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An Unsteady MHD Mixed Convection Flow Pattern of Casson Fluid through Past Vertical Porous Plate with Radiation and Chemical Reaction

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ABSTRACT

This manuscript reveals a detailed numerical study on the influence of radiation, radiation absorption and chemical reaction on unsteady magneto hydrodynamic free convective flow of a heat generating Casson fluid past an oscillating vertical plate surrounded in a porous medium in the presence of constant wall temperature and concentration. The non dimensional governing equations along with the corresponding boundary conditions are solved using finite difference method numerically. The impact of various emerging flow parameters on velocity, temperature and concentration are presented graphically and analyzed. A comparison is done with published results in order to validate the present methodology. Expressions for skin-friction, Nusselt number and Sherwood number are also obtained.

Keywords : Casson fluid, MHD, porous medium, heat and mass transfer, chemical reaction, radiation and heat generation.

I. INTRODUCTION

Several natural phenomena as well as technological issues are liable to the analysis of MHD concepts. Geophysics encounters magnetohydrodynamic characteristics in the relations of magnetic fields and conducting fluids. Engineers make use of MHD principle in pumps designing, space vehicle propulsion, thermal enrichment, organize and re-entry in creating latest systems of power generation etc. The study of MHD is relatively significant in the field of aerodynamics due to the fact that the temperature that occurs in such flight speeds is adequate to disconnect or ionize the air considerably and the motion of such ionized air may be restricted by applying proper magnetic field. During the last decades, incompressible viscous fluid flows and heat transfer phenomena in the presence of porous medium have acknowledged great attention owing to the plenty of practical applications in manufacturing and chemical processes. The curiosity in mixed convection boundary layer flows of visco-elastic fluid is growing to a large extent due to its numerous practical issues in industry, chemical and manufacturing technology. Based on this literature a large number of studies appeared. Chandra Reddy et al. [1] reported an analytical and numerical study on thermal and solutal buoyancy effect on MHD boundary layer flow of a visco-elastic fluid past a porous plate with varying suction and heat source in the presence of thermal diffusion. Muthucumaraswamy and Velmurugan [2] considered theoretical study of heat transfer effects on flow past a parabolic started vertical plate in the presence of chemical reaction of first order. Guruvi Reddy et al. [3] established and analyzed magneto-convetive and radiation absorption fluid flow past an exponentially accelerated vertical porous plate with variable temperature and concentration. Umamaheswar et al. [4] examined unsteady magneto hydrodynamic free convective double-diffusive viscoelastic fluid flow past an inclined permeable plate in the presence of viscous dissipation and heat absorption. Kandasamy et al. [5] considered and studied chemical reaction heat and mass transfer on MHD flow over a vertical stretching surface with heat source and thermal stratification effects. Lin et al. [6] established and analyzed magneto hydrodynamics thermo capillary Marangoni convection heat transfer of power-law fluids driven by temperature gradient. Sinha and Mondal [7] investigated influence of slip velocity on magneto hydrodynamic flow of blood and heat transfer through a permeable capacity. Sulochana and Samrat [8] considered an unsteady MHD radiative flow of a nanoliquid past a permeable stretching sheet. Chamkha et al. [9] considered unsteady MHD free convection flow past an exponentially accelerated vertical plate with mass transfer, chemical reaction and thermal radiation. MadhusudhanRao et al. [10] put an unsteady MHD free convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate with heat absorption radiation chemical reaction and Soret effects. Misra and Sinha [11] analyzed an effect of thermal radiation on MHD flow of blood and heat transfer in a permeable capillary in stretching motion. Tokis[12] discussed free convection and mass transfer effects on the magneto hydrodynamic flows near a moving plate in a rotating medium. Ahmed and Kalita [13] considered magneto hydrodynamic transient flow through a porous medium bounded by a hot vertical plate in presence of radiation: a theoretical analysis. Mehdi et al.[14] examined the free convective heat and mass transfer for MHD fluid flow over a permeable vertical stretching sheet in the presence of the radiation and buoyancy effects. Aziz and Aziz [15] analyzed MHD flow of a third grade fluid in a porous half space with plate suction or injection

