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Effect of Cosmological Constant on the Periods of Vibrating System In the Reissner-Nordstrom Space-Time

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ABSTRACT

In this paper considering circular trajectory $r =$ constant in the plane $\theta = \frac{\pi}{2}$ in the Reissner-Nordstrom (anti) de-Sitter space-time, a relation analogous to a relation between perihelic shift is obtained by using Shirokov's technique. Thereby our result supports the conclusion that the cosmological constant term Λ on gravitating particle instead of helping the matter to curve the space-time mode, decurves the space-time which means that nature of gravitational fields due to the matter and charges matter with cosmological constant Λ may be of different type. In our case θ- vibrations lie further behind r or (ϕ) - vibrations as an effect of positive cosmological constant on the periods of vibrating system.

Keywords: Cosmological constant, R-N field, θ- vibrations, r or (ϕ) - vibrations.

I. INTRODUCTION

According to Einstein, all the forms of matter and energy are under the influence of gravitation and hence the universe filled with matter and energy is under the action of the attractive force of gravitation. Moreover, the universe is static; therefore it is bound to collapse under gravity. So to prevent the collapse, Einstein (1917) introduced a cosmological constant Λ having the dimensions of space curvature. In 1922, Friedmann solved the Einstein's gravitational field equations and found a cosmological solution, which prevents non-static model of an expanding universe.

In 1922, Edwin Hubble's observations convinced astrophysicists that the universe is not at all static by observing the red shift of distant galaxies. Therefore, Einstein rejected the term Λ , which does not have a direct physical meaning. The term $\Lambda g_{\mu\nu}$ which is added to the energy momentum tensor suggests an interpretation in terms of constant pressure. This pressure would be responsible for avoiding the cosmological collapse seen in the case of the non-zero density of matter. But then it is necessary to explain the existence of such universal pressure by some macroscopic phenomenon.

Sakharov, Wheeler, Landau, Pomeranchak and others have proposed number of such explanations. In their papers, the authors evaluate the constant Λ , as being of order of $10^{-56} cm^{-1}$. The value of Λ along with its physical interpretation is deduced by supposing that the vacuum is endowed with very high elasticity (of order Λ^{-1}), which is due to the quantum fluctuations of energy in the vacuum. Due to the physically plausible reasons Λ is retained in the modern cosmology.

Establishing the criteria for existence and stability of circular orbits, Howes (1981) has studied the effect of a positive cosmological constant Λ on the circular orbits in the R-N field and Kerr field, with the help of geodesic deviation equation.

Taking into account the importance of Λ in the modern cosmology, we have studied the effect of a positive cosmological constant Λ , on the periods of vibrating system in this paper. In the section 2.2, expressions for the frequencies of a vibrating system are derived in the R-N field with cosmological constant Λ . In the section 2.3, the periods of vibrating system in the Schl'd field with cosmological constant Λ on the periods of vibrations is discussed. In the section 2.4, conclusions are drawn.

II. FREQUENCIES OF VIBRATIONS

In the GTR, the equation of deviation from the geodesic is

$\frac{d^2\xi^i}{ds^2} + 2\,\Gamma^i_{jk}\, u^j \frac{d\xi^k}{ds} + \frac{\partial \Gamma^i_{jk}}{\partial x^l}\, u^j u^k \xi^l = 0$, (2.1)

where ξ^i is the infinitesimal 4-vector giving the deviation from the basic geodesic, $u^i = \frac{dx^i}{ds}$ is the 4-velocity vector tangential to the basic geodesic and Γ^i_{jk} are Christoffel symbols defined as

$\Gamma^i_{jk} = \frac{1}{2} g^{li}\left(\frac{\partial g_{lj}}{\partial x^k} + \frac{\partial g_{lk}}{\partial x^j} - \frac{\partial g_{jk}}{\partial x^l}\right)$.

