{\infty}\sum_{j=1}^{\infty} p_{4,i,j}z^i y^j$,

$$G_5(z,y) = \sum_{i=0}^{\infty}\sum_{j=1}^{\infty} p_{5,i,j}z^i y^j,$$

$R_i(z) = \sum_{i=0}^{\infty} p_{4,i,j}z^i$ and $S_j(z) = \sum_{i=0}^{\infty} p_{5,i,j}z^i$, $\quad |z|,|y| \leq 1$

From using equations (1) to (12)

$$G_2(Z) = \frac{\lambda}{C}p_{0,0,0} \tag{13}$$

$$G_0(Z) = \frac{(1-z^N)}{(1-z)}p_{0,0,0} \tag{14}$$

$$G_1(Z) = \frac{\lambda z^N p_{0,0,0}}{[\lambda(1-z)+\theta]} \tag{15}$$

$$p_{4,0,1} = \frac{\lambda(\lambda+C)p_{0,0,0}}{\mu C} \tag{16}$$

$$[\lambda(1-z)+\beta]G_3(z) = \mu R_1(z) + \theta G_1(z) + \frac{\lambda}{C}[\lambda(z-1)-C]p_{0,0,0} \tag{17}$$

$$[\lambda(1-z)+\mu+\alpha]R_j(z) = \mu R_{j+1}(z) + \gamma S_j(z) + \beta p_{3,j,0}$$

$$[\lambda y(1-z)+\mu(y-1)+\alpha y]G_4(z,y) = \gamma y G_5(z,y) + \beta y G_3(y) - \mu y R_1(z) \tag{18}$$

$$(\lambda+\gamma)S_j(z) = \alpha R_j(z) + \lambda z S_j(z)$$

$$[\lambda(1-z)+\gamma]G_5(z,y) = \alpha G_4(z,y) \tag{19}$$

The G(z,y) is total p.g.f and given by

$$G(z,y) = G_0(z) + G_1(z) + G_2(z) + G_3(z) + G_4(z,y) + G_5(z,y)$$

The normalizing condition is

$$G(1,1) = G_0(1) + G_1(1) + G_2(1) + G_3(1) + G_4(1,1) + G_5(1,1) = 1 \tag{20}$$

On solving 13 -19 expressions

$$G_0(1) = Np_{0,0,0} \tag{21}$$

$$G_1(1) = \frac{\lambda}{\theta}p_{0,0,0} \tag{22}$$

$$G_2(1) = \frac{\lambda}{C}p_{0,0,0} \tag{23}$$

$$G_3(1) = \frac{\mu}{\beta} R_1(1) \tag{24}$$

$$G_4(1,1) = \frac{\lambda\mu\gamma R_1(1) + \beta\theta\gamma G_1^1(1)}{\beta[\mu\gamma - \lambda(\alpha+\gamma)]} + \frac{\gamma\lambda^2}{C[\mu\gamma - \lambda(\alpha+\gamma)]} p_{0,0,0} \tag{25}$$

$$G_5(1,1) = \frac{\alpha}{\gamma} G_4(1,1) \tag{26}$$

Where

$$p_{0,0,0} = \frac{1 - \left[\frac{\lambda}{\mu}\left(1+\frac{\alpha}{\gamma}\right) + \frac{\lambda}{\beta}\right]}{\left[N + \frac{\lambda}{\theta} + \frac{\lambda}{C}\right]} \tag{27}$$

Normalizing condition (20) leads to

$$R_1(1) = \frac{\lambda}{\mu}$$

And is substituting in equations (24),(25) and (26) leads to

$$G_3(1) = \frac{\lambda}{\beta}$$

$$G_4(1,1) = \frac{\lambda}{\mu}$$

And

$$G_5(1,1) = \frac{\lambda\alpha}{\mu\gamma}$$

The steady state probabilities that the server is in different modes are

P_o, P_1, P_2, P_3, P_4 and P_5 are equals to equations (21), (22), (23), (24), (25) and (26) respectively.

