



Approximate value of a irrational number using Duplex

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ABSTRACT

Swami Bharthi Krshnatrthji Maharaj is the author of book Vedic Mathematics. This book was first published in 1965. This book contains many mathematical calculations in various fields of mathematics. In this book Swamiji explained by coating examples in each chapter. All the formulas in Vedic Mathematics have a logical mathematical application. Some important most useful chapters in Vedic Mathematics are Division by Paravartya method or Dwajank method, recurring decimals, Complex Mergers, Partial Fractions etc. In this paper we will discuss the method of straight squaring (to obtain square of a number) by creating duplex (Dwandwa Yoga). This method is very powerful to obtain the square of a number. In the same chapter Swamiji explained that the reverse process of finding a square of a number (By creating duplex) gives us square root of a number. Swamiji developed this concept of duplex (Dwandwa Yoga) from algebra. In the expansion of $(ax + b)^2$, $(ax^2 + bx + c)^2$, $(ax^3 + bx^2 + cx + d)^2$,..... the coefficients of descending powers are always a^2 , $2ab$, $(b^2 + 2ac)$, $(2ad + 2bc)$,..... Swamiji called these coefficients as duplex. So duplex of single digit number is square of itself, square of two digit number is two times the product of two numbers, duplex of three digit number is sum of square of the middle number and two times the product of first and third number, and so on. Putting these duplex at proper place we obtain the square of a number. Similarly if we subtract the duplex from the given number we obtain the square root of a number. This method has a powerful application of finding the approximate value of irrational numbers (a number which have non recurring and non repeating decimal numbers). Irrational numbers are real numbers and we never obtain its exact value. So we obtain its approximate value always. Here we discuss how to find the approximate value of irrational number by creating duplex (Dwandwa Yoga).

I. INTRODUCTION

This method of finding the approximate value of irrational number is very simple. This method is applicable for the computer programming. This method of finding the square of a number is the application of algebra in arithmetic. Finding the square root is the reverse process of finding the square.

II. METHOD AND PROCEDURE

The Dwandwa Yoga or The Duplex combination process

How to generate the duplex ?

- 1) For single digit number 'a' duplex of 'a' = $D(a) = a^2$
- 2) For two digit number 'ab' duplex of 'ab' = $D(ab) = 2ab$

- 3) For three digit number 'abc' duplex of 'abc'
 $=D(abc) = 2ac + b^2$
- 4) For four digit number 'abcd' duplex of 'abcd'
 $=D(abcd) = 2ad + 2bc$

Square of two digit number ab that is $ax + b$ $x = 10$

$$(ab)^2 = (ax + b)^2 = a^2x^2 + 2abx + b^2 = a^2 / 2ab / b^2 = D(a) / D(ab) / D(b)$$

Find the square of following numbers by duplex method

- (1) 23, (2) 78, (3) 28, (4) 31, (5) 54, (6) 83, (7) 7.4, (8) 44, (9) 91, (10) 39

Square of three digit number abc that is $ax^2 + bx + c$

$$(abc)^2 = (ax^2 + bx + c)^2 = a^2x^4 + 2abx^3 + (2ac + b^2)x^2 + 2bcx + c^2$$

$$= a^2 / 2ab / (2ac + b^2) / 2bc / c^2$$

$$= D(a) / D(ab) / D(abc) / D(bc) / D(c)$$

Find the square of following numbers by duplex method

- (1) 103, (2) 624, (3) 567, (4) 9.38, (5) 220, (6) 396, (7) 481, (8) 808, (9) 514, (10) 286

Square of four digit number abcd

$$(abcd)^2 = (ax^3 + bx^2 + cx + d)^2$$

$$= a^2 / 2ab / (2ac + b^2) / (2ad + 2bc) / (2bc + c^2) / 2cd / d^2$$

$$= D(a) / D(ab) / D(abc) / D(abcd) / D(bcd) / D(cd) / D(d)$$

Find the square of following numbers by duplex method

- (1) 1111, (2) 2308, (3) 8568, (4) 36.68, (5) 2219, (6) 2694, (7) 4186, (8) 7059

Special cases

1) **Yavadunam** used when numbers are nearer to 10, 100, 1000,.....

$$(104)^2 = (104 + 4) / 4^2 = 10816$$

Find the square of the numbers 80 to 99 and 101 to 120.