II. FORMULATION OF THE PROBLEM

The unsteady free convection heat and mass transfer flow of a well-known non-Newtonian fluid, namely Walters B visco-elastic fluid past an infinite vertical porous plate, embedded in a porous medium in the presence of thermal radiation, oscillatory suction as well as variable permeability is considered. In addition to this the existence of heat generation / absorption is also considered. A uniform magnetic field of strength B0 is applied perpendicular to the plate. Let x^* axis be taken along with the plate in the direction of the flow and y^* axis is normal to it. Let us consider the magnetic Reynolds number is much less than unity so that the induced magnetic field is neglected in comparison with the applied transverse magnetic field. The basic flow in the medium is, therefore, entirely due to the buoyancy force caused by the temperature difference between the wall and the medium. It is assumed that initially, at $t^* \leqslant 0$, the plate as fluids are at the same temperature and concentration. When $t^*>0$, the temperature of the plate is instantaneously raised to T * and the concentration of the species is set to C^*. Under the above assumption with usual Boussinesq's approximation, the governing equations and boundary conditions are given by V= UH(t)cos(ωt)i (or) V =Usin(ωt)i Where H(t) is the unit step function, constant U is the amplitude of the plate oscillations, i is the unit vector in the vertical flow direction and ω is the frequency of oscillation of the plate. At the same time , the plate temperature is raised to Tw which is thereafter maintained constant.The tensor of the Casson fluid can be written as , where π= eijeij and eij is the (i,j)th component of deformation rate, π is the product of the component of deformation rate with itself , πc is the critical value of this product based on the non-

Newtonian fluid, μB is the plastic dynamic viscosity of its fluid and is yield stress of the non-Newtonian fluid. Before forming the governing equations we have taken some assumptions that are unidirectional flow, one dimensional flow, free convection, rigid plate, incompressible flow, unsteady flow, non-Newtonian flow, oscillating vertical plate and viscous dissipation term in the energy equation is neglected. Considering the above assumptions we have formed the following set of partial differential equations.

$$\rho\frac{\partial u'}{\partial t'}=\mu_\beta\left(1+\frac{1}{\gamma}\right)\frac{\partial^2 u'}{\partial y'^2}-\sigma B_0^2u'-\frac{\mu\phi}{k_1}u'+\rho g\beta(T'-T_\infty)+\rho g\beta^*(C'-C_\infty)\quad(1)$$

$$\rho C_p\frac{\partial T'}{\partial t'}=k\frac{\partial^2 T'}{\partial y'^2}-\frac{\partial q_r}{\partial y'}+Q'(T'-T_\infty)+Q_1(C'-C_\infty)+\sigma B_0^2u'^2\quad(2)$$

$$\frac{\partial C'}{\partial t'}=D\frac{\partial^2 C'}{\partial y'^2}-Kr'(C'-C_\infty)\quad(3)$$

Cogley et al. have shown that, in the optically thin limit for a non-gray gas near equilibrium, the radiative heat flux is represented by the following form:

$$\frac{\partial q_r}{\partial y'}=4(T'-T_\infty)I \text{ Where } I=\int K_{\lambda w}\frac{\partial e_{b\lambda}}{\partial T^*}d\lambda$$

The initial and boundary conditions are

$$\left.\begin{array}{ll} t'<0: u'=0, T'=T_\infty, C'=C_\infty & \text{for all } y'<0 \\ t'\geq 0: u'=u_0\sin(w't'), T'=T_w, C'=C_w & \text{at } y'=0 \\ \quad u'\to 0, T'\to T_\infty, C'\to C_\infty & \text{as } y'\to\infty \end{array}\right\}(4)$$

