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Finite Difference Methods for Weather Prediction in Abuja, Nigeria

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ABSTRACT

In this study, we simulate some finite difference schemes numerically to predict weather trends of Abuja Station, Nigeria. By evaluating the results from these schemes, it has shown that the best scheme in the finite difference method that gives a close accurate weather forecast is the trapezoidal scheme hence we use it to simulate numerical weather data obtained from Federal Airports Authority of Nigeria (FAAN), Abuja and corresponding numerical weather data obtained by the compatible finite difference schemes, using MATLAB (R2012a) software to obtain future numerical weather trends.

Keywords : finite different, trapezoidal scheme, Weather Prediction, forecasting, modelling

I. INTRODUCTION

Weather forecasting is one of the most complex and remarkably problems of modern science. In spite of evident advancement in the few decades and shift from manual forecasting methods to numerical ones, there are some significant problems that are yet to be solved either by manual methods or methods based on computer simulation posing interesting challenges for all those engaged in the field. An interminably need for in depth information on the actual meteorological conditions and problems associated to the use of traditional methods are responsible from intensive development of numerical weather prediction (NWP).

The history of numerical weather prediction considers how current weather conditions as input into mathematical models of the atmosphere and oceans to predict the weather and future sea state (the process of numerical weather prediction) has changed over the years. Though first attempted in the 1920s, it was not until the advent of the computer and computer simulation that computation time was reduced to less than the forecast period itself.

However, the vast range of available finite difference scheme is both a blessing and a curse, and many different combinations have been proposed, analysed and used for large scale geophysical fluid dynamics applications, particularly in the ocean modeling community Le Roux et al. (2005, 2007); Le Roux and Pouliot (2008); Danilov (2010); Cotter et al. (2009); Cotter and Ham (2011); Rostand and Le Roux (2008); Le Roux (2012); Comblen et al. (2010), whilst many other combinations have been used in engineering applications where different scales and modeling aspects are important.

In geophysical applications, the stability properties of compatible finite difference have long been recognized, leading to various choices being proposed and analyzed on triangular meshes, Walters and Casulli (1998); Rostand and Le Roux (2008). However, no explicit use was made of the compatible structure beyond stability until Cotter and Shipton (2012) used it, which proved that all compatible finite difference methods have exactly steady geotropic modes; this is considered a crucial property for numerical weather prediction Staniforth and Thuburn (2012).

The purpose of this paper is to obtain numerical weather data and weather trends using the various schemes of the Finite Difference Method because the Finite Difference Method is one of the most powerful numerical methods for obtaining the numerical solution of step-wise differential equations.

Finite Difference Approximations for Finite Difference Methods

The finite difference method involves using discrete approximations like

$$\frac{\partial u}{\partial x} \approx \frac{u_{i+1}-u_i}{\Delta x} \quad (1)$$

where the quantities on the right hand side are defined on the finite difference mesh. Approximations to the governing differential equation are obtained by replacing all continuous derivatives by discrete formulas such as those in Eq. (1).

Advection Equation

The advection equation is the major model used in this weather prediction meanwhile other schemes were derived based on their stability, conditional stability and neutrality as it affect the weather trends in a local station. Many of the important ideas can be illustrated by reference to the advection equation which we write in the form

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0 \quad (2)$$

where c is a constant. We divide the (x,t) −plane into a series of discrete points $(i\Delta x, n\Delta t)$ and denote the approximate solution for u at this point by u_i^n. The possible finite-difference scheme for the equation is

$$\frac{u_i^{n+1}-u_i^n}{\Delta t} + c\frac{u_i^n-u_{i-1}^n}{\Delta x} = 0 \quad (3)$$

We may rewrite (3) as

$$u_i^{n+1} = (1-\mu)u_i^n + \mu u_{i-1}^n, \quad (4)$$

where $\mu = c\Delta t/\Delta x$. The advection equation Eq. (2) has a possible finite-difference scheme given by Eq. (3) and hence an analytic solution of the advection equation in the form of a single harmonic is

$$u(x,t) = Re\left[U(t)e^{ikx}\right] \quad (5)$$

Here $U(t)$ is the wave amplitude and k the wavenumber. Substituting this result into Eq. (2) gives

$$\frac{dU}{dt} + ikcU = 0, \quad (6)$$

which has the solution

$$U(t) = U(0)e^{-ikct}, \quad (7)$$

$U(0)$ which is the initial amplitude. Hence

$$u(x,t) = Re\left[U(0)e^{-ik(x-ct)}\right] \quad (8)$$

as expected. The solution is finally expressed in Eq. (8).