Reissner-Nordstrom field with cosmological constant Λ , known as Reissner-Nordstrom (anti) de-Sitter space-time is

$$ds^2 = -\left(1 - \frac{2m}{r} + \frac{e^2}{r^2} - \frac{\Lambda}{3}r^2\right)^{-1} dr^2 - r^2\,(d\theta^2 + sin^2\theta\, d\phi^2) + \left(1 - \frac{2m}{r} + \frac{e^2}{r^2} - \frac{\Lambda}{3}r^2\right) dt^2 \quad (2.2)$$

where $r = x^1, \theta = x^2, \phi = x^3, t = x^4$.

For the field (2.2), metric tensors are

$g_{11} = -\left(1 - \frac{2m}{r} + \frac{e^2}{r^2} - \frac{\Lambda}{3}r^2\right)^{-1}, g_{22} = -r^2$,

$g_{33} = -r^2 sin^2\theta$,

$g_{44} = \left(1 - \frac{2m}{r} + \frac{e^2}{r^2} - \frac{\Lambda}{3}r^2\right), g_{ij} = 0$ for $i \neq j$ (2.3)

and the non-vanishing components of the Christoffel symbols are

$\Gamma^1_{11} = -\frac{1}{r}\left(\frac{m}{r} - \frac{e^2}{r^2} - \frac{\Lambda}{3}r^2\right)\left(1 - \frac{2m}{r} + \frac{e^2}{r^2} - \frac{\Lambda}{3}r^2\right)^{-1}$,

$\Gamma^1_{22} = -r\left(1 - \frac{2m}{r} + \frac{e^2}{r^2} - \frac{\Lambda}{3}r^2\right)^{-1}$,

$\Gamma^2_{21} = \frac{1}{r} = \Gamma^3_{31}$,

$\Gamma^1_{33} = -r\, sin^2\theta\left(1 - \frac{2m}{r} + \frac{e^2}{r^2} - \frac{\Lambda}{3}r^2\right)$,

$\Gamma^4_{41} = \frac{1}{r}\left(\frac{m}{r} - \frac{e^2}{r^2} - \frac{\Lambda}{3}r^2\right)\left(1 - \frac{2m}{r} + \frac{e^2}{r^2} - \frac{\Lambda}{3}r^2\right)^{-1}$,

$\Gamma^1_{44} = \frac{1}{r}\left(\frac{m}{r} - \frac{e^2}{r^2} - \frac{\Lambda}{3}r^2\right)\left(1 - \frac{2m}{r} + \frac{e^2}{r^2} - \frac{\Lambda}{3}r^2\right)$,

$\Gamma^2_{33} = -\sin\theta\cos\theta$, $\Gamma^3_{23} = \cot\theta$. (2.4)

We suppose that the basic geodesic is a circular trajectory with radius $r =$ constant in the plane $\theta = \frac{\pi}{2}$ in the field (2.2). Following Howes (1981), if the basic geodesic are circular in the axisymmetric stationary field, θ-disturbances are independent of r, ϕ, and t-perturbations.

Therefore for $i = 2$, equation (2.1) assumes the form

$\frac{d^2\xi^2}{ds^2} + \Gamma^2_{jk,2}\, u^j u^k \xi^2 = 0$ (2.5)

If we suppose that,

$\xi^2 = \xi^2_0\, e^{i\,\Omega\, s}$ (2.6)

(ξ^2_0 is the amplitude of θ-vibrations) then from (2.5), we obtain

$\Omega^2 = \Gamma^2_{jk,2}\, u^j u^k$, (2.7)

where comma in the Christoffel symbol denotes the partial differentiation and Ω is the frequency of θ-vibrations.

