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Fuzzy Sets and Graphs

Tarunika Sharma*[1], Rashi khubnani[2]

*[12] BSH (Mathematics), NHCE Bangalore, India
tarunika.sharma@gmail.com[1]
rashi.khubnani@gmail.com[2]

ABSTRACT

Fuzzy sets in mathematics correspond to uncertainty. We have defined fuzzy graph and its adjacency matrix and discussed about the bounds and energy of Fuzzy graphs. We have also extended the concept to social network to study strength of relationship.
Keywords: Adjacency matrix, Bounds, connected graph, Energy, vertex, edges, walk, path, strength, Fuzzy sets, Fuzzy graph.

I. INTRODUCTION

In real world problems, Mathematical model are stable till reasons, computing are crisp and precise in character, but when the complexity of a system increases behavior of system may not be very crisp and precise. Complexity, credibility and uncertainty are the major pillar of any model. Uncertainty has a pivotal role in maximizing the usefulness of a system.

Lotfi A. Zadeh gave framework to describe this phenomenon of uncertainty in his paper "Fuzzy Sets". Fuzzy principle is "Everything is a matter of degree" and the logic is called Fuzzy logic. Fuzzy subset of set P is a map called membership function .Value assigned to a individual represents its grade of membership in fuzzy sets.

Fuzzy set $P^{\sim}$ define as a set of ordered pairs. Mathematically it is represented as –

$$P^{\sim} = \{y, \mu P^{\sim}(y) \text{ such that } y \in U \}$$

Where U is the universal set and $\mu P^{\sim}(y)$ = degree of membership of y in {P} and the values of this set is in the range from 0 to 1.[1]

Graph is a presentation of number of points that are linked by lines. It is a study of points and lines. Each point is called a vertex and the lines are called edges. Graph is a tool for modeling relationships. They are used to find solutions to various problems. Graphical models are used to represent traffic network, communication network, airlines network, railway network, social networks etc. Many times graph can not represent all the systems properly, because of some uncertainty of the parameters of system, that is the reason why Fuzzy graph is been discussed. [2]

II. LOGICS OF FUZZY GRAPH

Fuzzy graph called f-graph is a combination of G: (σ, μ) here σ as fuzzy subset of P and μ as fuzzy relation on σ. Here P is nonempty finite set. Here μ is a relation named reflexive and symmetric on σ. Where $\sigma : P \rightarrow [0,1]$ and

$\mu : \times P \rightarrow [0,1]$: $\mu(u,v) \leq \sigma\{u\} \wedge \sigma\{v\} \forall$ u and v are the elements of set P. The crisp graph of the Fuzzy graph G: (σ, μ) is expressed as G*: (σ^*, μ^*) where σ^* is non empty set P of nodes and μ^* = E subset of $P \times P$. [3][4]

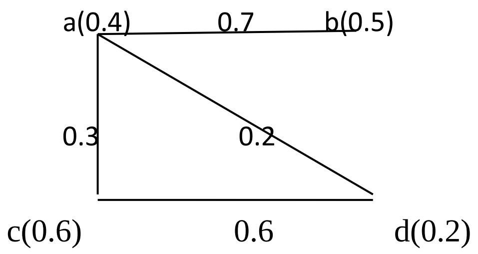
Fuzzy graph T : (τ, ω) said as partial fuzzy sub graph of G: (σ, μ) if $\tau(u) \leq \sigma(u)$ $\forall$ u and $\omega(u,v) \leq \mu(u,v)$ $\forall$ u, v. Also we can say T : (τ, ω) a fuzzy sub graph of G: (σ, μ) if $\tau(u) = \sigma(u)$ for all u $\epsilon \tau^*$ and

$\omega(u,v) = \mu(u,v)\ \forall\ u, v \in P^*$ [5]

Further we will discuss about order and size of Fuzzy graph with degree of its vertices.

For fuzzy graph G: (σ, μ): order of G is defined as $\sum \sigma(x)$ where x belongs to underlying set P. It is denoted by O (G). Fuzzy graph G: (σ, μ) : size of G is defined as $\sum \mu(x, y)$ where x , y belongs to underlying set P. It is denoted by S (G). For a fuzzy graph G: (σ, μ) : degree of a vertex u is define as $d_G(u) = \sum \mu(u, v)$. [6]

Let us consider Fuzzy graph G: (σ, μ) with a = 0.4, b = 0.5, c = 0.6, d = 0.2 and edges ab = 0.7, ad = 0.2, cd = 0.6, ac = 0.3



Here O (G) =1.8 S (G) = 1.7 $d_G(a) =$ 1.2, $d_G(b) =$ 0.7, $d_G(c) =$ 0.9, $d_G(d) =$0.8

