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Totally Umbilical Slant Submanifolds of S- manifolds

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ABSTRACT

In this paper, we study slant submanifolds of S-manifolds which are totally umbilical. We show that every totally umbilical proper slant submanifold of a S-manifold is either totally geodesic or if submanifold is not totally geodesic then we derive a formula for slant angle.

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I. INTRODUCTION

Blair [2], introduced the notion of an S-manifold equipped with a normal framed metric structure as a generalization of an almost Hermitian structure and almost contact metric structures. We refer to [30] for geometry of framed metric structures and related references cited therein.

On the other hand, as a natural generalization to the holomorphic and totally real subman- ifolds, Chen [9], introduced and studied slant submanifolds of an almost Hermitian manifolds. The contact version of slant submanifolds was introduced by Lotta [18]. Later, the study of slant submanifolds was enriched by the authors of [7, 11, 12, 14, 21, 23, 28] and many others. Recently Carraizo et. al. [6] defined and studied slant submanifolds of S-manifolds, when the second fundamental form is totally geodesic, f-umbilical, totally umbilical and austere submanifolds. Motivated by the above studies, in this paper we study totally umbilical slant submanifolds and is organized as follows: In section-2, we recall the notion of S-manifold, submanifold and formulas, basic ideas regarding it. Section 3 is the main section of this paper. Here, we derived the classification results of slant submanifolds of S-manifold, when the submanifold is totally umbilical.

II. PRELIMINARIES

Let M˜ be a (2n + s)-dimensional Riemannian manifold. It is said to be an S-manifold, if there exist on M˜ an J-structure [29] of rank 2n and s global vector fields ξ_1,ξ_2,⋯,ξ_s (structure vector fields), η^1,η^2,⋯,η^s are s 1-forms and g is a Riemannian metric on M such that [29]

$$J^2 = -I + \eta^\alpha \otimes \xi_\alpha, \eta(\xi) = 1, J\xi_\alpha = 0, \eta^\alpha \cdot J = 0, \quad (2.1)$$

$$g(JU, JV) = g(U,V) - \sum_\alpha \eta^\alpha(U)\eta^\alpha(V),\ g(U,\xi_\alpha) = \eta^\alpha(U), \quad (2.2)$$

$$\Omega(U,V) = g(U,JV) = -\Omega(V,U) \quad (2.3)$$

For any $U,V \in T\widetilde{M}, \alpha = 1,2,\cdots,s$.

An J-structure is called normal if

$$[J,J] + 2d\eta^\alpha \otimes \xi_\alpha = 0, \quad (2.4)$$

And an S-structure [2] if it is normal and

$$\Omega = d\eta^\alpha, \quad (2.5)$$

$\forall \alpha \in 1,2,\cdots,s$.

When s = 1, a framed metric structure is an almost contact metric structure, while an S-structure is a Sasakian structure. When s = 0, a framed metric structure is an almost Hermitian structure, a normal framed metric structure is a Hermitian structure, while an S-structure is a Kaehler structure.

If an J-structure on M˜ is an S-structure then it is known that [2]

$$(\tilde{\nabla}_U J)V = \sum_\alpha (g(JU,JV)\xi_\alpha + \eta^\alpha(V)J^2U), \quad (2.7)$$

$$\tilde{\nabla}_U \xi_\alpha = -JU. \quad (2.8)$$

Let M be an isometrically immersed submanifold of an S-manifold M˜, we denote by the same symbol g the induced metric on M . Let TM be the set of all vector fields tangent to M and T^⊥ M is the set of all vector fields normal to M. Then, the Gauss and Weingarten formulae are given by

$$\tilde{\nabla}_U V = \nabla_U V + \sigma(U,V), \quad \tilde{\nabla}_U Y = -A_Y U + \nabla^\perp{}_U Y, \quad (2.8)$$

for any U,V ∈ TM ,Y ∈ T^⊥ M , where ∇ (resp. ∇ ⊥) is the induced connection on the tangent bundle TM (resp. normal bundle T^⊥ M) [10]. The shape operator A is related to the second fundamental form σ of M by

$$g(A_Y U, V) = g(\sigma(U, V), Y), \quad (2.9)$$

Now, for any $x \in M$, $U \in T_xM$ and $\in T_x{}^\perp M$, we put

$$\varphi U = TX + NX, \quad \varphi Y = tV + nV, \quad (2.10)$$

where TX (resp. NX) is the tangential (resp. normal) component of φU, and tV (resp. nV) is the tangential (resp. normal) component of φY. From (2.3) and (2.10)