On introducing the following non-dimensional quantities

$$u=\frac{u'}{u_0},\ t=\frac{t'u_0^2}{\nu}, y=\frac{y'u_0}{\nu},\ \theta=\frac{T'-T_\infty}{T_w-T_\infty},\ C=\frac{C'-C_\infty}{C_w-C_\infty}$$

$Gr=\dfrac{\nu g\beta(T_w-T_\infty)}{u_0^3}$, (Grashof number),

$Gc=Gr=\dfrac{\nu g\beta^*(C_w-C_\infty)}{u_0^3}$, (Modified Grashof number)

$K=\dfrac{k_1u_0^2}{\phi\nu^2}$, (Permeability parameter),

$M=\dfrac{\sigma B_0^2\nu}{\rho u_0^2}$, (Magnetic parameter)

$\Pr=\dfrac{\nu\rho C_p}{k}$, (Prandtl number),

$Ec=\dfrac{u_0^2}{C_p(T_w-T_\infty)}$, (Eckert number),

$Sc=\dfrac{\nu}{D}$, (Schmidt number)

$Q=\dfrac{Q'\nu}{\rho C_p u_0^2}$, (Heat absorption parameter),

$R=\dfrac{4\nu I}{\rho C_p u_0^2}$, (Radiation parameter)

$\chi=\dfrac{Q_1\nu(C_w-C_\infty)}{\rho C_p u_0^2(T_w-T_\infty)}$, (Radiation absorption parameter),

$Kr=\dfrac{Kr'\nu}{u_0^2}$, (Chemical reaction parameter),

γ (Casson parameter)

In terms of the above non-dimension quantities, Equations (1)-(3) reduces to

$$\frac{\partial u}{\partial t}=\left(1+\frac{1}{\gamma}\right)\frac{\partial^2 u}{\partial y^2}-Mu-\frac{1}{K}u+Gr\theta+GmC\quad(5)$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}-R\theta+Q\theta+\chi C+M\,Ecu^2\quad(6)$$

$$\frac{\partial C}{\partial t}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}-Kr\,C\quad(7)$$

The corresponding initial and boundary conditions are:

$$\left.\begin{array}{ll} t<0: u=0, T=0, C=0 & \text{for all } y<0 \\ t\geq 0: u=\sin(wt), \theta=1, C=1 & \text{at } y=0 \\ \quad u\to 0, T\to 0, C^*\to 0 & \text{as } y\to\infty \end{array}\right\}\quad(8)$$

III. METHOD OF SOLUTION

Equations (5)-(7) are coupled non-linear partial differential equations and are to be solved by using the initial and boundary conditions (8). However exact solution is not possible for this set of equations and hence we solve these equations by finite-difference method. The equivalent finite difference schemes of equations for (5)-(7) are as follows:

$$\frac{u_{i,j+1}-u_{i,j}}{\Delta t}=\left(1+\frac{1}{\gamma}\right)\left(\frac{u_{i-1,j}-2u_{i,j}+u_{i+1,j}}{(\Delta y)^2}\right)-Mu_{i,j}-\frac{1}{K}u_{i,j}+Gr\theta_{i,j}+GcC_{i,j}\quad(9)$$

$$\frac{\theta_{i,j+1}-\theta_{i,j}}{\Delta t}=\frac{1}{\Pr}\left(\frac{\theta_{i-1,j}-2\theta_{i,j}+\theta_{i+1,j}}{(\Delta y)^2}\right)-R\theta_{i,j}+Q\theta_{i,j}+\chi C_{i,j}+MEc(u_{i,j})^2\quad(10)$$

$$\frac{C_{i,j+1}-C_{i,j}}{\Delta t}=\frac{1}{Sc}\left(\frac{C_{i-1,j}-2C_{i,j}+C_{i+1,j}}{(\Delta y)^2}\right)-KrC_{i,j}\quad(11)$$

