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Bounds on Rayleigh-Benard-Marangoni Convection in a Composite Layer with Conducting Plates

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ABSTRACT

Boundary effects on Rayleigh-Benard-Marangoni stability in a layer of composite scheme in which a liquid layer overlies a saturates porous material bounded by slabs of finite thermal conductivity and finite thickness has been investigated by means of linear stability analysis. The eigen value problem resulting from the stability analysis is solved by regular perturbation technique. It has been found the stability characteristics in terms of the critical Rayleigh number critical Marangoni number is profoundly influenced by the conductivity and slab thickness. Dependency of thermal conductivity ratio, and depth ratio is graphically discussed. The current findings may provide useful data in the solidification phase of alloys to understand the convective movement of the melt.

Key words: Thermal Conductivity: Rayleigh-Benard-Marangoni Convection: Boundary Slab.

I. INTRODUCTION

Thermal convection within a two-layer system constructed by a layer of fluid overlying a porous material saturated with the same fluid has numerous geophysical and industrial applications, such as the manufacturing of composite materials used in the aircraft and automobile industries, flow of water under the Earth's surface, flow of oil in underground reservoirs and growing of compound films in thermal chemical vapour deposition reactors. A detailed review is given by Nield & Bejan (2006), with current highly relevant literature including Chen &Chen (1988), Ewing &Weekes (1998), Blest et al. (1999), Straughan (2002, 2008),Carr (2004), Chang (2004, 2005, 2006), Hirata et al. (2007), Hoppe et al. (2007), Mu & Xu (2007) and Hill & Straughan (2009).

Chen and Chen (1988) produced a classical paper in which they have studied the thermal convection in two-layer system composed of a porous layer saturated with fluid over which lay the same fluid. The work of Chen and Chen (1988) employed the fundamental model for convection in a porous-fluid-layer system developed originally by Nield (1987). He reported that the relative thickness of the two layers determined whether this convection is concentrated in the fluid layer or in the porous layer. We examine the linear stability of Rayleigh-Benard-Marangoni convection between slabs of finite thermal conductivity and finite thickness due to an applied pressure gradient in the presence of an applied vertical temperature gradient. We believe that this problem is paradigmatic to the very general problem involving the interaction between a non-uniform applied temperature gradient and a variableviscosity flow. The results are relevant to current industrial applications involving chemical vapour deposition or the cooling of electronic equipment; see e.g. (Nicolas 2002, Ruan et al.2004, Hill 2004, Straughan 2008 , Generalis and Busse (2008)). The objective of the present study is to investigate the influences of the solid plates of finite thickness and of finite conductivity. The linear stability theory is applied and the resulting eigen value problem is solved by analytically using regular perturbation technique. The critical Rayleigh number and The critical Marangoni number which depend on related physical parameters, are investigated.

II. CONCEPTUAL MODEL

The system under investigation consisting of an fluid layer of thickness d and saturating an underlying porous layer of thickness d_m and bounded by solid layers of thickness of d_s. Thus the z indicating distances vertically upwards the fluid-saturated porous medium interface is at $z = 0$.

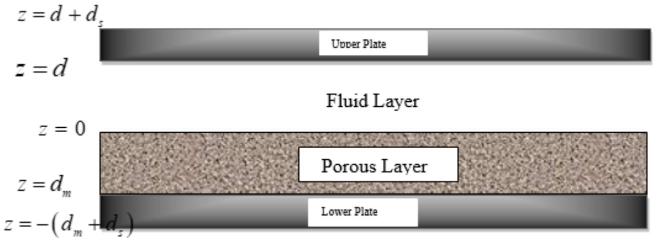

Fig.1.Physical configuration

III. Mathematical Formulation

The fluid-porous -solid layers governing equations are:

Fluid layer: $(0 \le z \le d)$

$$\nabla.\vec{q} = 0 \tag{1}$$

$$\rho_0\left[\frac{\partial \vec{q}}{\partial t} + (\vec{q}\cdot\nabla)\vec{q}\right] = -\nabla p + \rho_0 \vec{g}\left[1 - \alpha(T - T_0)\right] + \mu\nabla^2\vec{q} \tag{2}$$

$$\frac{\partial T}{\partial t} + (\vec{q}\cdot\nabla)T = \kappa\nabla^2 T \tag{3}$$

