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Queueing Theory - A Multifaceted Analytics Tool

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ABSTRACT

This paper is to present the Queueing theory as an analytical tool for various real time situations. First, we discuss different types of queues with the concepts vacation, Customer's impatience, server failure etc. Second, we look into different practical applications. It provides fundamental concepts of queueing models and their role in system's performance. As we seek to design more and more sophisticated processing systems, Statistical performance evaluation has got lot of significance. The capability to forecast a projected system's performance before one builds it is an extremely cost effective design instrument. In line to this, queueing theory is a tool for analysis of practical problems and has potential applications. Queueing models have wide range of applications in computer communication systems, manufacturing/production systems and inventory systems. There are many extensive works have been done in the Queueing theory over the past five decades. The motivation of this talk is to provide adequate information to analysts, industry people and others who are interested in using queuing theory to model congestion problems and want to locate the details of relevant models.

Keywords : Customer's impatience, Queue, System's performance

INTRODUCTION

Waiting lines or queues are a common phenomenon in life, especially in the province of organizations that are for profit making. Queues are common in such places as petrol or filling stations, supermarkets stores, clinics, hospitals, motor parks, manufacturing firms, to mention a but a few. An interesting aspect of queuing process resides in the measures of its system's performance, especially in terms of average service rate, systems, utilization and the costs implied for a given capacity level.

"Queuing models provide the analysis with a powerful tool for designing and evaluating the performance of queuing systems." Whenever customers arrive at a service facility, some of them have to wait before they receive the desired service. It means that the customer has to wait for his/her turn, may be in a line. Customers arrive at a service facility with several queues, each with one server (sales checkout counter). The customers choose a queue of a server according to some mechanism (e.g., shortest queue or shortest workload).

Delays and queuing problems are most common features not only in our daily-life situations such as at a bank or postal office, at a ticketing office, in public transportation or in a traffic jam but also in more technical environments, such as in manufacturing, computer networking and telecommunications. They play an essential role for business process reengineering purposes in administrative tasks. "Queuing models provide the analyst with a powerful tool for designing and evaluating the performance of queuing systems."

The main purpose of this paper is to review the application of queuing theory and to evaluate the parameters involved in the service unit of various systems.

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In this paper we try to give applications of it in some areas:

1. Inventory
2. Banking
3. Networking

Detailed Review literature is presented as follows:

Queueing in Inventory: Research on queueing systems with inventory control has captured much attention of researchers over the last decades. A wide range of literature that is available on Queueing Inventory System is presented. There are many real time problems arising in the worldwide involving inventory with queueing discipline solved by using the suitable methods.

In this system, customers arrive at the service facility one by one and require service. In order to complete the customer service, an item from the inventory is needed. A served customer departs immediately from the system and the on-hand inventory decreases by one at the moment of service completion.

The inventory is supplied by an outside supplier. This system is called a queueing-inventory system. The queueing-inventory system is different from the traditional queueing system because the attached inventory influences the service. If there is no inventory on hand, the service will be interrupted. Also, it is different from the traditional inventory management because the inventory is consumed at the serving rate rather than the customers' arrival rate when there are customers queued up for service.

(i) Single Server Queueing Systems with Inventory

Maike Schwarz et. al.(2006) proposed the model M/M/1 Queueing systems with inventory. They derived stationary distributions of joint queue length and inventory processes in explicit product form for various M/M/1-systems with inventory under continuous review and different inventory management policies, and with lost sales. Here demand follows Poisson distribution, service times and lead times exponential distribution.

Mohammad Saffari et. al. (2011) considered an M/M/1 queueing system with inventory under the (r, Q) policy and with lost sales, in which demands occur according to a Poisson process and service times are exponentially distributed. All arriving customers during stock out are lost. They derived the stationary distributions of the joint queue length and on hand inventory when lead times are random variables and can take various distributions.

