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Non-Classical Thermoelasticity in a Half Space under the influence of a Heat Source

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ABSTRACT

A two dimensional problem for an infinite half space is formulated, to study the thermoelastic response due to the presence of a heat source varying periodically with time. The Lord-Shulman theory of thermoelasticity with one relaxation time is considered. The bounding surface is traction free and subjected to a known temperature distribution. Integral transform technique is developed to find the analytic solution in the transform domain by using direct approach. Inversion of transforms is done by employing Gaver-Stehfast algorithm. Mathematical model is prepared for Copper material and numerical results for temperature, displacements and stresses thus obtained are illustrated graphically.

Keywords: Thermoelastic; half-space; Lord-Shulman; heat source.

I. INTRODUCTION

Thermoelastic problems are used to study the thermal stresses in an elastic body under high temperature gradients. The problems of thermoelasticity are broadly classified into two categories, namely static and dynamic problems. The problems dealing with dynamic thermal stresses are fundamentally important in engineering processes and have paved the way for technologies which operate in high temperatures such as nuclear reactors, aerodynamic structures, etc. The classical coupled thermoelasticity theory finds its first mention in Biot [1]. In non-classical theories of thermoelasticity, the Fourier heat conduction equation is generalized with the introduction of one relaxation time obtained by Lord and Shulman [2]. Various authors [3-8] contributed to the problems on generalized thermoelasticity. Recently, a lot of interest has developed in fractional order theory of thermoelasticity [9-15].

In this paper, a non-classical thermoelastic problem in a half space with a heat source is studied. The bounding surfaces are free of all loadings and subjected to a known temperature distribution. Gaver-Stehfast algorithm [7-9] is used to invert the Laplace transforms. All the integrals were evaluated using Romberg's integration technique [10] with variable step size.

II. FORMULATION OF THE PROBLEM

Consider a homogeneous isotropic thermoelastic solid occupying the region $z \geq 0$ and $0 < r < \infty$ .The z-axis is perpendicular to the bounding plane. The problem formulation is under the perview of Lord-Shulman theory of generalized thermoelasticity with one relaxation time. We shall assume that the initial state of the medium is quiescent at a temperature T_0. The surface of the medium is free from mechanical loads and a known temperature distribution is applied. A heat source is applied on the domain. Cylindrical polar coordinates (r,φ,z) are used.

The problem is thus two-dimensional with all functions considered depending on the spatial variables r and z as well as on the time variable t.

The displacement vector, thus, has the form $\vec{u}=(u,0,w)$.

The equations of motion can be written as

$$\mu\nabla^2 u-\frac{\mu}{r^2}u+(\lambda+\mu)\frac{\partial e}{\partial r}-\gamma\frac{\partial T}{\partial r}=\rho\frac{\partial^2 u}{\partial t^2} \qquad (1)$$

$$\mu\nabla^2 w+(\lambda+\mu)\frac{\partial e}{\partial z}-\gamma\frac{\partial T}{\partial z}=\rho\frac{\partial^2 w}{\partial t^2} \qquad (2)$$

The generalized equation of heat conduction has the form

$$k\nabla^2 T=\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)(\rho C_E T+\gamma T_0 e) -\rho\left(1+\tau_0\frac{\partial}{\partial t}\right)Q \qquad (3)$$

where T is the absolute temperature, ρ is the density of the medium, τ_0 is the relaxation time, Q is the heat source and e is the cubical dilatation given by the relation

$$e=\frac{u}{r}+\frac{\partial u}{\partial r}+\frac{\partial w}{\partial z}=\frac{1}{r}\frac{\partial}{\partial r}(ru)+\frac{\partial w}{\partial z} \qquad (4)$$

$$\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2} \qquad (5)$$

The following constitutive relations supplement the above equations

$$\sigma_{rr}=2\mu\frac{\partial u}{\partial r}+\lambda e-\gamma(T-T_0) \qquad (6)$$

$$\sigma_{zz}=2\mu\frac{\partial w}{\partial z}+\lambda e-\gamma(T-T_0) \qquad (7)$$

$$\sigma_{rz}=\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}\right) \qquad (8)$$

We shall use the following non-dimensional variables

$$r'=c_1\eta r,\ z'=c_1\eta z,\ u'=c_1\eta u,\ w'=c_1\eta w,\ t'=c_1^2\eta t,$$

$$\tau_0'=c_1^2\eta\tau_0,\ \sigma_{ij}'=\frac{\sigma_{ij}}{\mu},\ \theta=\frac{\gamma(T-T_0)}{(\lambda+2\mu)},\ Q'=\frac{\rho\gamma Q}{k c_1^2\eta^2(\lambda+2\mu)}$$

where $\eta=\frac{\rho c_E}{k}$, $c_1=\sqrt{\frac{\lambda+2\mu}{\rho}}$ is the speed of propagation of isothermal elastic waves.