For $i = 1, 3, 4$, from (2.1), we get

$\frac{d^2\xi^1}{ds^2} + 2\,\Gamma^1_{j3}\, u^j \frac{d\xi^3}{ds} + 2\,\Gamma^1_{j4}\, u^j \frac{d\xi^4}{ds} + \Gamma^1_{jk,l}\, u^j u^k \xi^l = 0$,

$\frac{d^2\xi^3}{ds^2} + 2\,\Gamma^3_{j1}\, u^j \frac{d\xi^1}{ds} = 0$,

and $\frac{d^2\xi^4}{ds^2} + 2\,\Gamma^4_{j1}\, u^j \frac{d\xi^1}{ds} = 0$ (2.8)

Further, if we suppose that

$\xi^j = \xi^j_0\, e^{i\,\omega\, s}$, (j = 1, 3, 4) (2.9)

(ξ^j_0 is the amplitude of r, ϕ and t-vibrations), then from (2.8), we get

$\left(\Gamma^1_{jk,1}\, u^j u^k - \omega^2\right)\xi^1_0 + 2\,i\,\omega\,\Gamma^1_{j3}\, u^j \xi^3_0 + 2\,i\,\omega\,\Gamma^1_{j4}\, u^j \xi^4_0 = 0$,

$2\,i\,\omega\,\Gamma^3_{j1}\, u^j \xi^1_0 - \omega^2 \xi^3_0 = 0$,

and $2\,i\,\omega\,\Gamma^4_{j1}\, u^j \xi^1_0 - \omega^2 \xi^4_0 = 0$ (2.10)

where ω is the frequency of r, ϕ and t-vibrations.

For non-trivial solution of (2.10), we equate the determinant of coefficients to zero and obtain

$\omega^2 = u^j u^k\, \Gamma^1_{jk,1} - 4\, u^j u^k\, \Gamma^3_{j1}\Gamma^1_{k3} - 4\, u^j u^k\, \Gamma^4_{j1}\Gamma^1_{k4}$

or $\omega^2 = \left(\Gamma^1_{33,1} - 4\,\Gamma^3_{31}\Gamma^1_{33}\right)(u^3)^2 + \left(\Gamma^1_{44,1} - 4\,\Gamma^4_{41}\Gamma^1_{44}\right)(u^4)^2$ (2.11)

where all the symbols Γ^i_{jk} and their derivatives are evaluated at $\theta = \frac{\pi}{2}$.

To determine u^3, consider geodesic equation
$\frac{du^i}{ds} + \Gamma^i_{jk}\, u^j\, u^k = 0$, (i, j, k = 1, 2, 3, 4) (2.12)
in the Einstein's theory of gravitation.

For circular orbits in the equatorial plane from (2.12) we find that

$$\frac{dt}{d\phi} = \frac{u^4}{u^3} = \left(\frac{-\Gamma^1_{33}}{\Gamma^1_{44}}\right)^{\frac{1}{2}}, \quad (2.13)$$

which provides the angular velocity of the test particle as seen from the infinity.
Using (2.4) in (2.13), we get

$$(u^4)^2 = \frac{r^2}{\left(\frac{m}{r} - \frac{e^2}{r^2} - \frac{\Lambda}{3} r^2\right)} (u^3)^2. \quad (2.14)$$

For the circular orbit in the equatorial plane, using (2.13) in (2.2), we get

$$(u^3)^2 = \left(\frac{m}{r^3}\right)\left(1 - \frac{e^2}{m\,r} - \frac{\Lambda}{3}\frac{r^3}{m}\right)\left(1 - \frac{3m}{r} + \frac{2\,e^2}{r^2}\right)^{-1} \quad (2.15)$$