IV.EXPECTED SYSTEM LENGTH

When the server is in different modes the mean number of customers in the system are assumed L_O, L_1, L_2, L_3, L_4 and L_5 are given as

$$L_0 = G_0'(1) = \frac{N(N-1)}{2} p_{0,0,0} \tag{28}$$

$$L_1 = G_1'(1) = \frac{\lambda[\lambda + N\theta]}{\theta^2} p_{0,0,0} \tag{29}$$

$$L_2 = G_2'(1) = 0 \tag{30}$$

$$R_1'(1) = \frac{\lambda^2(\alpha+\gamma)}{\mu^2\gamma}$$

$$L_3 = G_3'(1) = \frac{\lambda}{\beta} \tag{31}$$

$$L_4 = G_4'(1,1) = \frac{\lambda[\lambda^2\beta\alpha+\mu\gamma^2(\lambda+\beta)]}{\gamma\mu\beta[\mu\gamma-\lambda(\alpha+\gamma)]} + \frac{\lambda\gamma[2\lambda(\lambda+N\theta)+\theta^2N(N-1)]}{2\theta^2[\mu\gamma-\lambda(\alpha+\gamma)]} p_{0,0,0} \quad (32)$$

$$L_5 = G_5'(1,1) = \frac{\lambda\alpha}{\gamma^2} G_4(1,1) + \frac{\alpha}{\gamma} G_4'(1,1) \quad (33)$$

The expected system length is

$$L(N) = L_0 + L_1 + L_2 + L_3 + L_4 + L_5 \quad (34)$$

V. SOME OTHER SYSTEM PERFORMANCE MEASURES

Let E_0(idle), E_1(startup), E_2(timeout), E_3(batch service), E_4(individual service) and E_5(breakdown) denotes the expected length of periods of different states and long run fractions are given bellow. Also the cycle expected length is given by

$$E_c = E_0+E_1+E_2+E_3+E_4+E_5 \quad (35)$$

The fractions of time that the server in these different modes is obtained as follows:

$$\frac{E_0}{E_C} = P_0 = G_0(1) = Np_{0,0,0} \quad (36)$$

$$\frac{E_1}{E_C} = P_1 = G_1(1) = \frac{\lambda}{\theta} p_{0,0,0} \quad (37)$$

$$\frac{E_2}{E_C} = P_2 = G_2(1) = \frac{\lambda}{C} p_{0,0,0} \quad (38)$$

$$\frac{E_3}{E_C} = P_3 = G_3(1) = \frac{\lambda}{\beta} \quad (39)$$

$$\frac{E_4}{E_C} = P_4 = G_4(1) = \frac{\lambda}{\mu} \quad (40)$$

And

$$\frac{E_5}{E_C} = P_5 = G_5(1) = \frac{\lambda\alpha}{\mu\gamma} \quad (41)$$

Idle period expected length is

$$E_0 = \frac{N}{\lambda}$$

Using it in (36)

$$\frac{1}{E_C} = \frac{\lambda\left[1-\frac{\lambda}{\mu}\left(1+\frac{\alpha}{\gamma}\right)-\frac{\lambda}{\beta}\right]}{\left[N+\frac{\lambda}{\theta}+\frac{\lambda}{C}\right]} \quad (42)$$

VI. EVALUATION OF OPTIMAL N-POLICY (N*)

We construct a cost function for the present queueing model with the objective to find N that can minimize this function.

For this, we define various costs that incur per unit of time as shown below:

C_h= holding cost for each customer

C_o= operational cost of server

C_m= pre service cost per cycle

C_t= timeout cost per cycle

C_s= setup cost per cycle

C_b= breakdown cost

C_r= reward for the server being on vacation

The function of total expected cost per unit time is given by

$$T(N) = c_h L(N) + c_o\left[\frac{E_3+E_4}{E_c}\right] + c_m\left[\frac{E_1}{E_c}\right] + c_t\left[\frac{E_2}{E_c}\right] + c_b\left[\frac{E_5}{E_c}\right] + c_s\left[\frac{1}{E_c}\right] - c_r\left[\frac{E_0}{E_c}\right] \quad (42)$$