$$g(TX, V) + g(U, TY) = 0, \quad (2.11)$$

for each U,V ∈ TM ,Y ∈ T^⊥ M . The covariant derivatives of the tensor fields T, N, t and n are defined as

$$(\tilde{\nabla}_U \varphi)V = \tilde{\nabla}_U \varphi V - \varphi(\tilde{\nabla}_U V), \quad (2.12)$$

$$(\tilde{\nabla}_U T)V = \nabla_U TV - T(\nabla_U V), \quad (2.13)$$

$$(\tilde{\nabla}_U N)V = \nabla_U NV - N(\nabla_U V). \quad (2.14)$$

$$(\tilde{\nabla}_U t)Y = \nabla_U tY - t(\nabla_U Y). \quad (2.15)$$

$$(\tilde{\nabla}_U n)Y = \nabla_U nY - n(\nabla_U Y). \quad (2.16)$$

Now, on a submanifold of an S-manifold by equations (2.7) and (2.8) we get

$\nabla_U \xi = -PU$ (2.17) And

$\sigma(U,\xi) = -FU,$ (2.18)

for each $U \in TM$. Further from equation (2.7) and (2.10)

$A_Y\xi = tV, \eta(A_Y U) = 0,$ (2.19)

for each $Y \in T^\perp M$. On using equations (2.6), (2.8), (2.10) and (2.12)-(2.14), we obtain

$(\tilde{\nabla}_U T)V = A_{NV}U + t\sigma(U,V) + \sum_\alpha\{g(TU,TV)\xi_\alpha + \eta^\alpha(V)(T^2U + tNU)\}$, (2.20)

$(\tilde{\nabla}_U N)V = n\sigma(U,V) - \sigma(U,TV) + \eta^\alpha(V)(NTU + nNU)$ (2.21)

submanifold M of an almost contact metric manifold M˜ is said to be totally umbilical if

$\sigma(U,V) = g(U,V)H,$ (2.22)

where H is the mean curvature vector of M. Furthermore, a submanifold M is called totally geodesic, if σ(U,V)=0 for all U, V ∈ Γ(TM) and if H = 0, then M is minimal in M˜.

III. SLANT SUBMANIFOLDS OF S-MANIFOLD

In this section, we consider M is a proper slant submanifold of an S-manifold M˜. We always consider such submanifolds tangent to the structure vector fields ξ_α.

An immersed submanifold M of an S-manifold M˜ is slant in M˜ if for any x ∈ M and any U ∈ TxM such that U, ξ_α are linearly independent, the angle θ(x) ∈ [0,π/2] between f U and TxM is a constant θ, i.e., θ does not depend on the choice of U and x ∈ M, θ is called the slant angle of M in M˜. Invariant and anti-invariant submanifolds are slant submanifolds with slant angle θ=0 and θ=π/2 respectively [6]. A slant submanifold which is neither invariant nor anti-invariant is called a proper slant submanifold.

We have the following theorem which characterize slant submanifolds of a f -metric manifold:

Theorem 3.1: Let M be a submanifold of a f-metric manifold M̃, such that ξ_α∈TM.

Then, M is slant if and only if there exists a constant λ ∈[0,1] such that

$T^2 = -\lambda(I - \sum_{\alpha=1}^{s} \eta^\alpha \otimes \xi_\alpha)$. (3.1)

Further more, if θ is the slant angle of M, then

$\lambda = \cos^2\theta$.

From [6], for any U, V tangent to M , we can easily obtain the results for an S-manifold M̃,

$$g(TU,TV) = \cos^2\theta\{g(U,V) - \sum_{\alpha=1}^{s}\eta^\alpha(U)\eta^\alpha(V)\} \quad (3.2)$$

$$g(NU,NV) = \sin^2\theta\{g(U,V) - \sum_{\alpha=1}^{s}\eta^\alpha(U)\eta^\alpha(V)\} \quad (3.3)$$

Theorem 3.2. Let M be a totally umbilical slant submanifold of an S-manifold M̃, then the following statements are equivalent:

H∈ μ ;

either M is trivial or Sasakian or Anti-invariant submanifold of M̃.