Porous layer: $(-d_m \le z \le 0)$

$$\nabla\cdot\vec{q}_m = 0 \tag{4}$$

$$\frac{\rho_0}{\phi}\frac{\partial \overrightarrow{q_m}}{\partial t} = -\nabla p_m - \frac{\mu_m}{K}\overrightarrow{q_m} + \rho_0\vec{g}\left[1 - \alpha(T_m - T_0)\right] \tag{5}$$

$$\frac{\partial T_m}{\partial t} = \kappa_m \nabla^2 T_m \tag{6}$$

Solid layer: $(-(d_s + d_m) \le z \le 0$ and $d \le z \le d + d_s)$**:**

$$\frac{\partial T_s}{\partial t} = D_s\nabla^2 T_s\,. \tag{7}$$

Where $\vec{q}$ is the velocity vector, T is the temperature, p is the pressure, κ is the thermal diffusivity, α is the thermal expansion coefficient, ϕ is the porosity of the porous medium, A is the ratio of heat capacities , ρ_0 is the reference fluid density and the subscript m refer to the quantity in the porous layer.

To investigate the stability of the basic state, infinitesimal disturbances are superimposed in the form

$$\vec{q} = \overrightarrow{q'},\ T = T_b(z) + \theta,\ p = p_b(z) + p',\ \overrightarrow{q_m} = \overrightarrow{q'_m} \tag{8}$$

$$T_m = T_{mb}(z_m) + \theta_m\,,\ p_m = p_{mb}(z_m) + p_m{}' \tag{9}$$

Where the primed quantities are the perturbed ones over their equilibrium counterparts.

Following the standard linear stability analysis procedure and noting that the principle of exchange of stability holds, we arrive at the following stability equations (for details see Chen F. 1990):

$$(D^2 - a^2)\Theta_s = 0 \tag{10}$$

$$(D^2 - a^2)^2 W = Ra^2\Theta \tag{11}$$

$$(D^2 - a^2)\Theta = -W \tag{12}$$

$$\left(D_m{}^2 - a_m{}^2\right)W_m = -R_m a_m{}^2 \Theta_m \quad (13)$$

$$\left(D_m{}^2 - a_m{}^2\right)\Theta_m = -W_m \quad (14)$$

$$\left(D_m{}^2 - a_m{}^2\right)\Theta_s = 0 \quad (15)$$

where $D = \frac{d}{dz},\ D_m = \frac{d}{dz_m}$, $a = \sqrt{l^2 + m^2},\ a_m = \sqrt{\tilde{l}^2 + \tilde{m}^2}$ are the overall horizontal wave numbers in fluid and porous layers respectively. $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$, $\nabla_m^2 = \partial^2/\partial x_m{}^2 + \partial^2/\partial y_m{}^2 + \partial^2/\partial z_m{}^2$ are the Laplacian operators in fluid and porous layers respectively.

The boundary conditions are:

$$\theta = \theta_s, \quad D\theta = k_r\, D\theta_s \qquad \text{at} \quad z = 1 \quad (16)$$

$$\theta_m = \theta_s, \quad D\theta_m = k_{rm}\, D\theta_s \qquad \text{at} \quad z = -1 \quad (17)$$

$$D\theta_s = 0 \qquad \text{at} \quad z = -(1 + d_{rm}),\ 1 + d_r. \quad (18)$$

Here, $d_r = d_s/d$ is the ratio of the solid plate thickness to the liquid layer thickness and $d_{rm} = d_s/d_m$ is the ratio conductivity of the solid plate to that of the fluid layer with $k_{rm} = \zeta k_r$. Solving Eq. (15) for the solid layer, together with the boundary conditions $(16)-(18)$, the thermal boundary condition at the solid-fluid interface becomes

$$D\Theta = k_{rm}\, a_m \tanh\left(a_m\, d_{rm}\right)\Theta_m. \qquad \text{at} \quad z = -1 \quad (19)$$

$$D\Theta = k_r\, a \tanh\left(a\left(2 + d_r\right)\right)\Theta. \qquad \text{at} \quad z = 1 \quad (20)$$