Using the above non-dimensional variables, the governing equations take the form (dropping the primes for convenience)

$$\nabla^2 u-\frac{u}{r^2}+\left(\beta^2-1\right)e-\beta^2\frac{\partial\theta}{\partial r}=\beta^2\frac{\partial^2 u}{\partial t^2} \qquad (9)$$

$$\nabla^2 w+\left(\beta^2-1\right)\frac{\partial e}{\partial z}-\beta^2\frac{\partial\theta}{\partial z}=\beta^2\frac{\partial^2 w}{\partial t^2} \qquad (10)$$

$$\nabla^2\theta=\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)(\theta+\varepsilon e)-\left(1+\tau_0\frac{\partial}{\partial t}\right)Q \qquad (11)$$

while the constitutive relations (6)-(8), becomes

$$\sigma_{rr}=2\frac{\partial u}{\partial r}+\left(\beta^2-2\right)e-\beta^2\theta \qquad (12)$$

$$\sigma_{zz}=2\frac{\partial w}{\partial z}+\left(\beta^2-2\right)e-\beta^2\theta \qquad (13)$$

$$\sigma_{rz}=\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}\right) \qquad (14)$$

Here $\beta^2=\frac{(\lambda+2\mu)}{\mu}$

Combining equations (9) and (11), we obtain upon using equation (5),

$$\nabla^2 e-\nabla^2\theta=\frac{\partial^2 e}{\partial t^2} \qquad (15)$$

We assume that the initial state is quiescent, that is, all the initial conditions of the problem are homogeneous.

The thermal and mechanical boundary conditions of the problem at $z=0$ are taken as

$$\theta(r,0,t)=f(r,t),\quad 0<r<\infty \qquad (16)$$

$$\sigma_{zz}(r,0,t)=0,\quad 0<r<\infty \qquad (17)$$

$$\sigma_{rz}(r,0,t)=0,\quad 0<r<\infty \qquad (18)$$

where $f(r,t)$ are known function of r and t.

Eqns. (1)-(18) constitute the generalized thermoelastic formulation of the problem on axisymmetric half space.

III. SOLUTION OF THE PROBLEM

Applying the Laplace transform defined by the relation,

$$\bar{f}(r,z,s) = L[f(r,z,t)] = \int_0^\infty e^{-st} f(r,z,t)dt \qquad (19)$$

to all the non-dimensional equations (9)-(18), we get,

$$\nabla^2\bar{u} - \frac{\bar{u}}{r^2} + (\beta^2 - 1)\,\bar{e} - \beta^2 \frac{\partial\bar{\theta}}{\partial r} = \beta^2 s^2\,\bar{u} \qquad (20)$$

$$\nabla^2\bar{w} + (\beta^2 - 1)\frac{\partial\bar{e}}{\partial z} - \beta^2\frac{\partial\bar{\theta}}{\partial z} = \beta^2 s^2\,\bar{w} \qquad (21)$$

$$(\nabla^2 - s - \tau_0 s^2)\bar{\theta} = (1+\tau_0 s)\,(\varepsilon s\bar{e} - \bar{Q}) \qquad (22)$$

$$(\nabla^2 - s^2)\ \bar{e} = \nabla^2\bar{\theta} \qquad (23)$$

$$\bar{\sigma}_{rr} = 2\frac{\partial\bar{u}}{\partial r} + (\beta^2 - 2)\,\bar{e} - \beta^2\bar{\theta} \qquad (24)$$

$$\bar{\sigma}_{zz} = 2\frac{\partial\bar{w}}{\partial z} + (\beta^2 - 2)\,\bar{e} - \beta^2\bar{\theta} \qquad (25)$$

$$\bar{\sigma}_{rz} = \left(\frac{\partial\bar{u}}{\partial z} + \frac{\partial\bar{w}}{\partial r}\right) \qquad (26)$$

$$\bar{\theta} = \bar{f}(r,s) \qquad (27)$$

$$\bar{\sigma}_{zz} = \bar{\sigma}_{rz} = 0 \qquad (28)$$

Eliminating $\bar{e}$ between the equations (22) and (23), one obtains,

$$\left\{\nabla^4 - \left(s^2 + s(1+\tau_0 s)(1+\varepsilon)\right)\nabla^2 + s^3(1+\tau_0 s)\right\}\bar{\theta} = -(1+\tau_0 s)\,(\nabla^2 - s^2)\bar{Q} \qquad (29)$$

