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A Novel Delay Dependent Stability Analysis of Neural Networks Using LMI Approach

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ABSTRACT

In this paper we designed and evaluated the asymptotic stability of Neural network with time Delays is investigated. Based on a novel "Lyapunov kravoski's functional method (LKF)" and "Linear matrix inequality (LMI)" a new delay dependent stability condition is derived. These stability conditions are formulated as linear matrix inequalities (LMIs) which can be easily write in the form of by various convex optimization algorithms and it can be effectively solved by the use of "MATLAB programming".

Keywords: Stability, Delay-dependent stability, Asymptotic stability, Linear Matrix Inequality, Lyapunov-Krasovskii functional, Time-varying delay.

I. INTRODUCTION

Stability is a very fundamental issue in control theory and has been widely applied to biological and engineering systems. For the successful work of any systems, stability is required. Suppose the system is in instability we will not able to expect the accurate results. There are many types of networks particularly fuzzy, stochastic …. For each system we can provide the conditions of stability in the form of LMI.

Over the history of few decades the systems particularly of dynamical systems with time delays have been of huge attention. In exacting, the importance in stability study of different neural networks has been rising quickly due to their flourishing applications in deriving lots of business problems such as sales forecasting, buyer investigation, data support and risk administration.

From the previous decades, neural systems have been accepting expanding research thoughts because of their likely applications to many of real world systems in an assortment of fields of science and engineering, for example, issue examination, design recognizable proof, signal handling and parallel calculation [1]-[9].In modern times, a huge number of important subjects "including stability investigation, synchronization and state estimation for neural systems have been exhibited [8]-[9]. It is realized that the time delay is typically remembered for some electronic executions of neural systems because of the limited exchanging velocity of the enhancers and communication time"

.Therefore, delayed neural networks were suggested and much attention was paid to them

[5]- [6]. Current efforts on the problem of stability of time delay systems of neutral type can be divided into two categories, namely delay independent criteria and delay dependent criteria. A number of delay independent sufficient conditions for the asymptotic stability of neutral delay differential systems have been presented by various researchers [1]-[2].

Collectively with matrix inequality technique, the new operator technique and the (LKF) is used to examine the issue of robust stability with time delay for neural networks. Some delaydependent stability requirements are obtained by applying descriptor model transformation and decomposition technique.

However, these results are only concerned with the asymptotic stability, without providing any conditions for exponential stability and any information about the decay rates. Throughout this paper, the notation * represents the elements below the main diagonal of a symmetric matrix. AT means the transpose of A. We say X > Y if X - Y is positive definite, where X and Y are symmetric matrices of same dimensions. ‖ . ‖ refers to the Euclidean norm for vectors

II. SYSTEM DESCRIPTION AND PRELIMINARIES

Consider a neural network with time varying delay is of the form:

$$\dot{x}\,(t) = -\,Ax(t) + Bx\big(t-\rho(t)\big) + C\dot{x}\,(t-\rho(t)) \quad (1)$$

Where A, B and C are constant matrices and ρ (t) denotes delay and it is time varying and it is assumed to satisfy $0 \le \rho(t) \le \rho_M \; and \; 0 \le \dot{\rho}\,(t) \le l \le 1$ Where 'ρ_M' and 'l' are positive constants.

Lemma 2.1 (Schur complement [1])

Let E, F, G be the given matrices such that G > 0, then $\begin{pmatrix} F & E^T \\ E & -G \end{pmatrix} < 0 \leftrightarrow F + E^T G^{-1} E < 0$

Lemma 2.2

For any vectors x; y ∈ R^n and scalar ∈> 0 , , we have

$$2x^T y \le \in x^T x + \in^{-1} y^T y$$

Lemma 2.3

For any constant matrix $G \in R^{n*n} > 0, G = G^T > 0$ scalar $\rho > 0$, vector function $y\colon [0,\rho] \to R^n$ such that the integrations concerned are well defined, then

$$\left(\int_0^\rho z(s)ds\right) N \left(\int_0^\rho z(s)ds\right)^T \le \rho \int_0^\rho z^T(s)\, N\, z(s)ds"$$

Definition 2.1 Stability:

The equilibrium point x=0 and let $V\colon D \to R$ is continuous differentiable function for which

$(i)V(0) = 0$

$(ii)V(X(t)) > 0$ then x=0 is said to be stable if

$(iii)\dot{\boldsymbol{V}}(\boldsymbol{X}(\boldsymbol{t})) \le 0$ in D – 0.

Definition 2.2 Asymptotically Stable:

The equilibrium point x=0 and let $V\colon R^n \to R$ be a function which is continuously differentiable such that:

$(i)V(0) = 0$

$(ii)V(X(t)) > 0$ then x=0 is said to be asymptotically stable if

$(iii)\dot{\boldsymbol{V}}(\boldsymbol{X}(\boldsymbol{t})) < 0$

This leads to the celebrated theorem of Lyapunov.

III. ASYMPTOTIC STABILITY RESULTS

In this part, we will execute asymptotic stability analysis of neural networks with time varying delay described by (1).