However, in the von Neumann method we looked for an analogous solution of the finite-difference equation Eq. (4) which after substituting $u_j^n = Re\left[U^{(n)}e^{ikj\Delta x}\right]$, this reduces the entire scheme to the amplitude equation;

$$U^{(n+1)} = \lambda U^{(n)} \quad (9)$$

which properly defines the amplification factor $|\lambda|$ and hence we can now study the behavior of the amplitude $U^{(n)}$ as n increases, the stability of the scheme and the frequency of the stability is given by;

$$p = \omega\Delta t \quad (10)$$

$$\Delta t \le \frac{1}{|\omega|} \quad (11)$$

where p is the stability of the scheme, λ is the wavelength ω is the frequency and Δt the time interval and $\omega = 1,2,\ldots,n$.

For Euler Scheme

$$\lambda = 1 + ip, \quad |\lambda| = (1+p^2)^{\frac{1}{2}}. \qquad (12)$$

at $p = 1$, we have

$$\lambda = 1 + i$$

This scheme is unstable$|\lambda| > 1$ for any $p > 0$

For Backward Scheme

$$\lambda = \frac{\left(1+\frac{1}{4}ip\right)}{(1+p^2)}, \quad |\lambda| = (1+p^2)^{-\frac{1}{2}} \qquad (13)$$

at $p = 1$, we have

$$\lambda = 0.5 + 0.125i$$

This scheme is stable

For Trapezoidal Scheme

$$\lambda = \frac{\left(1+\frac{1}{4}p^2+ip\right)}{\left(1+\frac{1}{4}p^2\right)}, \quad |\lambda| = 1.$$

at $p = 1$, we have $\lambda = 1 + i/1.25$

This scheme is always neutral.

For Matsuno Scheme

$$\lambda = 1 - p^2 + ip, \ |\lambda| = (1 - p^2 + p^4)^{\frac{1}{2}} \qquad (15)$$

at $p = 1$, we have

$$\lambda = i$$

This scheme is stable, if $|p| \le 1$.

For Heun Scheme

$$\lambda = 1 - \frac{1}{2}p^2 + ip, \ |\lambda| = \left(1 + \frac{1}{4}p^4\right)^{\frac{1}{2}}. \qquad (16)$$

at $p = 1$, we have

$$\lambda = 0.5 + i$$

This is always >1 so that the Heun scheme is always unstable.

However, we select the real part minus the product of the imaginary part of the deduced wavelength with itself for the resultant solution of

$$U^{(n+1)} = \lambda U^{(n)}$$

as

$$U^{(n+1)} = Re\left[\lambda U^{(n)}\right]. \qquad (17)$$

II. NUMERICAL SOLUTIONS

Summary of Weather Data Set from Federal Airport Authority of Nigeria, Abuja Station

Table 1 : Dataset from the Federal Airport Authority of Nigeria for Abuja Station

Annual Climatological Summary **Year: 2015**

Station: ABUJA, NG **Elev: 343.1ft. Lat: 09.15°N Lon: 07.00°E**

STATION #	STATION NAME	ELEV	LAT	LONG	DATE	RelHum	TMAX	TMIN	RAINFALL	SUNSHINE HRS	WIND SPEED	WIND DIR.
65125	Abuja	343.1	09.15'N	07.00'E	201501	43	35.5	19.3	0	7.3	2.9	N
65125	Abuja	343.1	09.15'N	07.00'E	201502	50	37.4	23.2	0.6	7.5	3.7	NE
65125	Abuja	343.1	09.15'N	07.00'E	201503	62	37.7	25	7.5	8.2	3.5	NE
65125	Abuja	343.1	09.15'N	07.00'E	201504	62	36.6	25.7	74.2	7.5	5	E
65125	Abuja	343.1	09.15'N	07.00'E	201505	76	35.8	24.6	109.2	7.4	4.9	SW
65125	Abuja	343.1	09.15'N	07.00'E	201506	81	30.2	23.2	267.2	7.5	4.7	S
65125	Abuja	343.1	09.15'N	07.00'E	201507	86	28.7	22.3	314.8	4.5	3.7	SW
65125	Abuja	343.1	09.15'N	07.00'E	201508	87	28.7	22.5	278.3	5.2	4.2	NW
65125	Abuja	343.1	09.15'N	07.00'E	201509	83	29.5	22.2	258.4	5.2	4.1	W
65125	Abuja	343.1	09.15'N	07.00'E	201510	78	30	21.8	238.2	6.8	3.3	NW
65125	Abuja	343.1	09.15'N	07.00'E	201511	64	33.7	21.6	Trace	9.2	3	E
65125	Abuja	343.1	09.15'N	07.00'E	201512	36	35	17.2	0	8.8	3.2	NE

Source: FAAN

Solution of Sunshine Hours Prediction Using Finite Difference Scheme

Using Eq. (17) and sunshine hours value from Table 1 for the first month, we compute the predicted values for the different schemes.