III. ENERGY AND BOUNDS OF A GRAPH

Graph Energy : It is the sum of absolute values of Eigen values in adjacency matrix of any graph . Graph G with vertices andedges as {n,m}, say $v_1, v_2, \ldots, v_n$ be the vertices. Adjacency matrix A = A (G) of any graph G is the square matrix with order n such as--

$$a_{ij} = \begin{cases} 1 & \text{if } i \neq j : v_i \text{ and } v_j \text{ are adjacent} \\ 0 & \text{if } i \neq j : v_i, v_j \text{ are not adjacent} \\ 0 & \text{if } i = j \end{cases}$$

If $\lambda 1, \lambda 2, \ldots, \lambda n$ are the Eigen values for A(G). Then Graph energy for G is

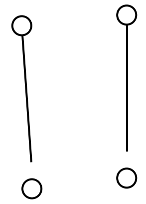
$$E = E(G) = \sum_{i=1}^{n} |\lambda i|$$

In view of the fact that A is symmetric matrix having zero trace, sum of Eigen values is equal to zero and all Eigen values are real. [7][8][9]

$\lambda 1 \geq \lambda 2 \geq \cdots \geq \lambda n$,

$\lambda 1 + \lambda 2 + \cdots + \lambda n = 0$

EXAMPLE: Let G be a disconnected graph



Then $A(G) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

Characteristic polynomial is det $(xI_4 - A) = (x^2-1)(x^2-1)$ with Eigen values 1,-1,1,-1

Energy of the graph E (G) = 4

If G_1 and G_2 are two components of above disconnected graph then we can easily verify that

$E(G) = E(G_1) + E(G_2)$

BOUNDS:

The bounds of graph are defined on the basis of vertices, edges, and determinants of adjacency matrix etc. Upper bound and lower bounds are explored, which define graphs with both extreme energies and is considered as field of upcoming growing area for research under spectral graph theory. [10][11][12]

Lower bounds on the energy of a graph are-

1.On the basis of Edges — (With equality holding iff G contains complete bipartite graph Ka, b : a · b = m, arbitrarily a lot of isolated vertices.
$E(G) \geq 2\sqrt{m}$

2. On the basis of vertices and edges and the determinant---

$$E(G) \geq \sqrt{2m + n(n-1)|\det A|^{2/n}}$$

3. As per non singular graphs

$$E(G) \geq \frac{2m}{n} + n - 1 + \ln\left(\frac{n|\det A|}{2m}\right)$$

4. As per simple graph with order n > 2 having m edges--(with equality holds iff $G \cong \frac{n}{2}K_2$ (n as even) or $G \cong \overrightarrow{K_n}$.)

$$E(G) \geq \sqrt{2m + n(n-1)|\det A|^{\frac{n}{2}} + \frac{4}{(n+1)(n+4)}\left[\sqrt{\frac{2m}{n}} - \left(\frac{2m}{n}\right)^{1/4}\right]^2}$$

5. As per the vertices

$$E(G) \geq 2(n-1)$$

Upper bounds

1. On the basis of vertices and edges-(with equality achieved only in the cases where G is consider as empty graph or else one regular graph)

$$E(G) \leq \sqrt{2mn}$$

2. If $2m \geq n$ and G as graph with n vertices having m edges-(With equality holding if and only if G is either $\frac{n}{2}K2$, Kn or else non complete connected regular graph with 2 non-trivial eigen values mutually along with absolute value) i.e $\sqrt{\frac{\left[2m-\left(\frac{2m}{n}\right)^2\right]}{n-1}}$

$$E(g) \leq \frac{2m}{n} + \sqrt{(n-1)\left[2m-\left(\frac{2m}{n}\right)^2\right]}$$

3. If $2m \leq n$ and G is graph with vertices and Edges as {n,m} (With equality holding iff G is put out of joint union of edges and cut off vertices)

$$E(G) \leq 2m$$

4. In terms of vertices

$$E(G) \leq \frac{n(\sqrt{1}+\sqrt{n})}{2}$$

5. As per G as bipartite graph with n vertices, where $n > 2$

$$E(G) \leq \frac{n(\sqrt{n}+\sqrt{2})}{\sqrt{8}}$$

IV. ENERGY AND BOUNDS OF FUZZY GRAPH

Consider P as a nonempty set. The fuzzy subset of P is a function $\sigma : P \to [0,1]$ where σ is called the membership function $\sigma(v)$ is called the membership of v where v belongs to the set P. Fuzzy relation is defined as matrix which is known to be Fuzzy relation matrix

$$M = [m_{ij}] \text{ here } m_{ij} = \mu(v_i, v_j)$$

Here $\mu(v_i, v_j)$ represents the strength of relationship among v_i and v_j. [13]

Adjacency matrix A of fuzzy graph G = (P, σ, μ) is a (n x n) medium matrix denoted as

$$A = [m_{ij}] \text{ here } m_{ij} = \mu(v_i, v_j)$$

Let us take a fuzzy graph G: (P, σ, μ)

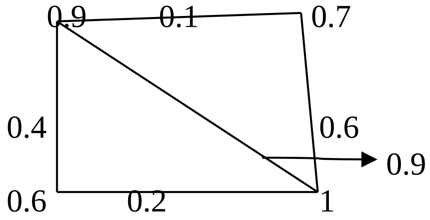


Figure (1)

Adjacency matrix of above figure (1) is

$$A = \begin{bmatrix} 0 & 0.1 & 0.9 & 0.4 \\ 0.1 & 0 & 0.6 & 0 \\ 0.9 & 0.6 & 0 & 0.2 \\ 0.4 & 0 & 0.2 & 0 \end{bmatrix}$$

∗ The Eigen values of a Fuzzy graph G known as the Eigen values of its own adjacency matrix A.