ToktasPalut and U ¨lengin(2015) coordinated the inventory policies in a two-stage decentralized supply chain, where each supplier has been considered as an M/M/1 queue and the manufacture has been assumed as a GI/M/1 queue. Alimardani et al.applied continuous review (S1,S) policy for inventory control and supposed a bi-product three-echelon supply chain which is modelled as an (M/M/ 1) queue model for each type of products offered through the developed network.

(ii) Queueing Inventory System with Stochastic Environment

Chang and Lu(2011)considered a serial production system controlled by the base-stock policy and presented a phase-type approximation for a controlled base-stock serial production system and proposed a cost model to determine the optimal base stock level.

Stephen C.Graves (1982) developed two distinct models for studying inventory systems with continuous production and perishable items. The perishable items have a deterministic usable life after which they must be outdated. For each of the models, analytical expressions derived from queueing theory, are found for the steady-state distribution of system inventory. Knowledge of this steady-state behaviour may be used for evaluation of system performance, and for onsideration of alternatives for improving system performance. The analysis for both models exploits the similarity of the inventory system with a single-server queueing system.

(iii).Queueing Inventory System with Substitution Flexibility

S.M. Seyedhoseiniet.al. (2015)introduced an application of queueing theory in inventory systems with substitution flexibility which can improve profits in many multi-product inventory systems. They prepared a comprehensive substitution inventory model, where an inventory system with two substitute products with ignorable lead time has been considered, and effects of simultaneous ordering have been examined and demands of customers for both of the products have been regarded as stochastic parameters, and queuing theory has been used to construct a mathematical model.

Ning Zhos et.al.(2009) introduced a queueing inventory system with two classes of customers. Customers arrive at a service facility according to Poisson process. Service time follows exponential distribution. Each service uses one item in the attached inventory supplied by an outside supplied by an outside supplier with exponentially distributed lead time. They find a priority service rule to minimize the long run expected waiting cost of dynamic programming method and obtain the necessary and sufficient condition for the priority queueing inventory system being stable. Using Bright- Tailor algorithm, they computed the steady state probability distribution by formulating the model as a level-dependent-quasi-birth-death-process.

Berman and Sapna(2001) studied queueing-inventory systems with Poisson arrivals, arbitrary distribution service times and zero lead times. The optimal value of the maximum allowable inventory which minimizes the long-run expected cost rate has been obtained. Berman and Sapna discussed a finite capacity system with Poisson arrivals, exponential distributed lead times and service times. The existence of a stationary optimal service policy has been proved.

Berman and Kim addressed an infinite capacity queuing-inventory system with Poisson arrivals, exponential service times and exponential lead times. The authors identified a replenishment policy which maximized the system profit. Berman and Kim studied internet based supply chains with Poisson arrivals, exponential service times and the Erlang lead times and found that the optimal ordering policy has a monotonic threshold structure.

(iv).Queueing System with Production-Inventory

Some related works in the production industry are He and Jewkes and He et. al.. He et. al.(2002) studied the inventory replenishment policy of an M/M/1 maketo- order inventory-production system with zero lead times. They explored the structure of the optimal replenishment policy which minimizes the average total cost per product. For the M/PH/1 make-to-order inventory-production system with Erlang distributed lead times, Heet. al. quantified the value of information used in inventory control. A logically related model has been studied by He et al.who analyzed a Markovian inventory production system, where customer demand occurs at a workshop comprising a single machine in a batch of size one.

Jung Woo Baekand Seung Ki Moon(2014) proposed the new model which is “The M/M/1 queue with a production inventory system and lost sales”. The authors considered an extension of the queueing system with inventory in which the stocks are delivered both by an outside supplier and an internal production and called the proposed queueing system as an M/M/1 queue with an attached production-inventory system. Customers arrive in the system according to a Poisson process, and a single server serves the customers. The customers leave the system with exactly one item from the inventory at his service completion epoch. If there is no inventory item, all arriving customers are lost. The stocks are replenished by an external order under (r, Q)-policy, or an internal production. The internal production process is assumed to be a Poisson process. They derived the stationary joint distribution of the queue length and the on-hand inventory in product form. Using the joint distribution, they introduced long-run performance measures and a cost model. Then, they established numerical examples, which minimize the long-run cost per unit time.