$$U^{(n+1)} = Re\left[\lambda U^{(n)}\right]$$

For Heun Scheme
where $\lambda = 0.5 + i$ and $U^{(n)} = 7.3$ for sunshine hours
then

$U^{(n+1)} = Re[7.3(0.5 + i)] = Re[3.65 + 7.3i] = 3.65 - 1 = 2.65$

For Matsuno Scheme

where $\lambda = i$ and $U^{(n)} = 7.3$ for sunshine hours
then

$$U^{(n+1)} = Re[7.3(i)] = Re[7.3i] = 0$$

For Trapezoidal Scheme
where $\lambda = 1 + i/1.25$ and $U^{(n)} = 7.3$ for sunshine hours
then

$$U^{(n+1)} = Re[7.3(1 + i/1.25)] = 7.66 - 1 = 6.66$$

For Backward Scheme
where $\lambda = 0.5 + 0.125i$ and $U^{(n)} = 7.3$ for sunshine hours
then

$$U^{(n+1)} = Re[7.3(0.5 + 0.125i)] = 4.6 - 1 = 3.6$$

For Euler Scheme
where $\lambda = 1 + i$ and $U^{(n)} = 7.3$ for sunshine hours
then

$$U^{(n+1)} = Re[7.3(1 + i)] = 7.3 - 1 = 6.3$$

The results of predicted sunshine hours for all the 12 months of the year are shown in Table 2

Table 2 : Sunshine Hours 2016 (SHRs 2016)

Months	Wavelength λ					Amplitude $U^{(n)}$	Schemes $U^{(n+1)}$(SHRs 2016)				
ω	Heun	Matsuno	Trapezoidal	Backward	Euler	Sunshine 2015	Heun	Matsuno	Trapezoidal	Backward	Euler
1	$0.5+i$	i	$1+i/1.25$	$0.5+0.125i$	$1+i$	7.3	2.65	0	6.66	3.6	6.3
2	$0.5+i$	i	$1+i/1.25$	$0.5+0.125i$	$1+i$	7.5	2.75	0	6.86	3.7	6.5
3	$0.5+i$	i	$1+i/1.25$	$0.5+0.125i$	$1+i$	8.2	3.1	0	7.56	4.08	7.2
4	$0.5+i$	i	$1+i/1.25$	$0.5+0.125i$	$1+i$	7.5	2.75	0	6.86	3.7	6.5
5	$0.5+i$	i	$1+i/1.25$	$0.5+0.125i$	$1+i$	7.4	2.7	0	6.76	3.68	6.4
6	$0.5+i$	i	$1+i/1.25$	$0.5+0.125i$	$1+i$	7.5	2.75	0	6.86	3.7	6.5
7	$0.5+i$	i	$1+i/1.25$	$0.5+0.125i$	$1+i$	4.5	1.25	0	3.86	2.2	3.5
8	$0.5+i$	i	$1+i/1.25$	$0.5+0.125i$	$1+i$	5.2	1.6	0	4.56	2.58	4.2
9	$0.5+i$	i	$1+i/1.25$	$0.5+0.125i$	$1+i$	5.2	1.6	0	4.56	2.58	4.2
10	$0.5+i$	i	$1+i/1.25$	$0.5+0.125i$	$1+i$	6.8	2.4	0	6.16	3.38	5.8
11	$0.5+i$	i	$1+i/1.25$	$0.5+0.125i$	$1+i$	9.2	3.6	0	8.56	4.58	8.2
12	$0.5+i$	i	$1+i/1.25$	$0.5+0.125i$	$1+i$	8.8	3.4	0	8.16	4.38	7.8

From the above table, the selection of the scheme to represent the model forecasting for the sunshine hours for 2016 is based on the trend of the scheme whose result is closest to the previous year i.e. 2015 and hence among all five schemes in the table it is very obvious that aside Euler's (Forward) Scheme

which is the second closest, the Trapezoidal Scheme is the closest to the given sunshine hours in 2015. Hence we use the Sunshine Hours predicted using the Trapezoidal Scheme.