2) **Ekaadhikena Purvena** means one more than the previous

Square of number ending with 5

$$(35)^2 = 3 \cdot 4 / 25 = 1225$$

Find the square of following numbers

- (1) 65, (2) 475, (3) 1235, (4) 105, (5) 55, (6) 26.5 (7) 355, (8) 6065, (9) 135, (10) 215

3) Squares of numbers near 50, 500 or 5000

Table 1

Base	Number N Near to	Deviation (a)	LP	RP	Digits in RP
100	50	a = N-50	25 + a	a ²	2
1000	500	a = N-500	250 + a	a ²	3
10000	5000	a = N-5000	25 00 + a	a ²	4

LP = left part, RP = right part

$$54^2 = 25 + 4 / 4^2 = 29 / 16 = 2916 \quad 48^2 = 25 + \bar{2} / \bar{2}^2 = 23/04 = 2304$$

Find the square of following numbers

- (1) 510, (2) 497, (3) 5015, (4) 4992, (5) 58, (6) 49, (7) 502, (8) 482, (9) 491, (10) 4995, (11) 5007, (12) 5011

Find the square of following numbers

- (1) 39, (2) 199, (3) 604, (4) 2011, (5) 7992, (6) 788, (7) 685, (8) 3021

Find by duplex method

- (1) $42^2 + 612^2$, (2) $72^2 - 37^2$, (3) $213^2 + 65^2 - 189^2$, (4) $3241^2 + 4035^2$, (5) $44^2 + 88^2$, (6) $34.6^2 + 2.35^2$, (7) $49^2 + 14^2 - 49^2$, (8) $231^2 + 425^2$, (9) $45.6^2 + 38.9^2$,

III. SQUARE ROOT OF A NUMBER

There are two methods to find square root of number.

- 1) Vilokanam means by observation,
- 2) Dvandvayogah means by adding duplexes

Facts

1) The squares of first nine natural numbers are 1, 4, 9, 16, 25, 36, 49, 64, & 81. Observe that 1, 4, 5, 6, 9 are digits at unit place. Hence no perfect square no has 2, 3, 7 & 8 at its unit place.

2) Square of 1 and 9 has 1 at its unit place. Square of 2 and 8 has 4 at its unit place. Square of 3 and 7 has 9 at its unit place. Square of 4 and 6 has 6 at its unit place. Square of 5 has 5 at its unit place.

3) Conversely for any **perfect square number**, digit at its unit place and 'digit at units' place' of square root are related as follows.

Table 2

Unit place digit of Perfect square number	Unit place digit of Square root
1	1 or 9
4	2 or 8
5	5
6	4 or 6
9	3 or 7

4) Number with odd numbers of zeros as consecutive right most digits is not a perfect square number.

Square root by Vilokanam:- This method is most suitable for perfect square number which has atmost four digit. **Method:-** 1) Group the numbers in two parts. Right part (RP) will consists of unit and tens place and left part will have remaining one or two digits.

2) Select a number p (say) whose square is nearest less than last part.

3) Observe the unit place digit of RP and choose possible unit place digit of square root. Let d_1 and d_2 be the digits as in the table.