After factorization the above equation becomes,

$$(\nabla^2 - k_1^2)(\nabla^2 - k_2^2)\ \bar{\theta} = -(1+\tau_0 s)\,(\nabla^2 - s^2)\bar{Q} \qquad (30)$$

where k_1^2 and k_2^2 are the roots with positive real parts of the characteristic equation

$$k^4 - \left(s^2 + s(1+\tau_0 s)(1+\varepsilon)\right)\ k^2 + s^3(1+\tau_0 s) = 0 \qquad (31)$$

The solution of Eq. (30) is written in the form,

$$\bar{\theta} = \bar{\theta}_1 + \bar{\theta}_2 + \bar{\theta}_p \qquad (32)$$

where $\bar{\theta}_i$ is a solution of the homogenous equation,

$$(\nabla^2 - k_1^2)\,\bar{\theta}_i = 0 \quad, i = 1,2. \qquad (33)$$

and $\bar{\theta}_p$ is a particular integral of equation (30).

In order to solve the problem, the Hankel transform of order zero with respect to r is used. The Hankel transform of a function $\bar{f}(r,z,s)$ is defined by the relation,

$$\bar{f}^*(\alpha,z,s) = H\left[\bar{f}(r,z,s)\right] = \int_0^\infty \bar{f}(r,z,s)\,r\,J_0(\alpha r)dr \qquad (34)$$

where J_0 is the Bessel function of the first kind of order zero and α is the Hankel transform parameter.

The inversion of Hankel transform is given by the relation

$$\bar{f}(r,z,s) = H^{-1}\left[\bar{f}^*(\alpha,z,s)\right] = \int_0^\infty \bar{f}^*(\alpha,z,s)\,\alpha\,J_0(\alpha r)d\alpha \qquad (35)$$

Applying the Hankel transform to equation (33), we get,

$$\left\{D^2 - (k_1^2 + \alpha^2)\right\}\bar{\theta}_i^* = 0 \quad, i = 1,2., \text{ where } D = \partial/\partial z \qquad (36)$$

The solution of the above equation is written in the form

$$\bar{\theta}_i^* = A_i(\alpha,s)(k_i^2 - s^2)e^{-q_i z} \qquad (37)$$

where $q_i = \sqrt{\alpha^2 + k_i^2}$

Applying Hankel transform to the equation (30), we get,

$$(D^2 - q_1^2)(D^2 - q_2^2)\ \bar{\theta}_p^* = -(1+\tau_0 s)\ (D^2 - q^2)\,\bar{Q}^* \qquad (38)$$

where $q = \sqrt{\alpha^2 + s^2}$

The periodically varying heat source $Q(r,z,t)$ in cylindrical co-ordinates is taken in the following form

$$Q(r,z,t) = Q_0\frac{\delta(r)}{2\pi r}.\frac{\sin\pi t}{\tau},\quad 0 \le t \le \tau \qquad (39)$$
$$= 0 \qquad ,\quad t > \tau$$

where Q_0 is the strength of the heat source and $\delta(r)$ is the well known Dirac's delta function.

On applying Laplace transform and Hankel transform to equation (39), we get,

$$\bar{Q}^* = \frac{Q_0\pi\tau(1+e^{-s\tau})}{(s^2\tau^2 + \pi^2)} \qquad (40)$$

The solution of the equation (38) has the form,

$$\bar{\theta}_p^* = \frac{(1+\tau_0 s)q^2}{q_1^2 q_2^2}\frac{Q_0\pi\tau(1+e^{-s\tau})}{(s^2\tau^2 + \pi^2)} \qquad (41)$$

Then the complete solution in the transformed domain is obtained as

$$\bar{\theta}^*(\alpha,z,s)=A_i(\alpha,s)\left(k_i^2-s^2\right)e^{-q_i z}+\frac{(1+\tau_0 s)q^2}{q_1^2 q_2^2}\frac{Q_0\pi\tau\left(1+e^{-s\tau}\right)}{(s^2\tau^2+\pi^2)} \quad (42)$$