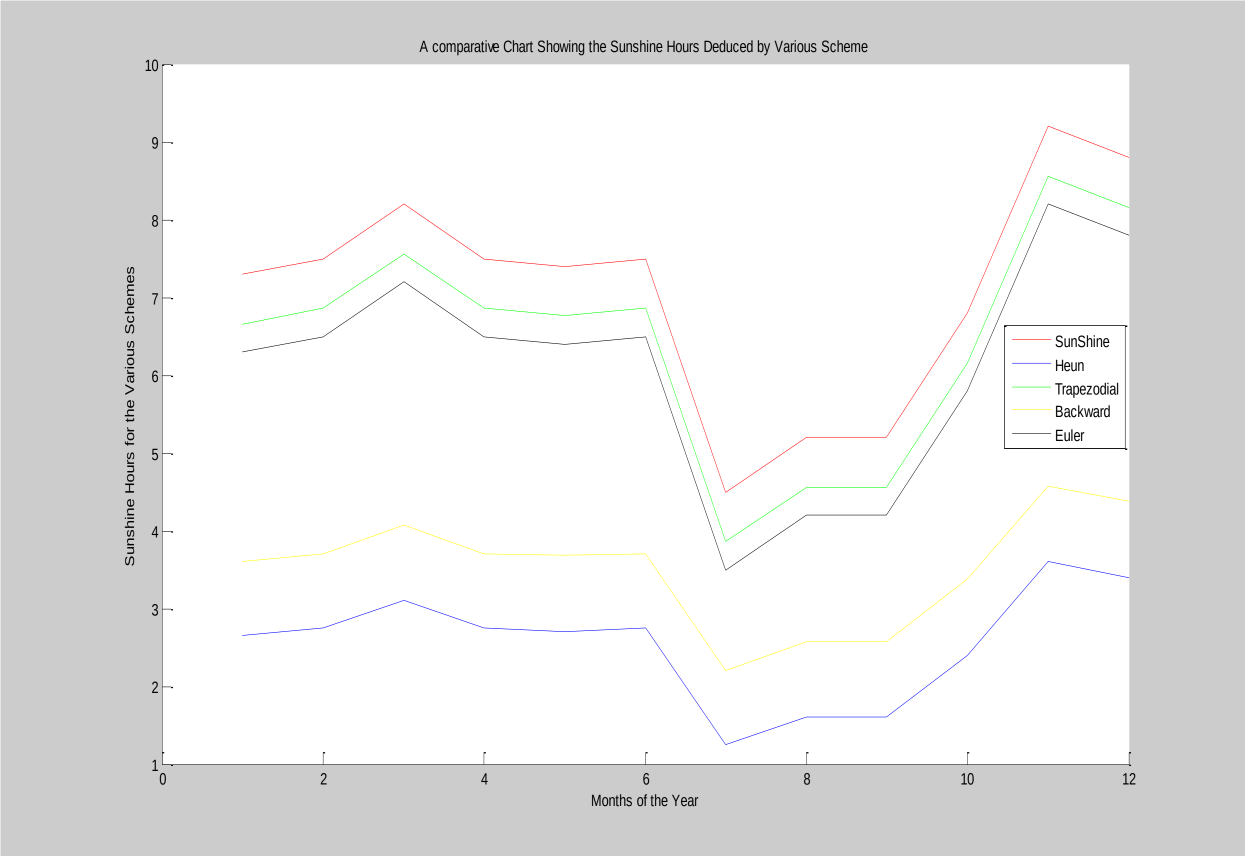

Figure 1 : A Comparative Chart Showing the Sunshine Hours Deduced by Various Schemes in one year

Observing our choice Trapezoidal Scheme from Figure 1 above it is obviously showing that the sunshine hours between January and May will be relatively high and will begin to decrease from June and start to rise again around September and falls again in December.

Solution of Wind Speed Prediction Using Finite Difference Scheme

Using equation (17) and wind speed value from Table 1 for the third month, we compute the predicted values for the different schemes.

$$U^{(n+1)} = Re[\lambda U^{(n)}]$$

For Heun Scheme

where $\lambda = 0.5 + i$ and $U^{(n)} = 3.5$ for wind speed

then

$$U^{(n+1)} = Re[3.5(0.5 + i)] = Re[1.75 + 3.5i] = 1.75 - 1 = 0.75$$

For Matsuno Scheme

where $\lambda = i$ and $U^{(n)} = 3.5$ for wind speed

then

$$U^{(n+1)} = Re[3.5(i)] = Re[3.5i] = 0$$

For Trapezoidal Scheme

where $\lambda = 1 + i/1.25$ and $U^{(n)} = 3.5$ for wind speed then

$$U^{(n+1)} = Re[3.5(1 + i/1.25)] = 3.86 - 1 = 2.86$$

For Backward Scheme

where $\lambda = 0.5 + 0.125i$ and $U^{(n)} = 3.5$ for wind speed then

$$U^{(n+1)} = Re[3.5(0.5 + 0.125i)] = 2.7 - 1 = 1.7$$

For Euler Scheme

where $\lambda = 1 + i$ and $U^{(n)} = 7.3$ for wind speed

then

$$U^{(n+1)} = Re[3.5(1 + i)] = 3.5 - 1 = 2.5$$

The results of predicted wind speed for all the 12 months of the year are shown in Table 3

Table 3 : Wind Speed 2016 (WS 2016)