Here, index i refer to y and j to time. The mesh system is divided by taking Δy = 0.04. From the initial condition in (8), we have the following equivalent:

$$u(i,0)=0,\ \theta(i,0)=0, C(i,0)=0 \ \ for\ all\ i \quad (12)$$

The boundary conditions from (8) are expressed in finite-difference form as follows

$$u(0,j)=1, \theta(0,j)=1, C(0,j)=1 \ for\ all\ j$$
$$u(i_{max},j)=\sin(w*(j-1)*\Delta t), \theta(i_{max},j)=1, C(i_{max},j)=1 \ for\ all\ j \quad (13)$$

(Here i_{max} was taken as 201)

First the velocity at the end of time step viz, u (i,j+1), (i=1,201) is computed from (9) in terms of velocity, temperature and concentration at points on the earlier time-step. Then θ (i, j +1) is computed from (10) and C (i, j +1) is computed from (11). The procedure is repeated until

t = 0.05 (i.e. j = 500). During computation Δt was chosen as 0.0001.

Skin-friction: The skin-friction in non-dimensional form is given by

$$\tau=-\left(1+\frac{1}{\gamma}\right)\left(\frac{du}{dy}\right)_{y=0}, where\ \tau^{*}=\frac{\tau}{\rho u_0^2}$$

Rate of heat transfer: The dimensionless rate of heat transfer is given by $Nu=-\left(\frac{d\theta}{dy}\right)_{y=0}$

Rate of mass transfer: The dimensionless rate of mass transfer is given by $Sh=-\left(\frac{dC}{dy}\right)_{y=0}$

IV. RESULT AND DISCUSSION

The influence of different physical parameters like Grashof number, modified Grashof number, magnetic parameter, thermal radiation, Prandtl number, Eckert number, Soret number and Schmidt number on velocity, temperature and concentration is discussed by using graphical representations. The general nature of the velocity profile is parabolic with picks near the plate. Figure 1 show that the velocity enhances for rising values of magnetic parameter. The effect of Prandtl number on velocity is displayed in figure 2. The Prandtl number is a dimensionless number approximating the ratio of momentum diffusivity (kinematic viscosity) and thermal diffusivity. The fluid velocity decreases for increasing values of Prandtl number. This is due to the effect of transverse magnetic field, which has the nature of reducing the velocity. These results are similar to that of Mishra et al. [13]. Figures 3, 4 depict the velocity variations under the effect of Grashof number and modified Grashof number respectively. The velocity of the flow grows when the values of these parameters increases. Figure 5 represents the impact of porous medium on velocity. It is evident that the velocity enhances for increasing values of porosity parameter. The changes in velocity under the existence of heat source / sink are depicted in figure 6. It is noticed that the velocity enhances in the presence of heat source where as it falls down in the case of heat sink. The variation in velocity under the influence of Schmidt number is shown in figure 7. It is evident that velocity comes down when the values of Schmidt number are increased. The existence of thermal diffusion results in improving the flow velocity which is clearfrom figure 8. These results coincide with that of Chandra Rddy et al.[1].The temperature decreases in the presence of thermal radiation which is shown in figure 9. The effect of Prandtl number on temperature is presented in figure 10. The temperature decreases for increasing values of Prandtl number. Figure 11 reveals the same nature as that of velocity under the influence of heat source/sink. Figure 12 depicts the influence of Eckert number on temperature. It is observed that the temperature rises with increasing values of Eckert number.