Therefore expressions for the frequencies of θ-vibrations and r (or ϕ)-vibrations in (2.7) and (2.11) simplify to

$$\Omega^2 = (u^3)^2 = \left(\frac{m}{r^3}\right)\left(1 - \frac{e^2}{m\,r} - \frac{\Lambda}{3}\frac{r^3}{m}\right)\left\{1 - \left(\frac{3m}{r} - \frac{2\,e^2}{r^2}\right)\right\}^{-1} \quad (2.16)$$

$$\&\ \omega^2 = (u^3)^2 \left\{1 - \frac{6m}{r} + \frac{3\,e^2}{r^2} + \frac{e^2}{m\,r} + \frac{e^4}{m^2 r^2} - \Lambda\left(\frac{4\,e^2 r}{m} + \frac{4\,r^3}{3\,m} - 5\,r^2\right)\right\} \times \left\{1 - \left(\frac{e^2}{m\,r} + \frac{\Lambda}{3}\frac{r^3}{m}\right)\right\}^{-1} (u^3)^2 \quad (2.17)$$

respectively.

III. PERIODS OF VIBRATIONS

The corresponding period of θ-vibration is

$$T_\theta = \frac{2\pi}{\Omega} = T_0\left(1 - \frac{3m}{2r} + \frac{e^2}{2mr} + \frac{\Lambda}{6}\frac{r^3}{m}\right) + o(\eta) \quad (3.1)$$

and that of r (or ϕ)-vibration is

$$T_r\ (or\ T_\phi) = T_0\left(1 + \frac{3m}{2r} + \frac{2\,\Lambda\,r^3}{3\ \ m}\right) + o(\eta) \quad (3.2)$$

in which $\frac{m}{r}, \frac{e}{r} = o(\eta)$, η is small and $T_0 = 2\pi\left(\frac{r^3}{m}\right)^{\frac{1}{2}}$ is the Newtonian period of test particle in the circular orbit of radius r.
The difference $\Delta T_{RN(\Lambda)}$ between the periods of θ-vibrations and r (or ϕ)-vibrations is

$$\Delta T_{RN(\Lambda)} = T_\theta - T_r = T_0\left(-\frac{3m}{r} + \frac{e^2}{2mr} - \frac{\Lambda}{2}\frac{r^3}{m}\right) \quad (3.3)$$

to the $1\frac{1}{2}$ order approximation.

For $\Lambda = 0$, (3.3) reduces to

$$\Delta T_{RN} = T_\theta - T_r = T_0\left(-\frac{3m}{r} + \frac{e^2}{2mr}\right) \quad (3.4)$$

Furthermore, for $e = 0$, (3.3) gives

$$\Delta T_{Schl'd(\Lambda)} = T_0\left(-\frac{3m}{r} - \frac{\Lambda}{2}\frac{r^3}{m}\right) \quad (3.5)$$

and for $\Lambda = 0$ from (3.5) we can recover the result

$$\Delta T_{Schl'd} = T_0\left(-\frac{3m}{r}\right) \quad (3.6)$$

which is analogous to the result obtained by Shirokov (1973) as a new effect of Einstein's Theory of Gravitation.

IV. CONCLUSION

From (3.3), (3.4), (3.5) and (3.6), we find relation between shifts in the periods of θ-vibration and r (or ϕ)-vibration in R-N field and Schwarzschild field with and without cosmological constant Λ, which is analogous to the relation between the perihelic shift in R-N field and Schwarzschild field obtained by H.J. Treder, H.H.V. Borzeszkowski, A.Van Der Merwe, W.Y.Yourgrau.

According to G.D. Rathod and T.M. Karade, the relation between perihelic shift $\delta\phi_{RN} < \delta\phi_{Schl'd}$ shows that charge on the gravitating particle instead of helping the matter to curve the space-time more, decurves the space-time.
Also according to Kalpana Pawar and G.D. Rathod, the similar relation between the periods of θ-vibration and r (or ϕ)-vibration is obtained which shows the effect of charge on gravitating particle is analogous to a relation obtained by using Shirokov's technique as $\Delta T_{RN} < \Delta T_{Schl'd}$.

In our case, from result (3.3), we observe that θ-vibrations lie further behind the r (or ϕ)-vibrations than the R-N field as an effect of positive cosmological constant Λ on the periods of vibrating system.

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