Months	Wavelength λ					Amplitude $U^{(n)}$	Schemes $U^{(n+1)}$(WS 2016)				
ω	Heun	Matsuno	Trapezoidal	Backward	Euler	Wind Speed'15	Heun	Matsuno	Trapezoidal	Backward	Euler
1	$0.5+i$	i	$1+i/1.25$	$0.5+0.125i$	$1+i$	2.9	0.45	0	2.26	1.4	1.9
2	$0.5+i$	i	$1+i/1.25$	$0.5+0.125i$	$1+i$	3.7	0.85	0	3.06	1.8	2.7
3	$0.5+i$	i	$1+i/1.25$	$0.5+0.125i$	$1+i$	3.5	0.75	0	2.86	1.7	2.5
4	$0.5+i$	i	$1+i/1.25$	$0.5+0.125i$	$1+i$	5	1.5	0	4.36	2.48	4
5	$0.5+i$	i	$1+i/1.25$	$0.5+0.125i$	$1+i$	4.9	1.45	0	4.26	2.4	3.9
6	$0.5+i$	i	$1+i/1.25$	$0.5+0.125i$	$1+i$	4.7	1.35	0	4.06	2.3	3.7
7	$0.5+i$	i	$1+i/1.25$	$0.5+0.125i$	$1+i$	3.7	0.85	0	3.06	1.8	2.7
8	$0.5+i$	i	$1+i/1.25$	$0.5+0.125i$	$1+i$	4.2	1.1	0	3.56	2.08	3.2
9	$0.5+i$	i	$1+i/1.25$	$0.5+0.125i$	$1+i$	4.1	1.05	0	3.46	2	3.1
10	$0.5+i$	i	$1+i/1.25$	$0.5+0.125i$	$1+i$	3.3	0.65	0	2.66	1.49	2.3
11	$0.5+i$	i	$1+i/1.25$	$0.5+0.125i$	$1+i$	3	0.5	0	2.36	1.48	2
12	$0.5+i$	i	$1+i/1.25$	$0.5+0.125i$	$1+i$	3.2	0.6	0	2.56	1.58	2.2

From the above table, the selection of the scheme to represent the model forecasting for the Wind Speed for 2016 is based on the trend of the scheme whose result is closest to the previous year i.e. 2015 and hence among all five schemes in the table it is very obvious that aside Euler's (Forward) Scheme which is the second most closest, the Trapezoidal Scheme is the most closest to the given Wind Speed in 2015. Hence we use the Wind Speed predicted using the Trapezoidal Scheme.