$$D^2 W + Ma^2\Theta = 0 \qquad \text{at} \quad z = 1 \quad (21)$$

$$W_m = 0 \qquad at\ z_m = -1. \quad (22)$$

At the interface (i.e., z = 0) the continuity of velocity, temperature, heat flux, normal stress and the Beavers and Joseph 1967 slip conditions are imposed. Accordingly, the conditions are:

$$W = \frac{\zeta}{\varepsilon_T} W_m \quad (23)$$

$$\Theta = \frac{\varepsilon_T}{\zeta}\Theta_m \quad (24)$$

$$D\Theta = D_m \Theta_m \quad (25)$$

$$\left[D^2 - 3a^2\right]DW = \frac{-\zeta^4}{\varepsilon_T Da} D_m W_m \quad (26)$$

$$\left[D^2 - \frac{\beta\zeta}{\sqrt{Da}} D\right]W = \frac{-\beta\zeta^3}{\varepsilon_T\sqrt{Da}} D_m W_m \quad (27)$$

Where $\zeta = \frac{d}{d_m}$ the thickness of fluid layer to porous layer and β is the Beavers-Joseph slip parameter.

3. Long wavelength asymptotic analysis

The solution of the Eqs. (10) – (15) and boundary conditions Eqs. (16) – (18) is obtained using a regular perturbation technique with wave number a as a perturbation parameter. For studying the validity of the small wave number analysis, the variables W, W_m and Θ, Θ_m are expressed in terms of the small wave number a,

$$\left(W, \Theta\right) = \sum_{i=0}^{N}\left(a^2\right)^i \left(W_i, \Theta_i\right) \quad (28)$$

$$\left(W_m, \Theta_m\right) = \sum_{i=0}^{N}\left(\frac{a^2}{\zeta^2}\right)^i \left(W_{mi}, \Theta_{mi}\right) \quad (29)$$

Substitution of Eqs. (28) and (29) into Eqs. (11) – (14) and collecting the terms of zeroth order, we obtain

$$D^4 W_0 = 0 \quad (30)$$

$$D^2\Theta_0 = -W_0 \quad (31)$$

$$D_m^2 W_{m0} = 0 \quad (32)$$

$$D_m^2 \Theta_{m0} = -W_{m0} \quad (33)$$

and the boundary conditions becomes

$$W_0 = 0, \quad D\Theta_0 = 0, \quad D^2 W_0 = 0 \qquad \text{at} \quad z = 1 \tag{34}$$

$$W_{m0} = 0, \quad D_m \Theta_{m0} = 0, \; D^2 W_{m0} = 0 \qquad \text{at} \quad z_m = -1. \tag{35}$$

And at the interface($i.e\ z = 0$)

$$W_0 = \frac{\zeta}{\varepsilon_T} W_{m0}, \; \Theta_0 = \frac{\varepsilon_T}{\zeta} \Theta_{m0}, \; D\Theta_0 = D_m \Theta_{m0} \tag{36}$$

$$D^3 W_0 - \eta D^2 W_0 = \frac{-\zeta^4}{Da\xi\varepsilon_T} D_m W_{m0} \tag{37}$$

$$D^2 W_0 - \frac{\beta\zeta}{\sqrt{Da\xi}} DW_0 = \frac{-\beta\zeta^3}{\varepsilon_T \sqrt{Da\xi}} D_m W_{m0}. \tag{38}$$

The solution to the zeroth order Eqs. (30) - (33) is given by

$$W_0 = 0, \quad \Theta_0 = \frac{\zeta}{\varepsilon_T}, \quad W_{m0} = 0, \quad \Theta_{m0} = 1 \tag{39}$$

At the first order in a^2 Eqs. (11) – (14) then reduces to

$$D^4 W_1 = R\frac{\varepsilon_T}{\zeta} \tag{40}$$

$$D^2\Theta_1 - \frac{\varepsilon_T}{\zeta} = -W_1 \tag{41}$$

$$D_m^2 W_{m1} = -R_m \tag{42}$$

$$D_m^2 \Theta_{m1} - 1 = W_{m1} \tag{43}$$

and the boundary conditions becomes

$$W_1 = 0, \quad D\Theta_1 = 2\left(1 + \zeta d_r\right)\zeta k_r \Theta_0, \; D^2 W_1 = -M\frac{\varepsilon_T}{\zeta} \qquad at \quad z = 1 \tag{44}$$

$$W_{m1} = 0, \quad D_m \Theta_{m1} = k_{rm} d_{rm}, \qquad at \quad z_m = -1. \tag{45}$$