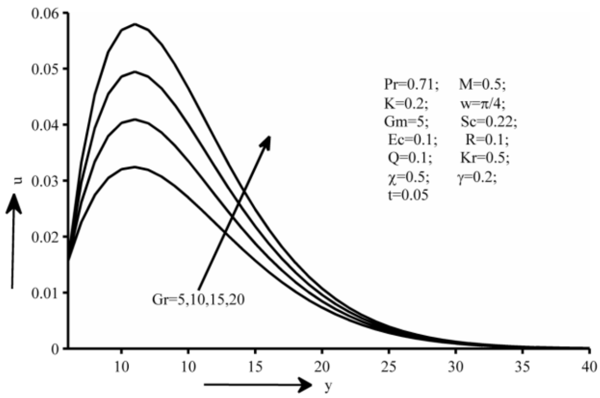

Fig 1: Effect of Grashof number on velocity

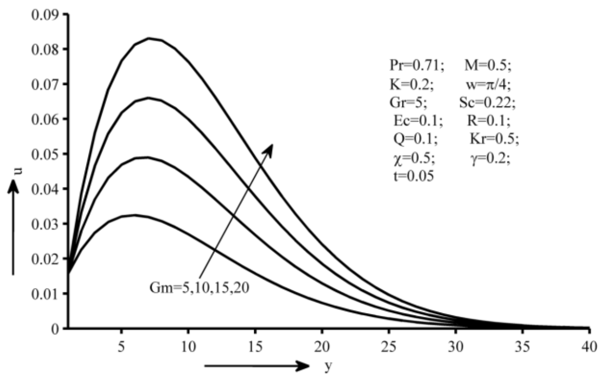

Fig 2: Effect of modified Grashof number on velocity

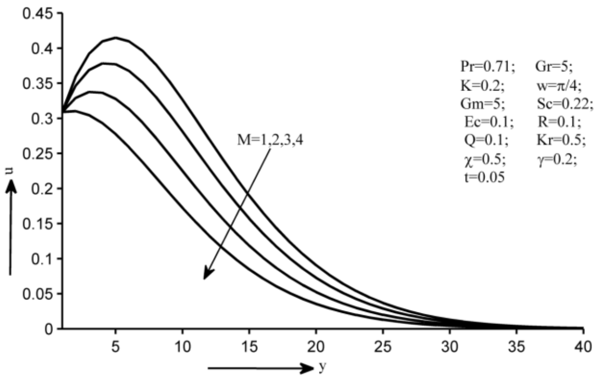

Fig 3: Effect of magnetic parameter on velocity

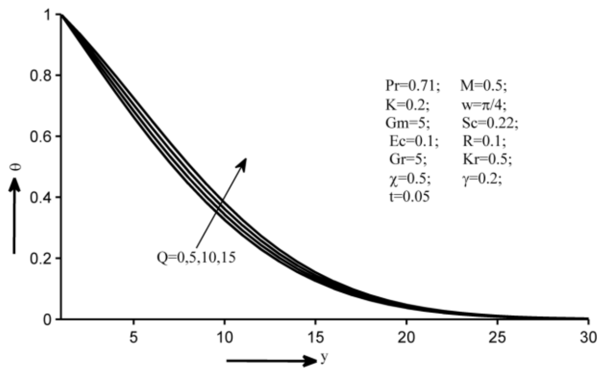

Fig 7: Effect of heat source on temperature

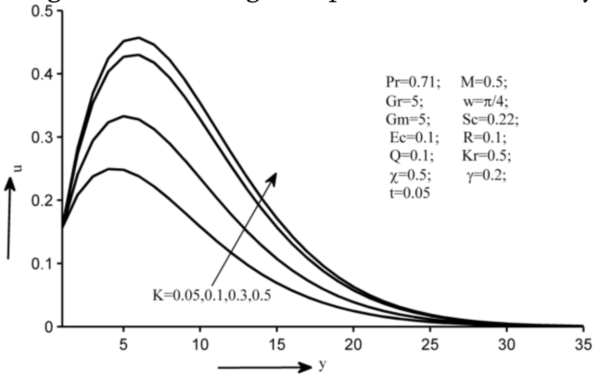

Fig 4: Effect of permeability parameter on velocity

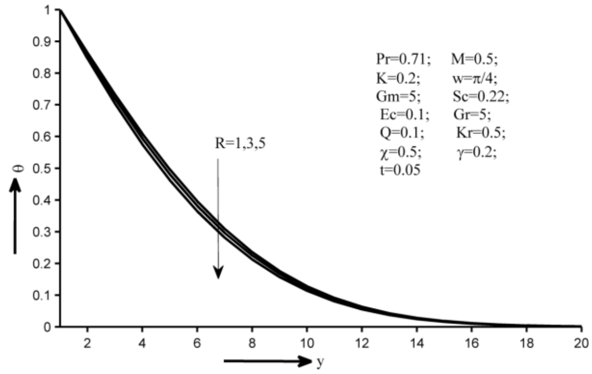

Fig 8: Effect of radiation parameter on velocity

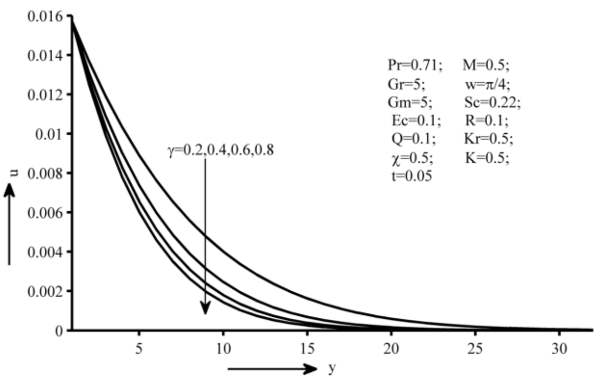

Fig 5: Effect of Casson parameter on velocity

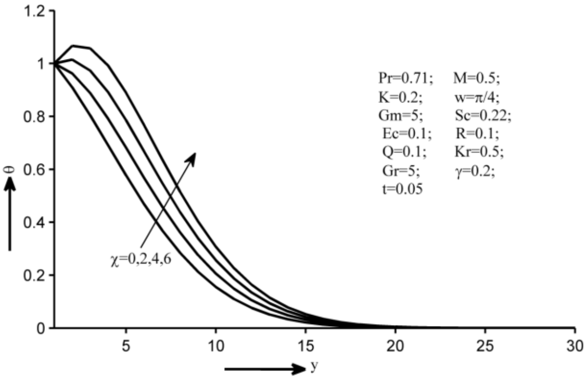

Fig 9: Effect of radiation absorption parameter

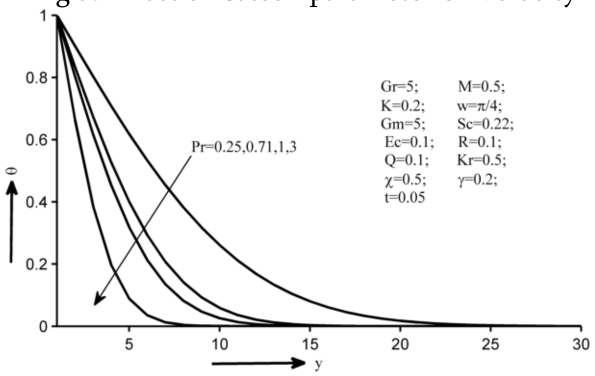

Fig 6: Effect of Prandtl number on temperature

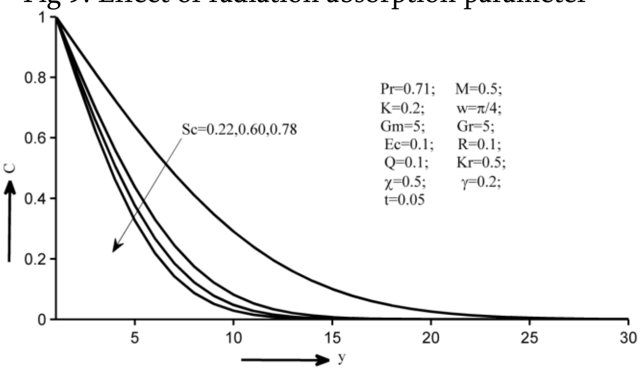

Fig 10: Effect of Schmidt number on concentration

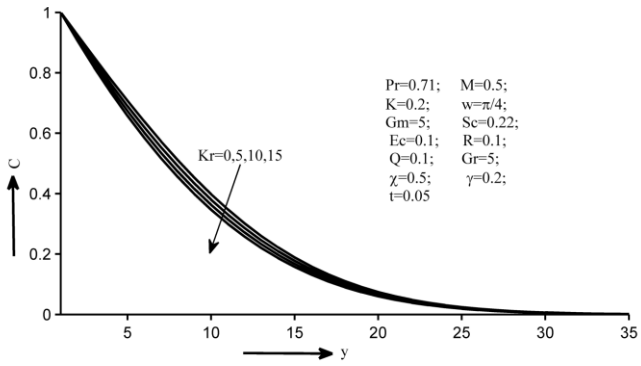

Fig 11: Effect of Chemical reaction parameter on concentration