(v).Queueing System with Service Inventory

In inventory management point of view, the assembly like queue can be applied to a service-inventory system in which the customers can be served only when the level of the attached inventory is positive. The simplest example is a retail market where customers spend time to pay (the service time) for the item (the inventory) that they want to purchase.

M. Geetha Rani and C. Elango(2012) developed a supply network model for a service facility system with perishable inventory by considering a two dimensional stochastic process of the form (L, X) = {(L(t), X(t)), t ≥ 0)} , where L (t) is the level of the on hand inventory and X (t) is the number of customers at time t. The inter-arrival time to the service station is assumed to be exponentially distributed with mean 1/λ and the service time for each customer is exponentially distributed with mean 1/ μ. The maximum inventory level is S and the maximum capacity of the waiting space is N. The replenishment process is assumed to be (S1, S) with a replenishment of only one unit at any level of the inventory. Lead time is exponentially distributed with parameter β. The items are replenished at a rate of β whose mean replenishment time is 1/β. Item in inventory is perishable when it's utility drops to zero or the inventory item become worthless while in storage. Perishable of any item occurs at a rate of γ. Once entered a queue, the customer may choose to leave the queue at a rate of α if they have not been served after a certain time. They derived the steady state probability distributions for the system states.

Berman and Kim(1999) analysed the situation in a stochastic environment where customers arrive at service facilities in a Poisson process and service times are exponentially distributed with mean inter-arrival times greater than the mean service time and each service require one item from inventory. A logically related model was studied by, He, Q-M, E.M. Jewes and Buzzacott, who analysed a complete Markov production – inventory system, where demands arrive at a workshop and are processed one by one in order. They also considered a service system with an attached inventory, with Poisson customer arrival process, exponential service times and Erlang distribution of replenishment. Their formulation was a Markov decision problem to characterize an optimal inventory policy as a monotonic threshold structure which minimizes system costs.

Saffari.et.al.(2013)considered an arbitrary distribution for the lead time with customers arriving during stock out period lost to the system. They derived the system state distribution as the product of the marginal distributions of the components. Following these pioneering works, there had been numerous other studies on inventory with positive service time.

(vi).Continuous Review Inventory Systems with Server Vacation

Daniel and Ramanarayanan (1987) introduced the concept of server vacation in the inventory system with two servers. In Daniel and Ramanarayanan, they had studied an (s,S) inventory system in which the server took a rest when the level of the inventory became zero. They assumed that the demands that occurred during stock-out period or the server rest periods were lost. The inter-occurrence times between successive demands, the lead times, and the rest times were assumed to follow general distributions. Using renewal and convolution techniques, they obtained expressions involving the steady state transition probabilities.

Narayanan et. al.(2008) considered an inventory system with random positive service time. Customers arrived at the service station according to a Markovian arrival process and the service time for each customer had phase-type distribution. They assumed correlated lead time for the orders and an infinite waiting hall for the customers. The customers who wait for service may renege after a random time. The server took multiple vacations whenever there was no customer waiting in the system or the inventory level was zero. Under the above assumptions, they analysed the level dependent quasi birth-death process

(vii).Queueing Inventory System with Postponed Demands/ Customers.