Proof: For any U, V ∈ TM , from equation (2.20), we have

$$(\tilde{\nabla}_U T)V = A_{NV}U + t\sigma(U,V) + \sum_\alpha \{g(TU,TV)\xi_\alpha + \eta^\alpha(V)(T^2U + tNX)\}. \quad (3.4)$$

Taking the inner product in (3.4) with ξ_α, by virtue of (2.11) we get

$$g(\nabla_U TV, \xi_\alpha) = g(\sigma(U,\xi_\alpha), NV) + g(t\sigma(U,V),\xi_\alpha) + sg(TU,TV).$$

As M is totally umbilical slant submanifold of M̃ , then from equation (2.22) the above equation becomes

$$-g(TV,\nabla_U\xi_\alpha) = g(H,NV)\eta^\alpha(U) + g(U,V)g(tH,\xi_\alpha) + sg(TU,TV).$$

Then from equations (2.10), (2.17) and (3.2), the above equation takes the form

$$\cos^2\theta(s-1)\left[g(U,V) - \sum_\alpha \eta^\alpha(U)\eta^\alpha(V)\right] = -g(H,NV)\eta^\alpha(U). \quad (3.5)$$

If H ∈ μ, then right hand side of the equation (3.5) is identically zero. Hence statement (ii) holds. Conversely, if (ii) holds then from (3.5) we get H ∈ μ. This completes the proof of the theorem.

Theorem 3.3. Let M be a totally umbilical proper slant submanifold of an S-manifold M̃ , such that H, ∇⊥X H ∈ μ, for all X ∈ TM . Then,

either M is totally geodesic;

(i) or the slant angle $\theta = \tan^{-1}\left(\sqrt{\frac{-g(U,V)}{\eta^\alpha(U)\eta^\alpha(V)}}\right)$.

Proof: Let $U, V \in TM$, we have

$$\tilde{\nabla}_U\phi V - \phi(\tilde{\nabla}_U V) = -\phi U.$$

Applying (2.8), (2.10) and from the fact that M is totally umbilical proper slant submanifold, we obtain

$$\nabla_U TV + g(U,TV)H - A_{NV}U + \nabla_U^\perp - T\nabla_U V - N\nabla_U V - g(U,V)\phi H = -TU - NU \quad (3.6)$$

Taking inner product with φH in equation (3.6) yields

$$g(U,TV)g(H,\phi H) + g(\nabla_U^\perp NV,\phi H) = g(N\nabla_U V,\phi H) + g(U,V)g(\varphi H,\varphi H) - g(NU,\phi H).$$

From the fact that H ∈ μ and by virtue of (2.2), we obtain

$$g(\nabla^{\perp}_{U} NV, \phi H) = g(U,V)\|H\|^2. \quad (3.7)$$

Now, for any U ∈ TM , we have

$$(\tilde{\nabla}_U \varphi)H = \tilde{\nabla}_U \varphi H - \varphi \tilde{\nabla}_U H.$$

Using (2.6) and the fact that H ∈ μ, we get

$$0 = \tilde{\nabla}_U \varphi H - \varphi \tilde{\nabla}_U H.$$

Using equations (2.8) and (2.10), we obtain

$$-A_{\varphi H} U + \nabla^{\perp}_{U} \varphi H = -TA_H U - NA_H U + t\nabla^{\perp}_{U} H + n\nabla^{\perp}_{U} H. \quad (3.8)$$

Taking inner product with NV in (3.8) for any V ∈ TM and using the fact that n∇⊥U H ∈ ν, (3.8) yields

$$g(\nabla^{\perp}_{U} \varphi H, NV) = -g(NA_H U, NV)$$

Applying (2.8) and (3.3), we get

$$g(\tilde{\nabla}_U NV, \phi H) = \sin^2\theta \left[g(U,V) - \sum_\alpha \eta^\alpha(U)\eta^\alpha(V)\|H\|^2\right]. \quad (3.9)$$

In view of (3.7) and (3.9), we obtain

$$\{\cos^2\theta g(U,V) - \sin^2\theta\, \eta^\alpha(U)\eta^\alpha(V)\}\|H\|^2 = 0. \quad (3.10)$$

Since M is proper slant submanifold, then from (3.10) it follows that either H = 0, that is M is totally geodesic in M or θ is acute angle , then $\theta = \tan^{-1}\left(\sqrt{\frac{-g(U,V)}{\eta^\alpha(U)\eta^\alpha(V)}}\right)$. This completes the proof of the theorem.

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