And at the interface ($i.e\ z = 0$)

$$W_1 = \frac{1}{\varepsilon T\zeta} W_{m1}, \; \Theta_1 = \frac{\varepsilon_T}{\zeta^3}\Theta_{m1}, \; D\Theta_1 = \frac{1}{\zeta^2} D_m \Theta_{m1} \tag{46}$$

$$D^3 W_1 = \frac{-\zeta^2}{\sqrt{Da\varepsilon T}} D_m W_{m1} \tag{47}$$

$$D^2 W_1 - \frac{\alpha\zeta^2}{\sqrt{Da}} DW_1 = \frac{-\alpha\zeta}{\varepsilon T\sqrt{Da}} D_m W_{m1} \tag{48}$$

The general solutions of Eq. (40) – (42) respectively given by

$$W_1 = R\left[c_1 + c_2 z + c_3\frac{z^2}{2} + c_4\frac{z^3}{6} + \frac{z^4}{24}\right] \tag{49}$$

$$W_{m1} = R\left[c_5 + c_6 z - \frac{z^2}{2}\frac{Da\,\varepsilon T^2}{\varsigma^4}\right] \tag{50}$$

$$c_1 = -\frac{-12Da^{3/2}R\alpha\,\varepsilon T - 24DaM\varepsilon T\varsigma^2 - 12DaR\varsigma^3 - 12Da^{3/2}R\,\varepsilon T\varsigma^3 - 12\sqrt{Da}M\alpha\varepsilon T\varsigma^4 - 5\sqrt{Da}R\alpha\varsigma^5 - 4DaR\alpha\varepsilon T\varsigma^5}{8R\varsigma^5\left(6\sqrt{Da}\alpha + 3\sqrt{Da}\varsigma^3 + \alpha\varsigma^5\right)}$$

$$c_2 = \frac{-12Da^{3/2}R\alpha\,\varepsilon T + 24DaM\varepsilon T\varsigma^2 + 12DaR\varsigma^3 + 12Da^{3/2}R\,\varepsilon T\varsigma^3 - 12\sqrt{Da}M\alpha\varepsilon T\varsigma^4 - 5\sqrt{Da}R\alpha\varsigma^5 - 4\sqrt{Da}M\varepsilon T\varsigma^7 - \sqrt{Da}R\varsigma^8}{8R\varsigma^5\left(6\sqrt{Da}\alpha + 3\sqrt{Da}\varsigma^3 + \alpha\varsigma^5\right)}$$

$$c_3 = \frac{-48\sqrt{Da}M\alpha\varepsilon T + 24\sqrt{Da}R\alpha\varsigma + 12DaR\alpha\varepsilon T\varsigma - 4M\alpha\varepsilon T\varsigma^5 - R\alpha\varsigma^6}{8R\varsigma\left(6\sqrt{Da}\alpha + 3\sqrt{Da}\varsigma^3 + \alpha\varsigma^5\right)}$$

$$c_4 = -\frac{-12DaR\alpha\varepsilon T + 24\sqrt{Da}M\varepsilon T\varsigma^2 + 12\sqrt{Da}R\varsigma^3 + 12M\alpha\varepsilon T\varsigma^4 + 5R\alpha\varsigma^5}{8R\left(6\sqrt{Da}\alpha + 3\sqrt{Da}\varsigma^3 + \alpha\varsigma^5\right)}$$

$$c_5 = \frac{Da\varepsilon T^2}{2\varsigma^4} - \frac{\sqrt{Da}\,\varepsilon T\left(12DaR\alpha\varepsilon T - 24\sqrt{Da}M\varepsilon T\varsigma^2 - 12\sqrt{Da}R\varsigma^3 - 12M\alpha\varepsilon T\varsigma^4 - 5R\alpha\varsigma^5\right)}{8R\varsigma^4\left(6\sqrt{Da}\alpha + 3\sqrt{Da}\varsigma + \alpha\varsigma^3\right)}$$

$$c_6 = \frac{-\sqrt{Da}\varepsilon T\left(12DaR\alpha\varepsilon T - 24\sqrt{Da}M\varepsilon T\varsigma^2 - 12\sqrt{Da}R\varsigma^3 - 12M\alpha\varepsilon T\varsigma^4 - 5R\alpha\varsigma^5\right)}{8R\varsigma^4\left(6\sqrt{Da}\alpha + 3\sqrt{Da}\varsigma + \alpha\varsigma^3\right)}$$