Padmavathi. I et. al .(2016)study a finite source (s,S) inventory system with postponed demands and server vacation. Their modified M vacation policy is defined as: Whenever the inventory level reaches zero, the server goes to inactive period which comprises the inactive-idle and vacation period. If replenishment occurs during the inactive-idle period, the server becomes active immediately, or otherwise he goes for a vacation period. The server can take at most M inactive

periods repeatedly until replenishment takes place. This inactive-idle time, the vacation time and lead time follow independent PH distributions. After the Mth inactive period, the server remains dormant in the system irrespective of the replenishment of order. Demands that occur during stock out or inactive periods, enter the pool and these demands are selected if the inventory level is above. The inter-selection time follows exponential distribution. The joint distribution

of the mode of the server, server status, the inventory level and the number of demands in the pool is obtained in the steady state. They have derived several system performance measures and total expected cost function.

Krishnamoorthy and Islam (2006)considered (s,S) inventory system with postponed demands in which arrival follows a Poisson process, exponential lead time.

They also assumed that the arriving demands who find the inventory level zero join the pool of finite size and these pooled demands are selected one by one according

to an exponential distributed time lag as long as the inventory level is greater than the reorder point.

Sivakumar and Arivarignan (2007)select the demand from the pool of infinite size until the inventory level drops to prefixed level N(1 ≤ N ≤ s). They assumed that primary and negative demand arrives according to independent Markovian arrival process and the life time, the lead time and the inter-selection time are independently distributed as exponential.

(viii).Queueing Systems with New Inventory Models

Narayanan et. al.(2008)studied a Markovian inventory system with positive service time and Krishnamoorthy et. al. considered a production inventory with service time. In both model, server took multiple vacation due to lack of inventory or the lack of customer or both. Queueing systems with server vacation have been widely studied in the literature.

Amirthakodi.M.et.al.(2015) considered a continuous review perishable inventory system with service facility consisting of finite waiting hall and a single server. The primary customers arrive according to a Markovian arrival process. An arriving customer, who finds the waiting hall is full, is considered to be lost. The individual customer's unit demand is satisfied after a random time of service which is distributed as exponential. The life time of each item is assumed to be exponentially distributed. The items are replenished based on variable ordering policy. The lead time is assumed to have phase type distribution. After the service completion, the primary customer may decide either to join the secondary (feedback) queue, which is of infinite size, or leave the system according to a Bernoulli trial and the server decides to serve either for primary or feedback customer according to a Bernoulli trail. The primary and secondary services are at different counters. The service time for feedback customers is assumed to be independent exponential distribution. After the service completion for feedback customer, the server starts immediately for primary customer's service, whenever the inventory level and the primary customer level is positive, otherwise the server becomes idle for an exponential duration. If the primary customer level and inventory level becomes positive during the server idle period then he starts service for primary customer immediately. After completing his idle period, the server goes to secondary counter to serve for feedback customer, if any. The joint probability distribution of the system is obtained in the steady state. Important system performance measures are derived and the long-run total expected cost rate is also calculated.

Maike Schwarz and Hans Daduna (2011)investigated M/M/1/∞-systems with inventory management, continuous review, exponentially distributed lead times and backordering. They compute performance measures and derive optimality conditions under different order policies. For performance measures, which are not explicitly at hand, we present an approximation scheme for all possible parameter combinations. Although they cannot completely determine analytically the steady state probabilities for the system, they are able to derive functional relations between interesting probabilities and show surprising insensitivity properties of several performance measures.

Queueing in Banking:

Nityangini Jhala and Pravin Bhathawala (2016) have described a methodology designed to support the decision-making process by the banks to meet the demand. In order to determine an optimum number of servers, queuing theory is applied. The effect of queuing in relation to the time spent by customers to access bank services is increasingly becoming a major source of concern. This is because keeping customers waiting too long could result to cost to them (waiting cost). Providing too much service capacity to operate a system involves excessive cost. But not providing enough service capacity results in excessive waiting time and cost. In this study, the queuing characteristics at XYZ bank was analyzed using a Multi-server queuing Model. The Waiting and service Costs were determined with a view to determining the optimal service level.