∗ Energy of Fuzzy graph G known as sum of absolute values of Eigen values of G. It is denoted by E(G). [14]

Eigen values of adjacency matrix A in the above matrix is given by -

[-1.0464, -0.3164, 0.1174, 1.2454]

For the above graph E (G) {figure 1} = 1.0464 + 0.3164 + 0.1174 + 1.2454 = 2.7256

BOUNDS

If we consider weighted graph G of order n with edges $e_1, e_2 \ldots\ldots e_n$, each edge with nonzero weight $w(e_i)$,then

$$E(G) \leq 2\sum_{i=1}^{m} |w(e_i)|$$

where equality holds iff each of connected part of G has the majority two vertices.

If w $(e_i) \in [0,1]$ then it become particular case of fuzzy graph.

We can restate theorem for fuzzy graph as---

If G= (P, σ, μ) fuzzy graph having $|P| = n$, $\mu^* = \{e_1, \ldots \ldots e_m\}$ then

$$E(G) \leq 2\sum_{i=1}^{m} \mu(e_i)$$

With respect to the membership values of vertices---

$$E(G) \leq (n-1)\sum_{i=1}^{n} \sigma(v_i)$$

If G^* as a cycle and $\mu^* = \{e_1, \ldots \ldots e_n\}$ then

$$E(G) \leq 2\sum_{i=1}^{m} \sigma(v_i) \ : \qquad v_i \in P, i = 1,2 \ldots n.$$

In terms of strength of the relation we can improve it as ----

If G = (P,σ, μ) as a fuzzy graph having $|P| = n$ and $\mu^* = \{e_1, \ldots \ldots e_m\}$.If $m_i = \mu(e_i)$ is the power of relation linked with i^{th} edge then equation (1) is----

$$\sqrt{2\sum_{i=1}^{m} m_i^2 + n(n-1)|A|^{\frac{2}{n}}} \leq E(G) \leq \sqrt{2(\sum_{i=1}^{m} m_i^2)n}$$

For the graph in figure (1) ---
E (G) = 2.7256
Lower bounds = 2.3238
Upper bounds = 3.3226
i.e. $2.3238 < 2.7256 < 3.3226$

So we have satisfied equation (1) for the graph in figure (1).

V. APPLICATION OF FUZZY GRAPH

Let us consider an example of Fuzzy graph from reality. Take an example of social network of five friends named A, B, C, D, E shown in figure (2) below.. Here the number represents the degree of friendship among each other. [15][16][17][18]

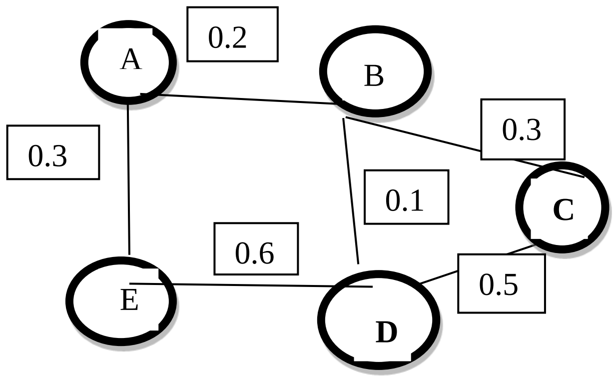


Figure (2)

Here number represents degree of friendship. The strong point of path is considered as weight (the membership value) of lowest arc of the path between two vertices.

Strength of connectedness among any two vertices r and s is definite as maximum of the strength of every paths between two vertices. It is denoted by $\mathrm{CONN}_G(x, y)$

In graph {figure (2)} ---

Path and strengths between two friends A and D are

Path number	Strength
A$\rightarrow B \rightarrow D$	0.1
A$\rightarrow B \rightarrow C \rightarrow D$	0.2
A$\rightarrow E \rightarrow D$	0.3

$\mathrm{CONN}_G(x, y) = \max\{0.1, 0.2, 0.3\} = 0.3$
So the strongest path is A$\rightarrow E \rightarrow D$

VI. CONCLUSION

We discussed basic concepts about fuzzy graphs, its energy and analysis in social network. In future this concept and spectra of fuzzy graphs may be applied to various real life problems and can be discussed in forthcoming papers.

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