We can modify system (1) to the following descriptor system.

$$\dot{x}\,(t) = y(t)$$
$$y(t) = -Ax(t) + Bx\big(t-\rho(t)\big) + Cy(t-\rho(t)) \quad (2)$$

Theorem 3.1: Under the above lemmas, the above system (1) is asymptotically stable if there exists some positive definite matrices P, Q, R, S and the positive diagonal matrices $M = diag\{m_1, m_2, m_{3,\ldots} m_n\}$,' such that the following LMI condition is satisfied,'

$$\Omega = \begin{pmatrix} \varphi_{11} & \varphi_{12} & \varphi_{13} \\ * & \varphi_{22} & \varphi_{22} \\ * & * & \varphi_{33} \end{pmatrix} < 0$$

Proof:

This theorem can be prove by considering the Lyapunov functions are

$V = V_1 + V_2 + V_3 + V_4$ are as follows. (3)

We define the Lyapunov functions as follows

$V_1(t) = x^T(t)\, P\, x(t)$

$V_2(t) = 2\sum_{i=1}^{m} m_i \int_0^{x_i} f_i(s)ds$

$V_3(t) = \int_{t-\rho}^{t} [x^T(s)\, Q\, x(s) + g^T(\mathrm{x}(s))\, R\, g(x(s))]ds$

$V_4(t) = \int_{t-\rho}^{t} (s - t + \dot{\rho}) h^T(\mathrm{x}(\theta))\, S\, h(x(\theta))\mathrm{d}\theta ds$

Let us define the derivative of the Lyapunov functions is as follows:

$$\dot{V}_1 = 2x^T(t)P\dot{x}(t) = 2x(t)y(t) = 2x^T[-Ax(t) + Bx(t-\rho(t)) + Cy(t-\rho(t))]$$

$$\dot{V}_2 = 2\sum_{i=1}^{m} m_i f_i(x_i(t))\dot{x}(t) = f^T(\mathrm{x}(t))[-2\mathrm{M}Ax(t) + 2\mathrm{M}Bf^T(\mathrm{x}(t))x(t-\rho(t)) + 2\mathrm{M}f^T(\mathrm{x}(t))Cy(t-\rho(t))]$$

$$\dot{V}_3 = x^T(t)Q\,x(t) - (1-\mathrm{d})x^T(t-\rho)Q\,x(t-\rho) + g^T(\mathrm{x}(t))\mathrm{g}(\mathrm{x}(t))\mathrm{R} - (1-\mathrm{d})g^T\,\mathrm{x}(\mathrm{t}-(\rho))\mathrm{R}g(\mathrm{x}(t-\rho))$$

$$\dot{V}_4 = \bar{\rho}h^T(\mathrm{x}(t))S\,h(x(t)) - \int_{t-\bar{\rho}}^{t} h(x(s))R\,h(x(s))\,ds$$

$$= \bar{\rho}h^T(\mathrm{x}(t))Sh(x(t)) - \left(\int_{t-\bar{\rho}}^{t} h(x(s))\,ds\right)^T \mathrm{S} \left(\int_{t-\bar{\rho}}^{t} h(x(s))\,ds\right)$$

On substituting all the values in the equation (3), we get

$$\dot{V} \le 2x^T(t)[-Ax(t) + Bx(t-\rho(t)) + Cy(t-\rho(t))] + f^T(\mathrm{x}(t))[-2\mathrm{M}Ax(t) + 2\mathrm{M}Bf^T(\mathrm{x}(t))\,x(t-\rho(t)) + 2\mathrm{M}f^T(\mathrm{x}(t))Cy(t-\rho(t))] + x^T(t)Q\,x(t) - (1-\mathrm{d})\,x^T(t-\rho)Q\,x(t-\rho) + g^T(\mathrm{x}(t))\mathrm{g}(\mathrm{x}(t))\mathrm{R} - (1-\mathrm{d})g^T\,\mathrm{x}(\mathrm{t}-(\rho))\mathrm{R}g(\mathrm{x}(t-\rho)) + \bar{\rho}h^T(\mathrm{x}(t))Sh(x(t)) - \left(\int_{t-\bar{\rho}}^{t} h(x(s))\,ds\right)^T \mathrm{S} \left(\int_{t-\bar{\rho}}^{t} h(x(s))\,ds\right)$$

$$\dot{V} \le \Pi^T \Omega \Pi$$

Where

$$\Omega = \begin{pmatrix} \varphi_{11} & \varphi_{12} & \varphi_{13} \\ * & \varphi_{22} & \varphi_{22} \\ * & * & \varphi_{33} \end{pmatrix}$$

By applying the previous Lemmas 2.2 in R with some attempt, we obtain

$\dot{\boldsymbol{V}} < \mathbf{0}$ since $\Omega < 0$

"Hence, by stability theorem of Lyapunov"

$$\dot{\boldsymbol{V}}(\boldsymbol{X}) < 0.$$

Hence we concluded that the system is asymptotic stable.

IV. CONCLUSION

We have offered the sufficient condition for the asymptotic stability for neural networks with time varying delay. Using the LKF concept and LMI the delay-dependent criterion for ensuring the asymptotic stability of neural networks with time delays was derived.

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