After simplification, this function attains minima where

$$N^* = \frac{-b+\sqrt{b^2-4ac}}{2a} \quad (43)$$

Where $a = \frac{C_h\mu\gamma\theta C}{2[\mu\gamma-\lambda(\alpha+\gamma)]}$, $\quad b = \frac{C_h\mu\gamma\lambda(C+\theta)}{[\mu\gamma-\lambda(\alpha+\gamma)]}$ and

$$c = \frac{C_h\lambda[2\lambda-\mu\gamma]}{2[\mu\gamma-\lambda(\alpha+\gamma)]} - \lambda[C(C_m + C_r) + \theta(C_t + C_r) + C_s\theta C]$$

VII. SENSITIVITY ANALYSIS

We perform numerical experiments to evaluate the impact of various parameters (both non-monetary and monetary) on system performance measures as well as threshold N* as follows:

The sensitivity analysis is carried over by fixing

Non-monetary parameters as λ=0.5, μ=2.5, β=2, Θ=2, ϑ=2, α=0.1, C=1

and monetary parameters as C_{h}=5, C_o=100, C_b=100, C_m=100, C_r=40, C_s=500, C_t=30.

Case I: Non-monetary parameters effect

- Table 1 and Figure 1 show that by increasing the value of λ, N^* and $T(N^*)$ are increases, where as $L(N^*)$ is a convex function of λ.
- Table 2 and Figure 2 show that by increasing the value of μ, N^* and $T(N^*)$ are increases and $L(N^*)$ is decrease.

- Table 3 and Figure 3 show that by increasing the value of β, N^* is constant, $T(N^*)$ is increase and $L(N^*)$ is decrease.
- Table 4 and Figure 4 show that by increasing the value of Θ, N^*, $T(N^*)$ and $L(N^*)$ are decreases.
- Table 5 and Figure 5 show that by increasing the value of ϑ, N^* and $T(N^*)$ are increases, and $L(N^*)$ is decrease.
- Table 6 and Figure 6 show that by increasing the value of α, N^* and $T(N^*)$ are decreases, and $L(N^*)$ is increase.
- Table 7 and Figure 7 show that by increasing the value of C, N^* and $T(N^*)$ are decreases and $L(N^*)$ is increase.
- The following tables: 1, 2, 3, 4, 5, 6 and 7 shows the effect of λ, μ, β, Θ, ϑ, α and C on N^*, $L(N^*)$ and $T(N^*)$ respectively.
- The following figures: 1, 2, 3, 4, 5, 6 and 7 shows the effect of λ, μ, β, Θ, ϑ, α and C on N^*, $L(N^*)$ and $T(N^*)$ respectively.

Table 1:

λ	0.2	0.4	0.6	0.8	1
N*	6.561574	8.658769	9.867117	10.53453	10.78943
L(N*)	2.656696	3.309946	3.345882	2.967855	2.259531
T(N*)	14.26564	45.54419	69.98695	89.3981	104.2638

Figure1:

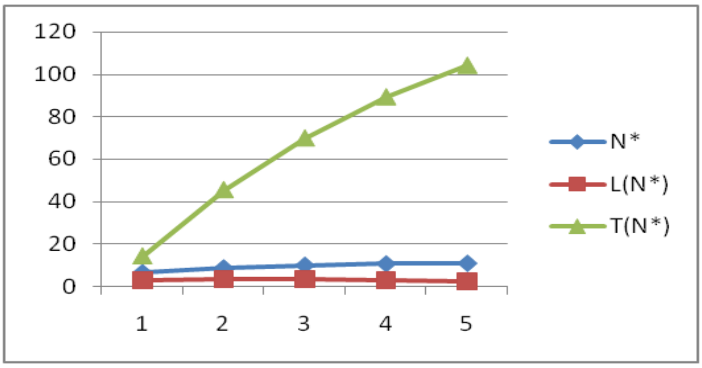


Table 2:

μ	2.5	3	3.5	4	4.5
N*	9.343686	9.563624	9.717893	9.832119	9.920117
L(N*)	3.387616	3.450749	3.497181	3.532626	3.560519
T(N*)	58.45769	54.59423	51.82771	49.74894	48.12979

Figure2:

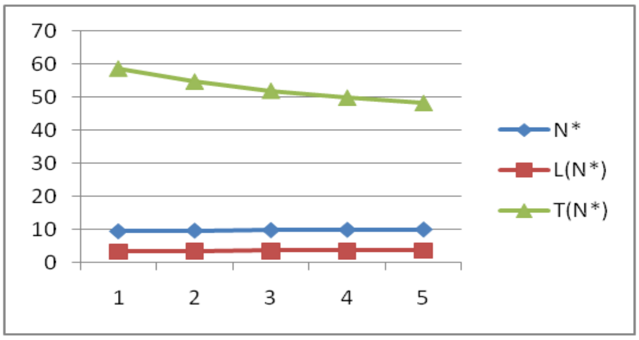


Table 3;

β	2	3	4	5	6
N*	9.343686	9.343686	9.343686	9.343686	9.343686
L(N*)	3.387616	3.714565	3.878039	3.976124	4.041513
T(N*)	58.45769	51.06769	47.37268	45.15568	43.67768

Figure3:

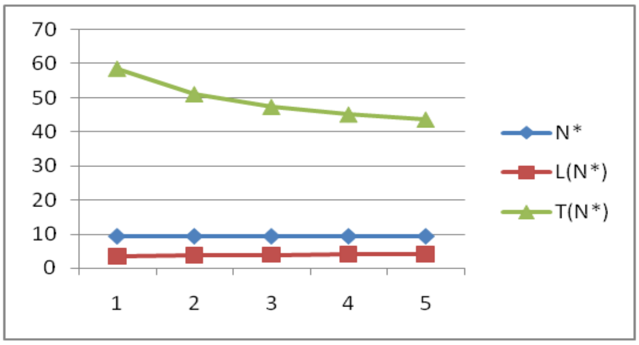


Table 4:

Θ	2	3	4	5	6
N*	9.343686	9.233389	9.177072	9.142895	9.119945
L(N*)	3.387616	3.318067	3.283028	3.261917	3.247806
T(N*)	58.45769	57.80842	57.47998	57.28166	57.14891

Figure4:

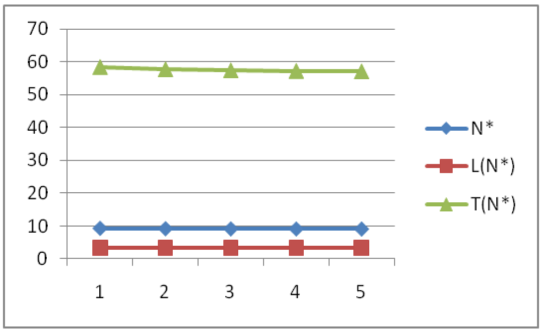


Table 5:

ϑ	2	3	4	5	6
N*	9.343686	9.365623	9.376574	9.383139	9.387513
L(N*)	3.387616	3.392088	3.394601	3.396196	3.397296
T(N*)	58.45769	58.08171	57.89502	57.78342	57.70919

Figure5:

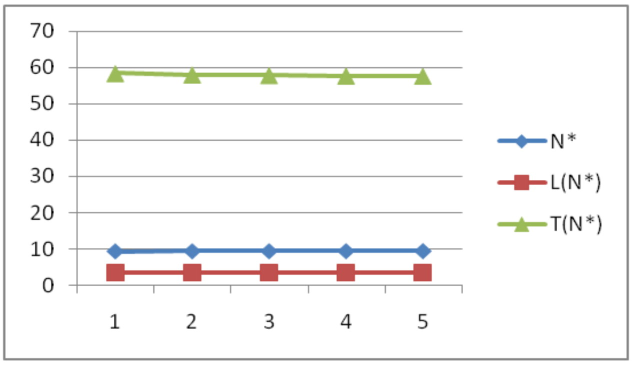


Table 6:

α	0.1	0.2	0.3	0.4	0.5
N*	9.343686	9.28008	9.216067	9.151641	9.086793
L(N*)	3.387616	3.373132	3.358957	3.345107	3.331602
T(N*)	58.45769	59.5748	60.69144	61.80765	62.92347

Figure6:

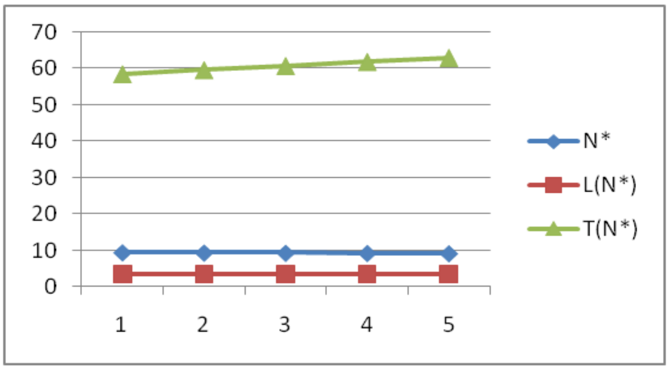
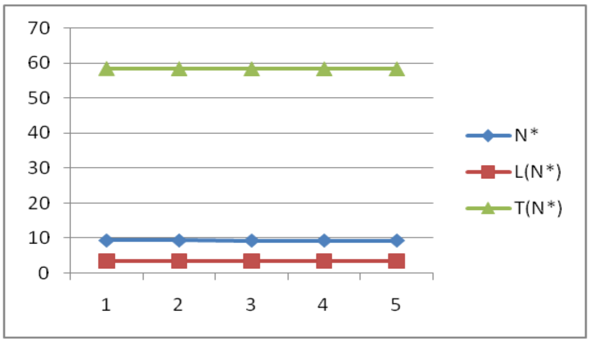


Table 7:

C	1	1.1	1.2	1.3	1.4
N*	9.343686	9.335014	9.327721	9.321502	9.316137
L(N*)	3.387616	3.397356	3.405536	3.412501	3.418505
T(N*)	58.45769	58.42755	58.40221	58.38059	58.36195

Figure7:



Case II: Monetary parameters effect

- Table 8 and Figure 8 show that by increasing the value of C_h, N^* and $L(N^*)$ are decreases, and $T(N^*)$ is increase.
- Table 9, Table 10 and Figure 9, Figure 10 show that by increasing the values of C_0 and C_b, N^* and $L(N^*)$ are constants, and $T(N^*)$ is increase.
- Table 11, Table 13, Table 14 and Figure 11, Figure 13, Table 14 show that by increasing the values of C_m, C_s and C_t, N^*, $L(N^*)$ and $T(N^*)$ are increases.
- Table 12 and Figure 12 show that by increasing the value of C_r, N^* and $L(N^*)$ are increases, and $T(N^*)$ is decrease.
- The following tables: 8, 9, 10, 11, 12, 13 and 14 shows the effect of C_h, C_0, C_b, C_m, C_r, C_s and C_t on N^*, $L(N^*)$ and $T(N^*)$ respectively.
- The following figures: 8, 9, 10, 11, 12, 13 and 14 shows the effect of C_h, C_0, C_b, C_m, C_r, C_s and C_t on N^*, $L(N^*)$ and $T(N^*)$ respectively.