Ahmed S. A. AL-Jumaily and Huda K. T. AL-Jobori (2011) have focused on the banks lines system, the different queuing algorithms that are used in banks to serve the customers, and the average waiting time. The aim of this paper is to build automatic queuing system for organizing the banks queuing system that can analyses the queue status and take decision which customer to serve. The new queuing architecture model can switch between different scheduling algorithms according to the testing results and the factor of the average waiting time. The main innovation of this work concerns the modeling of the average waiting time is taken into processing, in addition with the process of switching to the scheduling algorithm that gives the best average waiting time.

Queueing in Networking:

Advances in Queueing Theory and Network Applications presents several useful mathematical analyses in queueing theory and mathematical models of key technologies in wired and wireless communication networks such as channel access controls, Internet applications, topology construction, energy saving schemes, and transmission scheduling. In sixteen high-quality chapters, this work provides novel ideas, new analytical models, and simulation and experimental results by experts in the field of queueing theory and network applications. The text serves as a state-of-the-art reference for a wide range of researchers and engineers engaged in the fields of queueing theory and network applications, and can also serve as supplemental material for advanced courses in operations research, queueing theory, performance analysis, traffic theory, as well as theoretical design and management of communication networks.

Today's computer systems are more complex, more rapidly evolving, and more essential to the conduct of business than those of even a few years ago. The result is an increasing need for tools and techniques that assist in understanding the behavior of these systems. Such an understanding is necessary to provide intelligent answers to the questions of cost and performance that arise throughout the life of a system: 0 during design and implementation - An aerospace company is designing and building a computer-aided design system to allow several hundred aircraft designers simultaneous access to a distributed database through graphics workstations. Early in the design phase, fundamental decisions must be made on issues such as the database accessing mechanism and the process synchronization and communication mechanism. The relative merits of various mechanisms must be evaluated prior to implementation. - A computer manufacturer is considering various architectures and protocols for connecting terminals to mainframes using a packet oriented broadcast communications network. Should terminals be clustered? Should packets contain multiple characters? Should characters from multiple terminals destined for the same mainframe be multiplexed in a single packet? l during sizing and acquisition - The manufacturer of a turn-key medical information system needs an efficient way to size systems in preparing bids. Given estimates of the arrival rates of transactions of various types, this vendor must project the response times that the system will provide when running on various hardware configurations. B. Filipowicz And J. Kwiecień(2008) described queueing systems and queueing networks which are successfully used for performance analysis of different systems such as computer, communications, transportation networks and manufacturing. It incorporates classical Markovian systems with exponential service times and a Poisson arrival process, and queueing systems with individual service. Oscillating queueing systems and queueing systems with Cox and Weibull service time distribution as examples of non-Markovian systems are studied. Jackson's, Kelly's and BCMP networks are also briefly characterized. The model of Fork-Join systems applied to parallel processing analysis and the FES approximation making possible of Fork-Join analysis is also presented. Various types of blocking representing the systems with limited resources are briefly described. In addition, examples of queueing theory applications are given. The application of closed BCMP networks in the health care area and performance evaluation of the information system is presented. In recent years the application of queueing systems and queueing networks to modelling of human performance arouses researchers' interest. Finally this paper has given an architecture called the Queueing Network-Model Human Processor is presented.

References:

1. Maike Schwartz(2006),M/M/1 Queueing systems with inventory, *Queueing Systems* **volume 54**, pages55–78
2. Mohammad Safari et al(2011), A queueing system with inventory and mixed exponentially distributed lead times,*The International Journal of Advanced Manufacturing Technology* **volume 53**, pages1231–1237
3. J. Ren, Hualong Xie (2015),Coordinating a Multi-Retailer Decentralized Distribution System with Random Demand Based on Buyback and Compensation Contracts,Journal of Industrial Engineering and Management
4. Kuo-Hwa Chang(2011),Inventory management in a base-stock controlled serial production system with finite storage space,Mathematical and Computer Modelling ,54(11-12):2750-2759,10.1016/j.mcm.2011.06.063
5. Stephen C. Graves (1982),The Application of Queueing Theory to Continuous Perishable Inventory Systems,Management Science 28(4):400-406,DOI: 10.1287/mnsc.28.4.400
6. Liming Liu, et al (2016), Analysis and Optimization of a Multistage Inventory-Queue System, Management Science.50(3); 365-380.
7. S. M. Seyedhoseini et al (2015), Application of queuing theory in inventory systems with substitution flexibility,Journal of Industrial Engineering International volume 11, pages37–44