On applying the inverse Hankel transform to equation (42), we get,

$$\bar{\theta}(r,z,s)=\int_0^\infty\left\{\begin{array}{l}\sum_{i=1}^{n} A_i(\alpha,s)\left(k_i^2-s^2\right)e^{-q_i z}\\ +\frac{(1+\tau_0 s)q^2}{q_1^2 q_2^2}\frac{Q_0\pi\tau\left(1+e^{-s\tau}\right)}{(s^2\tau^2+\pi^2)}\end{array}\right\}\alpha J_0(\alpha r)\,d\alpha \quad (43)$$

Similarly eliminating θ between equations (22) and (23), we get,

$$\left(\nabla^2-k_1^2\right)\left(\nabla^2-k_2^2\right)\bar{e}=-(1+\tau_0 s)\nabla^2\overline{Q} \quad (44)$$

On applying Hankel transform to equation (44), we get,

$$\left(D^2-q_1^2\right)\left(D^2-q_2^2\right)\bar{e}^*=-(1+\tau_0 s)\left(D^2-\alpha^2\right)\overline{Q}^* \quad (45)$$

Complete solution of equation (45) is obtained as,

$$\bar{e}^*(\alpha,z,s)=\sum_{i=1}^{2}A_i(\alpha,s)k_i^2e^{-q_i z}+\frac{(1+\tau_0 s)\alpha^2}{q_1^2 q_2^2}\frac{Q_0\pi\tau\left(1+e^{-s\tau}\right)}{(s^2\tau^2+\pi^2)} \quad (46)$$

Taking the inverse Hankel Transform to equation (46), one obtains,

$$\bar{e}(r,z,s)=\int_0^\infty\left\{\begin{array}{l}\sum_{i=1}^{2}A_i(\alpha,s)k_i^2e^{-q_i z}\\ +\frac{(1+\tau_0 s)\alpha^2}{q_1^2 q_2^2}\frac{Q_0\pi\tau\left(1+e^{-s\tau}\right)}{(s^2\tau^2+\pi^2)}\end{array}\right\}\alpha J_0(\alpha r)\,d\alpha \quad (47)$$

Applying Hankel transform to equation (21) and then using equations (42) and (46), the axial displacement component is obtained as,

$$\bar{w}^*(\alpha,z,s)=B(\alpha,s)\,e^{-q_3 z}-\sum_{i=1}^{2}A_i(\alpha,s)q_ie^{-q_i z} \quad (48)$$

where $q_3=\sqrt{\alpha^2+\beta^2 s^2}$

On applying the inverse Hankel transform to equation (48), we get,

$$\bar{w}(r,z,s)=\int_0^\infty\left\{B(\alpha,s)\,e^{-q_3 z}-\sum_{i=1}^{2}A_i(\alpha,s)q_ie^{-q_i z}\right\}\alpha J_0(\alpha r)\,d\alpha \quad (49)$$

Applying the Hankel transform to equation (20) and using equations (42), (46) and (48), we get,

$$H\left[\frac{1}{r}\frac{\partial}{\partial r}(r\bar{u})\right]=\left\{\begin{array}{l}B(\alpha,s)\,q_3\,e^{-q_3 z}\\ -\alpha^2\left[\begin{array}{l}\sum_{i=1}^{2}A_i(\alpha,s)e^{-q_i z}\\ -\frac{(1+\tau_0 s)}{q_1^2 q_2^2}\frac{Q_0\pi\tau\left(1+e^{-s\tau}\right)}{(s^2\tau^2+\pi^2)}\end{array}\right]\end{array}\right\} \quad (50)$$

On applying inverse Hankel transform to equation (50), one obtains,

$$\bar{u}=\int_0^\infty\left\{\begin{array}{l}B(\alpha,s)\,q_3\,e^{-q_3 z}\\ -\alpha^2\left[\begin{array}{l}\sum_{i=1}^{2}A_i(\alpha,s)e^{-q_i z}\\ -\frac{(1+\tau_0 s)}{q_1^2 q_2^2}\frac{Q_0\pi\tau\left(1+e^{-s\tau}\right)}{(s^2\tau^2+\pi^2)}\end{array}\right]\end{array}\right\}J_1(\alpha r)\,d\alpha \quad (51)$$