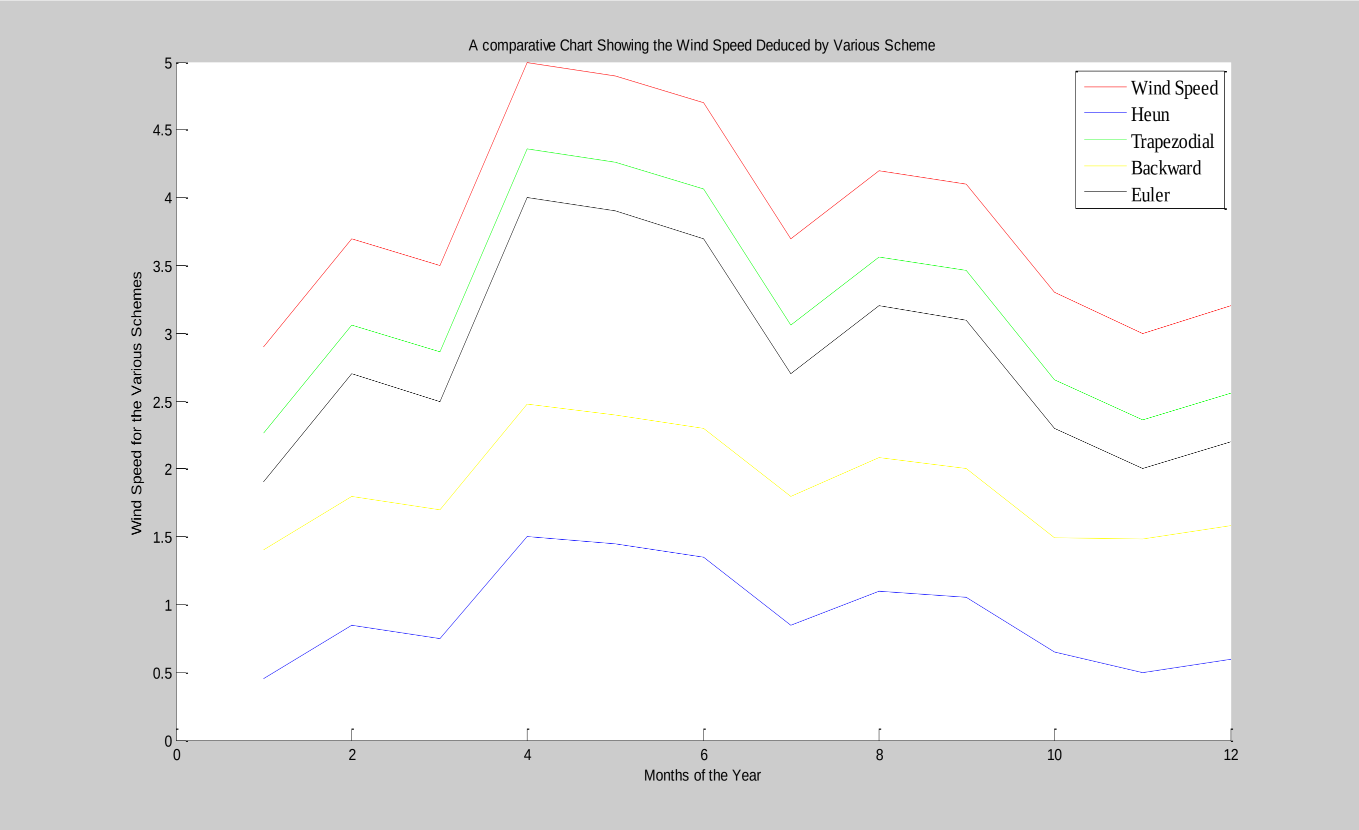

Figure 2: A Comparative Chart Showing the Wind Speed Deduced by Various Schemes in one year

Observing our choice Trapezoidal Scheme from Figure 2 above it is obviously showing that the wind speed will increase from January to May and will begin to decrease from June and start to rise again around September and falls again in November.

Solution of Rainfall Prediction Using Finite Difference Scheme

Using equation (17) and rainfall value from Table 1 for the second month, we compute the predicted values for the different schemes.

$$U^{(n+1)} = Re[\lambda U^{(n)}]$$

For Heun Scheme

where $\lambda = 0.5 + i$ and $U^{(n)} = 0.6$ for rainfall

then

$$U^{(n+1)} = Re[0.6(0.5 + i)] = Re[0.3 + 0.6i] = 0.3 - 1 = -0.7$$

For Matsuno Scheme

where $\lambda = i$ and $U^{(n)} = 0.6$ for rainfall

then

$$U^{(n+1)} = Re[0.6(i)] = Re[0.6i] = 0$$

For Trapezoidal Scheme

where $\lambda = 1 + i/1.25$ and $U^{(n)} = 0.6$ for rainfall

then

$$U^{(n+1)} = Re[0.6(1 + i/1.25)] = 0.36 - 0.8 = -0.04$$

For Backward Scheme

where $\lambda = 0.5 + 0.125i$ and $U^{(n)} = 0.6$ for rainfall

then

$$U^{(n+1)} = Re[0.6(0.5 + 0.125i)] = 0.3 - 0.015628 = 0.28$$

For Euler Scheme
where $\lambda = 1 + i$ and $U^{(n)} = 0.6$ for rainfall
then

$$U^{(n+1)} = Re[0.6(1 + i)] = 0.6 - 1 = -0.4$$

The results of predicted rainfall for all the 12 months of the year are shown in Table 4

Table 4: RainFall 2016 (RF 2016)

Months	Wavelength λ					Amplitude $U^{(n)}$	Schemes $U^{(n+1)}$(RF 2016)				
ω	Heun	Matsuno	Trapezoidal	Backward	Euler	Rain Fall '15	Heun	Matsuno	Trapezoidal	Backward	Euler
1	$0.5 + i$	i	$1 + i/1.25$	$0.5 + 0.125i$	$1 + i$	0	-1	0	-0.64	-0.02	-1
2	$0.5 + i$	i	$1 + i/1.25$	$0.5 + 0.125i$	$1 + i$	0.6	-0.7	0	-0.04	0.28	-0.4
3	$0.5 + i$	i	$1 + i/1.25$	$0.5 + 0.125i$	$1 + i$	7.5	2.75	0	6.86	3.7	6.5
4	$0.5 + i$	i	$1 + i/1.25$	$0.5 + 0.125i$	$1 + i$	74.2	36.1	0	73.56	37	73.2
5	$0.5 + i$	i	$1 + i/1.25$	$0.5 + 0.125i$	$1 + i$	109.2	53.6	0	108.56	54.58	108.2
6	$0.5 + i$	i	$1 + i/1.25$	$0.5 + 0.125i$	$1 + i$	267.2	132.6	0	266.56	133.58	266.2
7	$0.5 + i$	i	$1 + i/1.25$	$0.5 + 0.125i$	$1 + i$	314.8	156.4	0	314.16	157.38	313.8
8	$0.5 + i$	i	$1 + i/1.25$	$0.5 + 0.125i$	$1 + i$	278.3	138.15	0	277.66	139	277.2
9	$0.5 + i$	i	$1 + i/1.25$	$0.5 + 0.125i$	$1 + i$	258.4	128.2	0	257.76	129	257.4
10	$0.5 + i$	i	$1 + i/1.25$	$0.5 + 0.125i$	$1 + i$	238.2	118.1	0	237.56	119	237.2
11	$0.5 + i$	i	$1 + i/1.25$	$0.5 + 0.125i$	$1 + i$	Trace	Trace	Trace	Trace	Trace	Trace
12	$0.5 + i$	i	$1 + i/1.25$	$0.5 + 0.125i$	$1 + i$	0	-1	0	-0.64	-0.02	-1