Equations (41) and (43) involving $D^2\Theta_1$ and $D_m^2\Theta_{m1}$ respectively provide the solvability requirement which is given by

$$\int_0^1 W_1\,dz + \frac{1}{\zeta^2}\int_{-1}^0 W_{m1}\,dz = \frac{\varepsilon_T}{\zeta} + \frac{1}{\zeta^2} + \frac{kr\,dr}{\zeta^2} \tag{51}$$

The expressions W_1 and W_{m1} is back substituted in (51) and integrating , we obtain the expression for critical Rayleigh number R_c

$$R_c = \frac{1/\zeta^2 + kr\,dr/\zeta^2 + \varepsilon T/\zeta - M\alpha\,\varepsilon T\,\zeta^2 c_7 - 3Da\,M\,\varepsilon T\left(\varepsilon T + \zeta\right)c_8 - \sqrt{Da}\,M\varepsilon T\,\Delta_1 c_9}{\alpha\,\zeta^3 c_{10} + Da\,\Delta_2\,c_{11} + Da\Delta_3\,c_{12} + \sqrt{Da}\,\Delta_4\,c_9 + Da^{3/2}\,\varepsilon T\,\Delta_5\,c_{10}} \tag{52}$$

Where

$$c_7 = \frac{1}{48\left(\alpha\,\zeta^3 + 3\sqrt{Da}\left(2\alpha + \zeta\right)\right)},$$

$$c_8 = \frac{1}{2\zeta^4\left(\alpha\,\zeta^3 + 3\sqrt{Da}\left(2\alpha + \zeta\right)\right)},$$

$$\Delta_1 = 6\alpha\,\varepsilon T + 10\alpha\,\zeta + \zeta^2,$$

$$c_9 = \frac{1}{8\zeta^2\left(\alpha\,\zeta^3 + 3\sqrt{Da}\left(2\alpha + \zeta\right)\right)},$$

$$c_{10} = \frac{1}{320\left(\alpha\zeta^3 + 3\sqrt{Da}\left(2\alpha+\zeta\right)\right)},$$

$$\Delta_2 = 4\varepsilon T\left(9 + 4\alpha\varepsilon T\right),$$

$$c_{11} = \frac{1}{48\zeta^3\left(\alpha\zeta^3 + 3\sqrt{Da}\left(2\alpha+\zeta\right)\right)},$$

$$\Delta_3 = 3\left(12 + 5\alpha\varepsilon T\right)\zeta,\ \Delta_4 = 25\alpha\varepsilon T + 39\alpha\zeta + 2\zeta^2,$$

$$c_{12} = \frac{1}{80\zeta\left(\alpha\zeta^3 + 3\sqrt{Da}\left(2\alpha+\zeta\right)\right)},$$

$$\Delta_5 = 5\alpha\varepsilon T + 3\alpha\zeta + 4\varepsilon T\zeta + 3\zeta^2,$$

$$c_{13} = \frac{1}{4\zeta^6\left(\alpha\zeta^3 + 3\sqrt{Da}\left(2\alpha+\zeta\right)\right)}$$

IV. Results and Discussion

Rayleigh-Benard-Marangoni stability in a layer of composite scheme affected by walls of finite thickness and of finite conductivity were investigated by the linear analysis. The resulting eigen value problem is solved analytically using a regular perturbation technique with wave number a as a perturbation parameter. The marginal stability of the system considered in this investigation is given by equation (52). We can check this formula against known results for the following limiting case:

In the limit $M \to 0$ and $\zeta >>> 1$, equation (52) is simplified to the following result, which is the case of a single fluid layer between a solid walls of finite thickness and of finite conductivity,

$$R_c = 720\left(1 - \left(2 + d_r\right)k_r\right). \qquad (53)$$

As $k_r = 0$ or $d_r = 0,$ equation (53) can be reduced much further to the result $R_c \to 720$ which is the known exact value (Nield 1987).