On using equations (43), (47), (49) and (51) in equations (25) and (26), we obtain the stress components as,

$$\bar{\sigma}_{zz}=\int_0^\infty\left\{\begin{array}{l}-2B(\alpha,s)\,q_3\,e^{-q_3 z}\\ +\left(\alpha^2+q_3^2\right)\left[\begin{array}{l}\sum_{i=1}^{2}A_i(\alpha,s)e^{-q_i z}-\\ \frac{(1+\tau_0 s)}{q_1^2 q_2^2}\frac{Q_0\pi\tau\left(1+e^{-s\tau}\right)}{(s^2\tau^2+\pi^2)}\end{array}\right]\end{array}\right\}\alpha J_0(\alpha r)\,d\alpha$$

$$\bar{\sigma}_{rz}=\int_0^\infty\left\{\begin{array}{l}-\left(1+q_3^2\right)B(\alpha,s)\,e^{-q_3 z}\\ +\left[\sum_{i=1}^{2}A_i(\alpha,s)\,q_i(1+\alpha^2)e^{-q_i z}\right]\end{array}\right\}J_1(\alpha r)\,d\alpha \quad (53)$$

After applying the Hankel transform to equations (27) and (28), the boundary conditions take the form,

$$\bar{\theta}^*(\alpha,0,s)=\bar{f}^*(\alpha,s) \quad (54)$$

$$\bar{\sigma}_{zz}^*(\alpha,0,s)=\bar{\sigma}_{rz}^*(\alpha,0,s)=0 \quad (55)$$

On applying the boundary conditions (54) and (55) to equations (43), (52) and (53), the system of linear equations involving unknown parameters $A_1(\alpha,s), A_2(\alpha,s)$ and $B(\alpha,s)$ are obtained as follows,

$$\sum_{i=1}^{n}A_i(\alpha,s)\left(k_i^2-s^2\right)+\frac{(1+\tau_0 s)q^2}{q_1^2 q_2^2}\frac{Q_0\pi\tau\left(1+e^{-s\tau}\right)}{(s^2\tau^2+\pi^2)}=\bar{f}^*(\alpha,s) \quad (56)$$

$$-2B(\alpha,s)\,q_3 + \left(\alpha^2+{q_3}^2\right)\left[\begin{array}{l}\sum_{i=1}^{2} A_i(\alpha,s) \\ -\dfrac{(1+\tau_0 s)}{{q_1}^2{q_2}^2}\dfrac{Q_0\pi\tau\left(1+e^{-s\tau}\right)}{(s^2\tau^2+\pi^2)}\end{array}\right]=0 \qquad (57)$$

$$-\left(1+{q_3}^2\right)B(\alpha,s)+\left[\sum_{i=1}^{2} A_i(\alpha,s)\; q_i(1+\alpha^2)\right]=0 \qquad (58)$$

On solving the system of linear equations (56) - (58) unknown parameters are determined and the complete solution of the problem is obtained in the Laplace transform domain.

IV. INVERSION OF DOUBLE TRANSFORMS

Due to the complexities involved in the inversion of the Laplace transforms, we employ a numerical scheme based on Gaver-Stehfast algorithm. Gaver [16] and Stehfast [17 ,18] derived the formula given below. By this method the inverse $f(t)$ of the Laplace transform $\bar{f}(s)$ is approximated by,

$$f(t)=\frac{\ln 2}{t}\sum_{j=1}^{K} D(j,K)\,F\left(j\frac{\ln 2}{t}\right) \qquad (59)$$

With

$$D(j,K) =(-1)^{j+M}\sum_{n=m}^{\min(j,M)}\frac{n^M(2n)!}{(M-n)!n!(n-1)!(j-n)!(2n-j)!} \qquad (60)$$

where K is an even integer, whose value depends on the word length of the computer used. $M=K/2$ and m is the integer part of the $(j+1)/2$. The optimal value of K was chosen as described in Gaver-Stehfast algorithm, for the fast convergence of results with the desired accuracy. The Romberg numerical integration technique [19] with variable step size was used to evaluate the integrals involved. All the programs were made in mathematical software Matlab.

V. NUMERICAL CALCULATIONS

$$f(r,t)=\theta_0 H(a-r)H(t) \qquad (61)$$

where θ_0 is a constant temperature, $H(.)$ is a Heaviside unit step function.