Table 8:

C_h	5	10	15	20	25
N*	9.343686	6.413972	5.121045	4.353185	3.831102
L(N*)	3.387616	2.401034	1.97031	1.717183	1.54686
T(N*)	58.45769	72.53103	83.38723	92.60487	100.7917

Figure8:

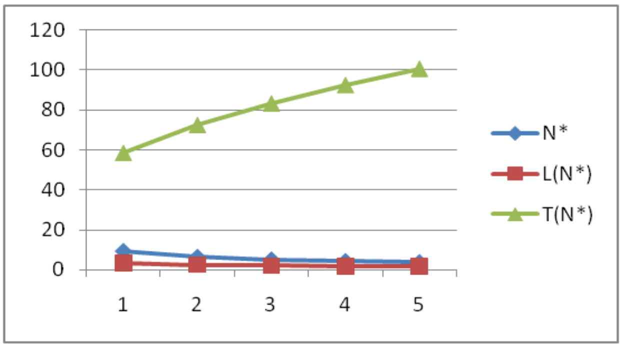


Table 9:

C_0	100	200	300	400	500
N*	9.343686	9.343686	9.343686	9.343686	9.343686
L(N*)	3.387616	3.387616	3.387616	3.387616	3.387616
T(N*)	58.45769	103.4577	148.4577	193.4577	238.4577

Figure9:

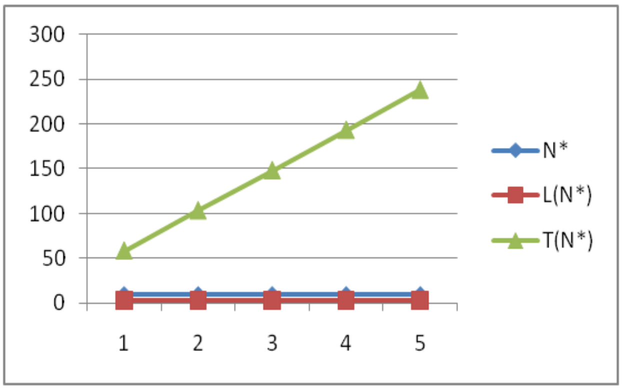


Table 10:

C_b	100	200	300	400	500
N*	9.343686	9.343686	9.343686	9.343686	9.343686
L(N*)	3.387616	3.387616	3.387616	3.387616	3.387616
T(N*)	58.45769	59.45769	60.45769	61.45769	62.45769

Figure10:

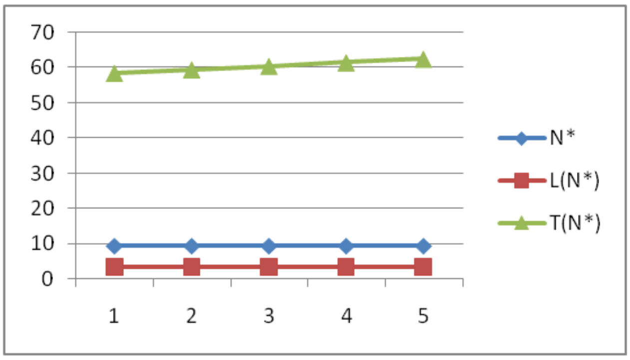


Table 11:

C_m	100	200	300	400	500
N*	9.343686	9.727714	10.09816	10.45636	10.80346
L(N*)	3.387616	3.517547	3.642971	3.764325	3.881981
T(N*)	58.45769	59.76833	61.03273	62.25546	63.44038

Figure11:

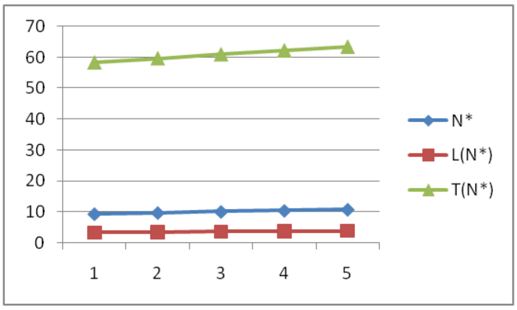


Table 12:

C_r	40	50	60	70	80
N*	9.343686	9.460411	9.575817	9.689947	9.802843
L(N*)	3.387616	3.427098	3.466143	3.504765	3.542978
T(N*)	58.45769	53.45605	48.44991	43.43943	38.42475

Figure12:

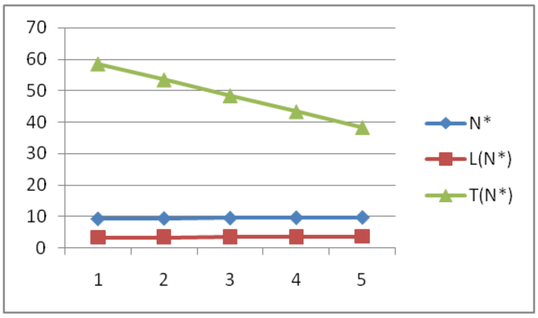


Table 13:

C_s	500	1000	1500	2000	2500
N*	9.343686	12.69926	15.37087	17.65876	19.69217
L(N*)	3.387616	4.52548	5.434089	6.213227	6.906229
T(N*)	58.45769	69.91342	79.03794	86.85336	93.80023

Figure13:

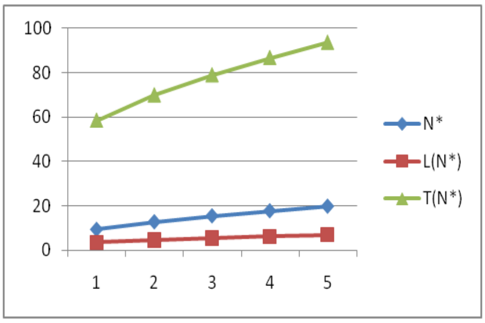
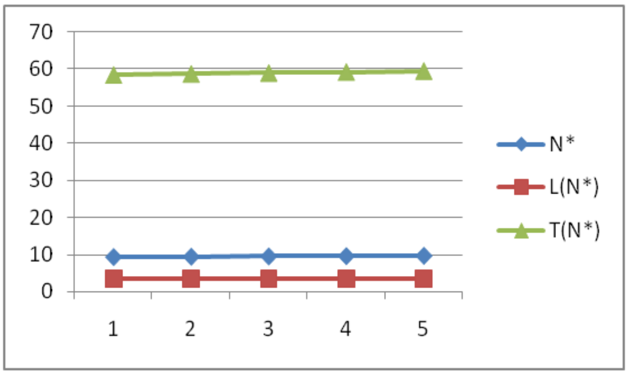


Table 14:

C_t	30	40	50	60	70
N*	9.343686	9.421652	9.499024	9.575817	9.652043
L(N*)	3.387616	3.413987	3.440161	3.466143	3.491937
T(N*)	58.45769	58.72377	58.98782	59.24991	59.51006

Figure14:



VII. CONCLUSIONS

In this model we have obtained explicit expressions for the system length of a queuing model in different modes. Sensitivity analysis made for and numerical values are presented for the different values of monetary and non-monetary parameters to illustrate the validity of the proposed model.

REFERENCES

[1] V.Vasanta Kumar and K.Chandan(2013), Optimal strategy analysis of an N-policy two- phase M/M/1 queueing system with server breakdowns.

[2] Krishna.C.M and Lee.Y.H. (1990). A study of two-phase service. Operation Research Letters,9,91-97

[3] Levy,Y. and Yechiali,U. (1975) Utilization of Idle Time in an M/G/1 Queueing System. Management Science, 22,202211.http://dx.doi. org/10.1287/mnsc.22.2.202

[4] Oliver C. Ibe and Olubukola A.Isijola (2014) M/M/1 multiple Vacation Queueing Systems with differentiated vacations, Modeling and Simulation in Engineering vol 2014(2014),Article ID 158247,6 pages http://dx.doi.org/10.1155/2014/158247.

[5] Oliver c. Ibe(2015) M/G/1 Vacation Queueing Systems with Server Timeout,American Journal,5, 7788.http://www.scrip.org/journal/ajor.

[6] Doshi,B.T(1986)Queueing Systems with Vacations, a Survey. Queueing Systems,1,29-66.

[7] Doshi,B.T.(1991).Analysis of two-phase queueing system with General Service Times.Operations Research Letters,10,265-272.

[8] Kim,T.S. and Chae,K.C.(1998).A two-phase queueing system with threshold,IEEE,502-507.

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