To gain physical insight into the onset of the convection, we illustrate the eigen functions of vertical velocity W and corresponding streamline patterns in Figure 2. Figure 2 present the analytically predicted velocity profile at the vertical center line of a system for $\zeta = 1$ and various values of the k_r *and* d_r for $\alpha = 0.1, \zeta = 1, \varepsilon_T = 0.75, Da = 0.001$. It shows that the major part of the flow is confined in the pure fluid layer $(0 \le z \le 1)$, while the fluid is almost at rest in the porous part. The variation of R_c obtained as a function of depth ratio ζ for different values of k_r, d_r are presented in a Fig.3. As expected, the effect of increase in k_r, d_r is to increase the critical Rayleigh number. Furthermore, the variation in has a significant effect on the onset of convection for the values of $\zeta \le 2.5,$ while the curves of different k_r, d_r merge into one when $\zeta > 6.$ The variation of M_c obtained as a function of depth ratio ζ for different values of k_r, d_r are presented in a Fig.4. As expected, the effect of increase in k_r, d_r is to increase the critical Rayleigh number. Furthermore, the variation is significant on the onset of convection for the values of $\zeta \le 2.5,$ while the curves of different k_r, d_r merge in to one when $\zeta > 6.$

A plot of M_c as a function of R_m^c is shown in Fig.6 for a several values of kr, d_r for, $\varepsilon_T = 0.725$ $Da = 4\times10^{-6}$ and $\zeta = 1.$ We notice from figure that when $M_c = 0$, the curve trend toward $R_m^c = 485$ for $k_r = d_r = 1$ the curve trend toward $R_m^c = 410$ for $M_c = 50$. This shows that the thermal buoyancy dominates the system over the effect of surface tension. It is evident from figure that the effect of thermal buoyancy increases so that the system is under the domination of the thermal mode.

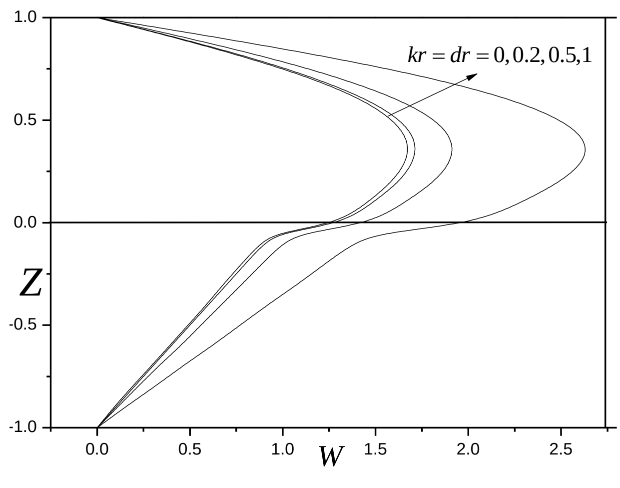

Fig.2. Vertical velocity profile for different values of kr, dr when $\zeta = 1$.