On applying Hankel and Laplace transform to equation (61), we get,

$$\bar{f}^*(\alpha,s)=\frac{a\theta_0 J_1(\alpha a)}{\alpha s} \qquad (62)$$

For the purpose of illustration, a mathematical model is prepared for a Copper material with the following material properties,

$k=386\,J.K^{-1}.m^{-1}.s^{-1}$, $\alpha_t=1.78\times10^{-5}K^{-1}$,$C_E=383.1J.Kg^{-1}.K^{-1}$, $\mu=3.86\times10^{10}N.m^{-2}$, $\lambda=7.76\times10^{10}N.m^{-2}$, $\rho=8954\ kg.m^{-3}$ $\tau_0=0.02s$, $T_0=293\ K$, $\varepsilon=0.0168\,N.m.J^{-1}$, $c_1=4.158\times10^3 m.s^{-1}$, $\eta=8886.73\,s.m^{-2}$, $\beta^2=4$, $a=1$, $\theta_0=1$, $b=1.$

The numerical values for temperature θ and the axial stress component σ_{zz} have been calculated for different time instants $t=0.1\,,0.4\,,1$, along the radial direction and are displayed graphically for Lord-Shulman theory (L-S theory) and the particular case of Classical Coupled thermoelasticity (CT theory) as shown in figure 1-2 respectively .

Figure 1 depicts the non-dimensional temperature distribution along the radial direction at different time instants. The variation in values observed for the two theories (CT and LS) in the plots. Due to the application of the heat source, it is observed that the values of non-dimensional temperature θ drops gradually along the radial direction till $r=5.2$ and then it increases till $r=7$.

Figure 2 describes the axial stress σ_{zz} along the radial at different time instants. Different profiles of axial stress are seen at small times (i.e. at $t=0.1\,,0.4$) and large times (i.e. at $t=1$). The difference in results for LS and CT is observed.

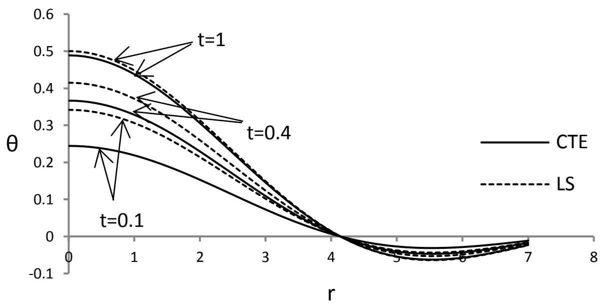

Figure 1. Distribution of dimensionless temperature along radial direction.

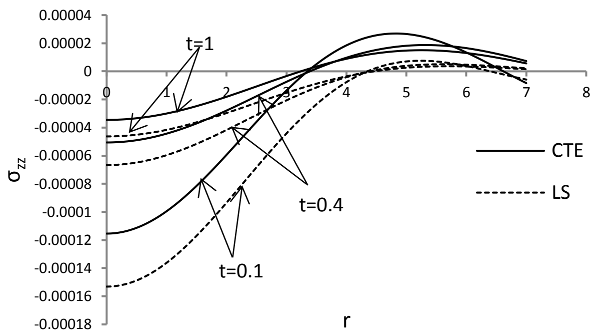

Figure 2. Distribution of dimensionless axial stress along radial direction.

VI. CONCLUSION

A problem in non-classical thermoelasticity (LS model) is formulated for half space with a heat source and the results are compared for the model with CT. It is observed that the non-dimensional temperature and axial stress component along the radial direction predicts changes for small and large times. This type of behaviour of the variables is observed due to the presence of the periodically varying heat source distributed over the radial direction. Due to the presence of relaxation parameter in the field equations, the heat wave assumes finite speed of propagation. Finally, it is concluded that the solutions in this problem will prove to be useful to determine the thermal behaviour in important engineering problems by using the more realistic non-classical model of thermoelasticity.

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Appendix:

Nomeclature:

r Radius

t Time

T Temperature

T_0 Reference temperature

θ Temperature change

$Q(r,z,t)$ Heat source

k Thermal conductivity of material

η Dimensionless characteristic length

c_1 Speed of propagation of the longitudinal wave

u Radial displacement component

w Axial displacement component

$\sigma_{rr}, \sigma_{zz}, \sigma_{rz}, \sigma_{\varphi\varphi}$ Components of stress function

E Young's modulus

ρ Density

C_E Specific heat at constant strain

μ, λ Lamé's constants

L Laplace transform

δ Dirac delta function

(r,ϕ,z) Cylindrical polar coordinates

τ_0 Relaxation times

e Cubical dilatation

$H(.)$ Heaviside unit step function