Find the Square root of 20439441

$$\begin{array}{r|l|l|l|l|l|l}
 & 2 & 0 & 4 & 3 & 9 & 4 & 4 & 1 \\
 & & & 40 & 40 & 20 & 10 & 0 & 0 \\
 & & & & \bar{2} \bar{5} & \bar{2} \bar{0} & \bar{1} \bar{4} & \bar{4} & \bar{1} \\
 2a = 8 & \underline{\hspace{1cm}} & \underline{44} & \underline{18} & \underline{9} & \underline{0} & \underline{0} & \underline{0} & \underline{0} \\
 & \mathbf{4} & \mathbf{5} & \mathbf{2} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}
 \end{array}$$

4) Then square root of given number is pd_1 or pd_2 . Observe $pd_1 \leq p5^2 \leq pd_2$.

5) This comparison gives exact square root.

Find the square root of the following numbers by vilokanam.

- (1) 784, (2) 2304, (3) 6561, (4) 5929, (5) 7056, (6) 2025, (7) 1369, (8) 2704, (9) 676, (10) 4356

Square root by Dwandvayoga:- This method is applicable to evaluate square root of any number. This is the reverse process to find the square of number by dwandvayoga.

1) Group the digits by selecting two at a time from right to left. The left part (LP) may contain one or two digits.

2) Select number 'a' (say) whose square is nearest less than LP and write 'a' as first digit in answer.

3) Find $r = LP - a^2$ and add $r*10$ to next right place. Select $2a$ as divisor. Divide the number obtain after adding $r*10$ by $2a$. The quotient is b which is second digit of answer. Remainder is r_1 . Add r_1*10 to next right place. Obtain the number by adding r_1*10 . Form which subtract b^2 . Divide the remaining number by $2a$. The quotient is c which third digit of answer. Remainder is r_2 . Add r_2*10 to next right place. Obtain the number by adding r_2*10 . Form which subtract $2bc$. Divide the remaining number by $2a$. The quotient is d which fourth digit of answer. And the process is continued till we obtain the answer.

Square root of 20439441 is ± 4521

Find the Square root of 1052 up to two decimal places.

$$\begin{array}{r}
 \begin{array}{|c|c|} \hline 1 & 0 \\ \hline \end{array} & \begin{array}{|c|c|} \hline 5 & 2 \\ \hline \end{array} & & & \\
 \begin{array}{|c|c|} \hline & \\ \hline \end{array} & \begin{array}{|c|c|} \hline 10 & 30 \\ \hline \end{array} & \begin{array}{|c|c|} \hline 40 & 60 \\ \hline \end{array} & & \\
 \begin{array}{|c|c|} \hline & \\ \hline \end{array} & \begin{array}{|c|c|} \hline & \bar{4} \\ \hline \end{array} & \begin{array}{|c|c|} \hline & \bar{16} \\ \hline \end{array} & \begin{array}{|c|c|} \hline & \bar{28} \\ \hline \end{array} & \\
 2a = 6 & \begin{array}{|c|c|} \hline & \\ \hline \end{array} & \begin{array}{|c|c|} \hline 15 & 28 \\ \hline \end{array} & \begin{array}{|c|c|} \hline 24 & 32 \\ \hline \end{array} & \\
 \begin{array}{|c|c|} \hline & \\ \hline \end{array} & \begin{array}{|c|c|} \hline 3 & \\ \hline \end{array} & \begin{array}{|c|c|} \hline 2 & 4 \\ \hline \end{array} & \begin{array}{|c|c|} \hline 3 & 0 \\ \hline \end{array} &
 \end{array}$$

Square root of 1052 is ± 32.43

Find the Square root of the following number.

- (1) 625, (2) 1521, (3) 61009, (4) 1849, (5) 46225, (6) 32761, (7) 6482116,
(8) 915849

Find the Square root of the following number up to two decimal places.

- (1) 58, (2) 45, (3) 652,

IV. CONCLUSIONS

This method has a powerful application of finding the approximate value of irrational numbers (a number which have non recurring and non repeating decimal numbers). Irrational numbers are real numbers and we never obtain its exact value. So we obtain its approximate value always.

V. REFERENCES

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