8. Ning Zhao et al (2009),A queueing-inventory system with two classes of customers, International Journal of Production Economics,Volume 129, Issue 1, January 2011, Pages 225-231
9. Berman, O, Sapna K.P(2001). Optimal control of service for facilities holding inventory. Computers and Operations Research. 28 (5); 2001: 429–441
10. He Q.M, Jewkes E M and Buzacott J. (2002) Optimal and near-optimal inventory policies for a make to order inventory-production system. European Journal of Operational Research. 141; : 113-132.
11. Jung Woo Baek, Seung Ki Moon. The M/M/1 queue with a production-inventory system and lost Sales. Applied Mathematics and Computation. 233; 534–544.
12. Saffari .M, Haji. R (2009),Queueing system with inventory for two echelon supply chain. CIE Int. Conference., 835–838.
13. Berman O, Kim, (1999)E. Stochastic models for inventory management at service facilities. Stochastic Models 15(4); 695–718.
14. Geetha rani. M and Elango.C.(2012) Markov Process for Service Facility systems with perishable inventory and analysis of a single server queue with reneging. Stochastic Model, International Journal of Computer Applications. 44(15); :18-23.
15. Narayanan V. C, Deepak T. G, Krishnamoorthy A and Krishnakumar, B(2008). On an (s,S) inventory policy with service time, vacation to server and correlated lead time. Quality Technology and Quantitative Management. 5(2);129–143.
16. Daniel JK, Ramanarayanan R.(1987)An inventory system with two servers and rest periods. In Cahiers du C.E.R.O Bruxelles: University Libre De Bruxelles. 29; 95–100.
17. Sivakumar B, Arivarignan G. (2007)An inventory system with postponed demands. Stoch. Anal. Appl. 26(1); 84–97.
18. Padmavathi B, Sivakumar G and Arivarignan(2015), A retrial inventory system with single and modified multiple vacation for server, Ann Oper Res. 233; 335–364. DOI 10.1007/s10479-013-1417-187.
19. Krishnamoorthy A, Deepak TG, Narayanan V C and Vineetha K.(2006), Effective utilization of idle time in an (s,S) inventory with positive service time. Journal of Applied Mathematics and Stochastic Analysis.
20. Maike Schwarz et al (2006), M/M/1 Queueing systems with inventory. Queueing systems. 54(1); pp.55-78.
21. Amirthakodi, M., Radhamani, V.,Sivakumar,B.A (2015), perishable inventory system with service facility and feedback customers, Ann. Oper. Res. 233; 25–55 DOI 10.1007/s10479-015-1821-9.
22. Nityangini Jhala(2016), Application Of Queuing Theory In Banking Sector, IOSR Journal of Mathematics (IOSR-JM),e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 12, Issue 2 Ver. II (Mar. - Apr. 2016), PP 73-75
23. Ahmed S. A. AL-Jumaily and Huda K. T. AL-Jobori(2011), Automatic Queuing Model for Banking Applications, International Journal of Advanced Computer Science and Applications(IJACSA), Volume 2 Issue 7, 2011.
24. B. FILIPOWICZ(2008), Queueing systems and networks. Models and applications, BULLETIN OF THE POLISH ACADEMY OF SCIENCES TECHNICAL SCIENCES Vol. 56, No. 4

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