From the above table, the selection of the scheme to represent the model forecasting for the Rain Fall for 2016 is based on the trend of the scheme whose result is closest to the previous year i.e. 2015 and hence among all five schemes in the table it is very obvious that aside Euler's (Forward) Scheme which is the second most closest, the Trapezoidal Scheme is the most closest to the given Rain Fall in 2015. Hence we use the Rain Fall predicted using the Trapezoidal Scheme.

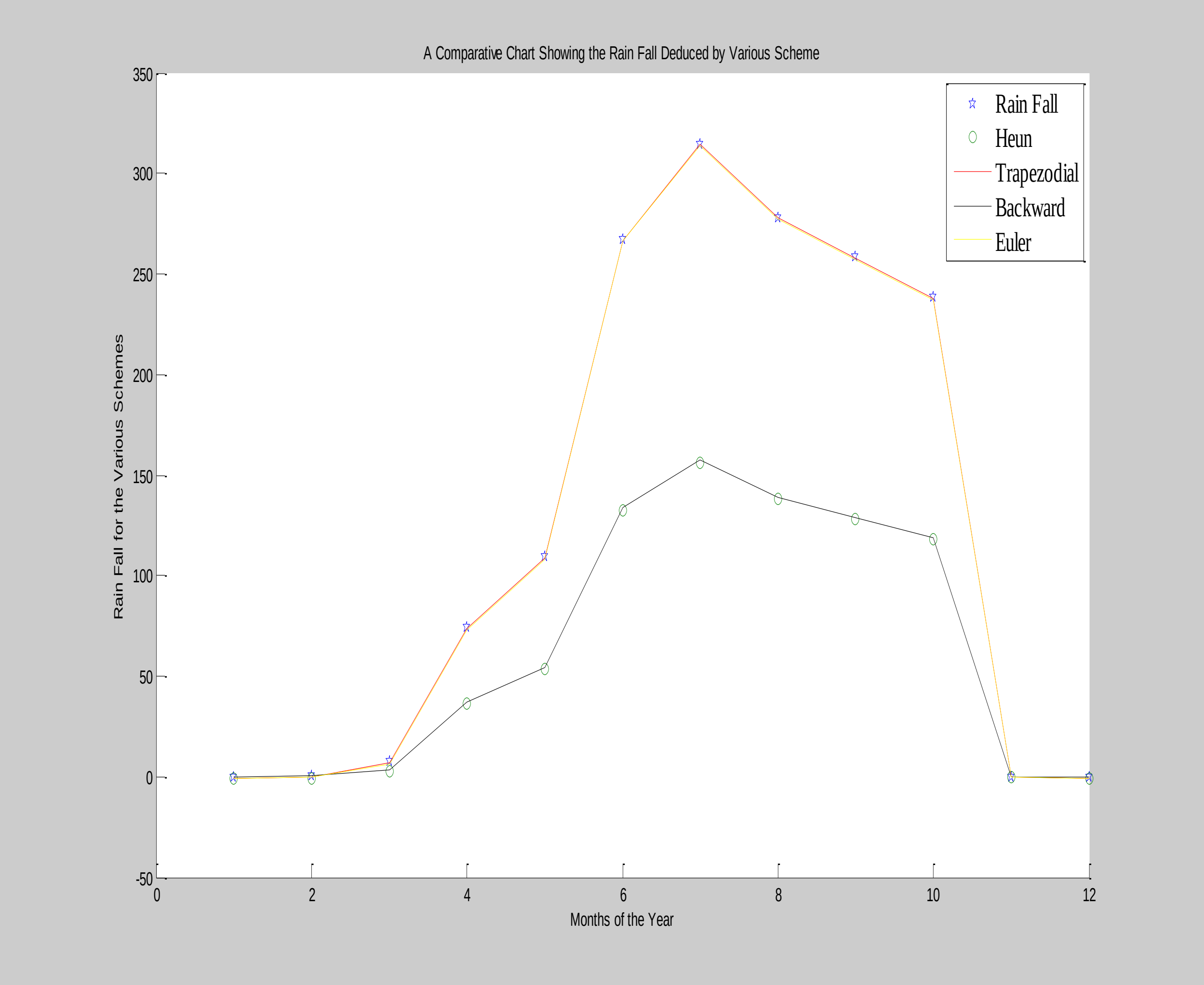

Figure 3 : A Comparative Chart Showing the Rain Fall Deduced by Various Schemes in one year

Observing our choice Trapezoidal Scheme from Figure 3 above it is obviously showing that the rain fall will start around late February and be very high in June till around October then will begin to reduce and dry season will set in from November.