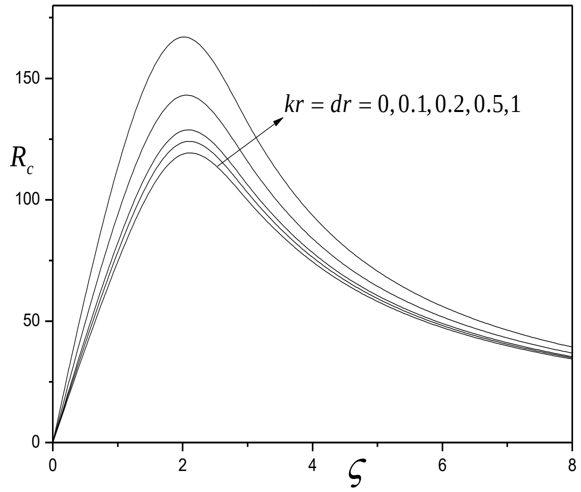

Fig.3. M_c versus ζ for different values of dr when $kr = 1$

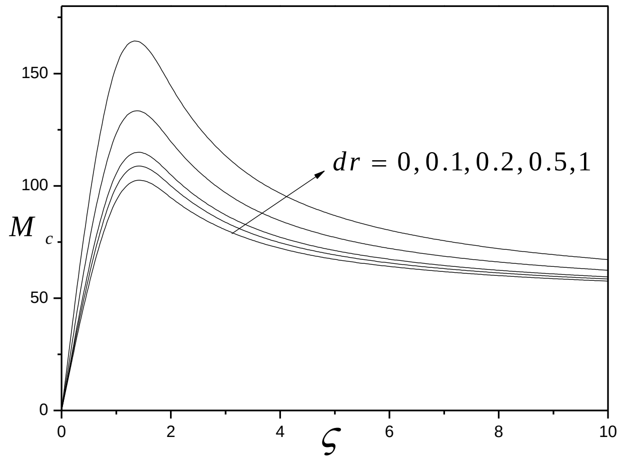

Fig.4. Variation of M_c with ζ for different values of dr .

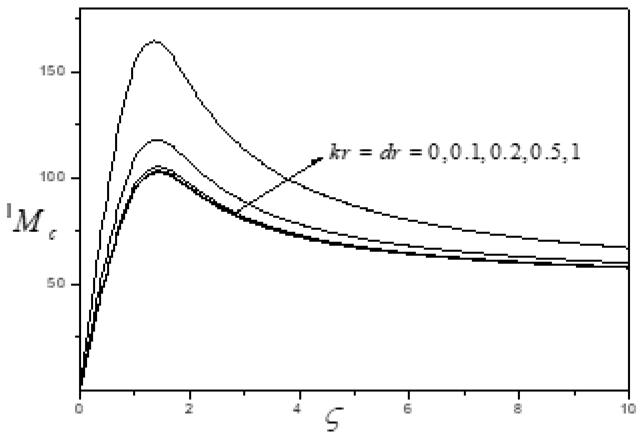

Fig.5. Variation of M_c with ζ for different values of kr, dr .

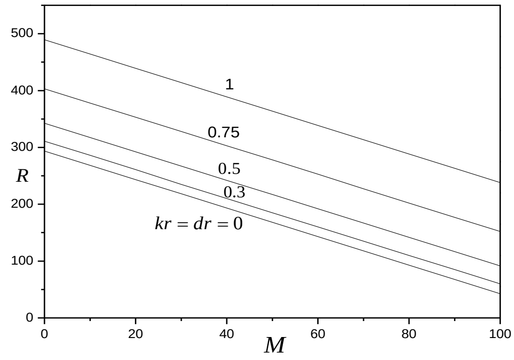

Fig.6. Variation of M with R for different values of kr, dr when $\zeta = 1$

V. CONCLUSION

Boundary effects on Rayleigh-Benard-Marangoni stability in a layer of composite scheme in which a liquid layer overlies a saturates porous material bounded by slabs of finite thermal conductivity and finite thickness has been investigated by means of linear stability analysis . From this we observed that it is possible to control the convection effectively by choosing various physical parameters .

In this investigation, an analytical study of Rayleigh-Bernard-Marangoni convection in a superposed fluid and porous layers with boundary slab. The simultaneous effect of the depth ratio, and the heat conductivity ratio and depth ratio (slab) were examined pictorially and compared to the constant viscosity model. The following main determinations are pointed out as follows throughout the above analysis.

- Stabilizes a larger depth ratio (slab) and the critical numbers of Marangoni and Rayleigh increases with .
- Increasing the heat conductivity ratio contributes to a stabilizing state as heat disturbances deep into the solid layer are easily dissipated and the critical numbers of Marangoni and Rayleigh rises.

VI. REFERENCES

[1] Nield, D. A. & Bejan, A. 2006 Convection in porous media, 3rd edn. New York, NY: Springer-Verlag.

[2] Chen, F.: Throughflow effects on convective instability in superposed fluid and porous layers. J. Fluid. Mech. 231, 113–133 (1990)

[3] Chen, F. & Chen, C. F. 1988 Onset of finger convection in a horizontal porous layer underlying a fluid layer. J. Heat Transfer 110, 403–409.