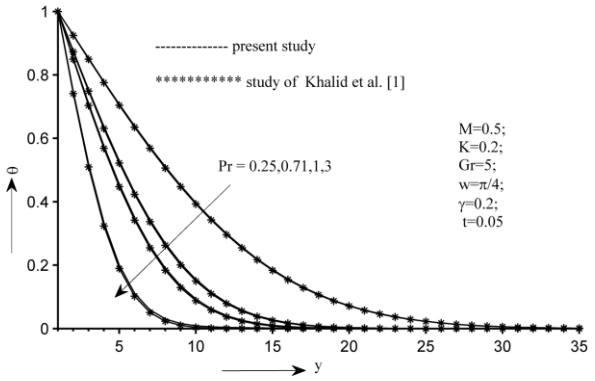

Fig 12: Comparison of present study with the study of in the absence of Gm,Q,R,Ec,χ,Sc,Kr

Table.2: Variations in Nusselt number

Pr	Q	R	χ	Nu
0.71	0.8	0.8	0.1	4.4626
1	0.8	0.8	0.1	5.3128
3	0.8	0.8	0.1	8.6469
7.1	0.8	0.8	0.1	11.1283
0.71	0.1	0.8	0.1	5.3356
0.71	0.3	0.8	0.1	5.3291
0.71	0.5	0.8	0.1	5.3256
0.71	1	0.8	0.1	5.3024
0.71	0.8	0.5	0.1	5.4492
0.71	0.8	1	0.1	5.6524
0.71	0.8	2	0.1	5.8672
0.71	0.8	0.8	1	4.4124
0.71	0.8	0.8	2	3.5264
0.71	0.8	0.8	3	3.0564
0.71	0.8	0.8	4	2.5648

Table.3: Variations in Sherwood number

Sc	Kr	Sh
0.22	0.8	3.5484
0.60	0.8	4.6542
0.78	0.8	4.8492
0.96	0.8	5.5321
0.22	0.1	3.5234
0.22	0.3	3.6134
0.22	0.5	3.7568
0.22	0.9	3.8829

V. CONCLUSION

An analytical solution is investigated on MHD boundary layer flow of a visco-elastic fluid past a porous plate with varying suction and heat source/sink in the presence of thermal radiation and diffusion. The governing equations for the velocity field, temperature and concentration by finite difference method. The main findings of this study are as follows.

- Velocity of the fluid reduces for increasing values of Prandtl number and magnetic parameter.
- Temperature of the fluid grows for rising values of Eckert number, but a reverse effect is noticed in the case of Prandtl number and radiation absorption parameter.
- The concentration reduces with an increase in Schmidt number.
- The existence of heat source leads to enhance the temperature and a reverse trend is observed in the presence of heat sink.
- Skin friction decreases with an increase of Eckert number and Schmidt number but a reverse effect is noticed in the case of radiation absorption parameter and magnetic parameter.
- Nusselt number increases as radiation absorption parameter increases but in the case of Eckert number it decreases.
- Sherwood number increases with an increase in Schmidt number.

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