[4] Ewing, E. & Weekes, S. 1998 Numerical methods for contaminant transport in porous media.

[5] Comput. Math. 202, 75–95.

[6] Blest, C., Duffy, B. R., Mckee, S. & Zulkfle, A. K. 1999 Curing simulation of thermoset composites.

[7] Composites Part A 30, 1289–1309. (doi:10.1016/S1359-835X(99)00032-9)

[8] Straughan, B. 2002 Sharp global nonlinear stability for temperature-dependent viscosity

[9] convection. Proc. R. Soc. Lond. A 458, 1773–1782. (doi:10.1098/rspa.2001.0945)

[10] Straughan, B. 2008 Stability and wave motion in porous media. Appl. Math. Sci. Series, vol. 165.

[11] New York, NY: Springer.

[12] Carr, M. 2004 Penetrative convection in a sup erp osed p orous-medium-fluid layer via internal

[13] heating. J. Fluid Mech. 509, 305–329. (doi:10.1017/S0022112004009413)

[14] Chang, M. H. 2004 Stability of convection induced by selective absorption of radiation in a fluid

[15] overlying a porous layer. Phys. Fluids 16, 3690–3698. (doi:10.1063/1.1789551)

[16] Chang, M. H. 2005 Thermal convection in superposed fluid and porous layers subjected to a

[17] horizontal plane Couette flow. Phys. Fluids 17, 064106-1–064106-7.

[18] Chang, M. H. 2006 Thermal convection in superposed fluid and porous layers subjected to a plane

[19] Hirata, S. C., Goyeau, B., Gobin, D., Carr, M. & Cotta, R. M. 2007 Linear stability of natural

[20] convection in superposed fluid and porous layers: influence of the interfacial modelling. Int. J.

[21] Heat Mass Transfer 50, 1356–1367. (doi:10.1016/j.ijheatmasstransfer.2006.09.038)

[22] Hoppe, R. H. W., Porta, P. & Vassilevski, Y. 2007 Computational issues related to iterative coupling

[23] of subsurface and channel flows. Calcolo 44, 1–20. (doi:10.1007/s10092-007-0126-z)

[24] Mu, M. & Xu, J. C. 2007 A two-grid method of a mixed Stokes–Darcy model for coupling fluid

[25] flow with porous medium flow. SIAM J. Numer. Anal. 45, 1801–1813. (doi:10.1137/050637820)

[26] Hill, A. A. & Straughan, B. 2009 Global stability for thermal convection in a fluid overlying a

[27] highly porous material. Proc. R. Soc. A 465, 207–217. (doi:10.1098/rspa.2008.0303)

[28] Chen, F. & Chen, C. F. 1988 Onset of finger convection in a horizontal porous layer underlying a

[29] fluid layer. J. Heat Transfer 110, 403–409.

[30] Nield, D. A.: “Throughflow effects on the Rayleigh-Benard convective instability problem”, J. Fluid Mech. 185, 353–360, 1987.

[31] X. Nicolas, Bibliographic review on the Poiseuillee Rayleigh Bénard flows; the mixed convection flows in horizontal rectangular ducts heated from below, International Journal of Thermal Sciences 41 (2002) 961e1016.

[32] Ruan, X. Ye., Chen., Doona, T., Yang, T.: Developments in Ohmic heating, in: P.S. Richardson (Ed.), Improving the Thermal Processing of Foods, Woodhead Publishing in Food Science and Technology, CRC Press, 224-252, 2004.

[33] Hill, A. A.: Penetrative convection induced by the absorption of radiation with nonlinear internal heat source, Dyn. Atmos. Oceans 38, 57–67, 2004.

[34] Straughan, B.,: “Stability and wave motion in porous media”, Appl. Math. Sci. Ser.vol. 165. Springer, New York, 2008.

[35] Generalis, S. C., Busse, F. H.: Transition in inclined internally heated fluid layers: G.G.M. Stoffels, T.H. van der Meer, A.A. van Steenhoven (Eds.), Proceedings 5th European Thermal-Sciences Conference (18-21) 1-8 Eindhoven (NL), 2008.