3.5 Solution of Relative Humidity Prediction Using Finite Difference Scheme

Using equation (17) and relative humidity value from Table 1 for the fourth month, we compute the predicted values for the different schemes.

$$U^{(n+1)} = Re\left[\lambda U^{(n)}\right]$$

For Heun Scheme

where $\lambda = 0.5 + i$ and $U^{(n)} = 62$ for relative humidity

then

$$U^{(n+1)} = Re[62(0.5 + i)] = Re[31 + 62i] = 31 - 1 = 30$$

For Matsuno Scheme

where $\lambda = i$ and $U^{(n)} = 62$ for relative humidity

then

$$U^{(n+1)} = Re[62(i)] = Re[62i] = 0$$

For Trapezoidal Scheme

where $\lambda = 1 + i/1.25$ and $U^{(n)} = 62$for relative humidity

then

$$U^{(n+1)} = Re[62(1 + i/1.25)] = 62 - 0.64 = 61.36$$

For Backward Scheme
where $\lambda = 0.5 + 0.125i$ and $U^{(n)} = 62$ for relative humidity
then

$$U^{(n+1)} = Re[62(0.5 + 0.125i)] = 31 - 0.015625 = 30.98$$

For Euler Scheme
where $\lambda = 1 + i$ and $U^{(n)} = 62$ for relative humidity
then

$$U^{(n+1)} = Re[62(1 + i)] = 62 - 1 = 61$$

The results of predicted relative humidity for all the 12 months of the year are shown in Table 5

Table 5 : Relative Humidity 2016 (RH 2016)

Months	Wavelength λ					Amplitude $U^{(n)}$			Schemes $U^{(n+1)}$(RH 2016)		
ω	Heun	Matsuno	Trapezoidal	Backward	Euler	Rel. Hum'15	Heun	Matsuno	Trapezoidal	Backward	Euler
1	$0.5 + i$	i	$1 + i/1.25$	$0.5 + 0.125i$	$1 + i$	43	20.5	0	42.36	21.48	42
2	$0.5 + i$	i	$1 + i/1.25$	$0.5 + 0.125i$	$1 + i$	50	24	0	49.36	24.98	49
3	$0.5 + i$	i	$1 + i/1.25$	$0.5 + 0.125i$	$1 + i$	62	30	0	61.36	30.98	61
4	$0.5 + i$	i	$1 + i/1.25$	$0.5 + 0.125i$	$1 + i$	62	30	0	61.36	30.98	61
5	$0.5 + i$	i	$1 + i/1.25$	$0.5 + 0.125i$	$1 + i$	76	37	0	75.36	37.98	75
6	$0.5 + i$	i	$1 + i/1.25$	$0.5 + 0.125i$	$1 + i$	81	39.5	0	80.36	40.48	80
7	$0.5 + i$	i	$1 + i/1.25$	$0.5 + 0.125i$	$1 + i$	86	42	0	85.36	42.98	85
8	$0.5 + i$	i	$1 + i/1.25$	$0.5 + 0.125i$	$1 + i$	87	42.5	0	86.36	43.48	86
9	$0.5 + i$	i	$1 + i/1.25$	$0.5 + 0.125i$	$1 + i$	83	40.5	0	82.36	41.48	82
10	$0.5 + i$	i	$1 + i/1.25$	$0.5 + 0.125i$	$1 + i$	78	38	0	77.36	38.98	77
11	$0.5 + i$	i	$1 + i/1.25$	$0.5 + 0.125i$	$1 + i$	64	31	0	63.36	31.98	63
12	$0.5 + i$	i	$1 + i/1.25$	$0.5 + 0.125i$	$1 + i$	36	17	0	35.36	17.98	35

From the above table, the selection of the scheme to represent the model forecasting for the Relative Humidity for 2016 is based on the trend of the scheme whose result is closest to the previous year i.e. 2015 and hence among all five schemes in the table it is very obvious that aside Euler's (Forward) Scheme which is the second most closest, the Trapezoidal Scheme is the most closest to the given Relative Humidity in 2015. Hence we use the Relative Humidity predicted using the Trapezoidal Scheme.

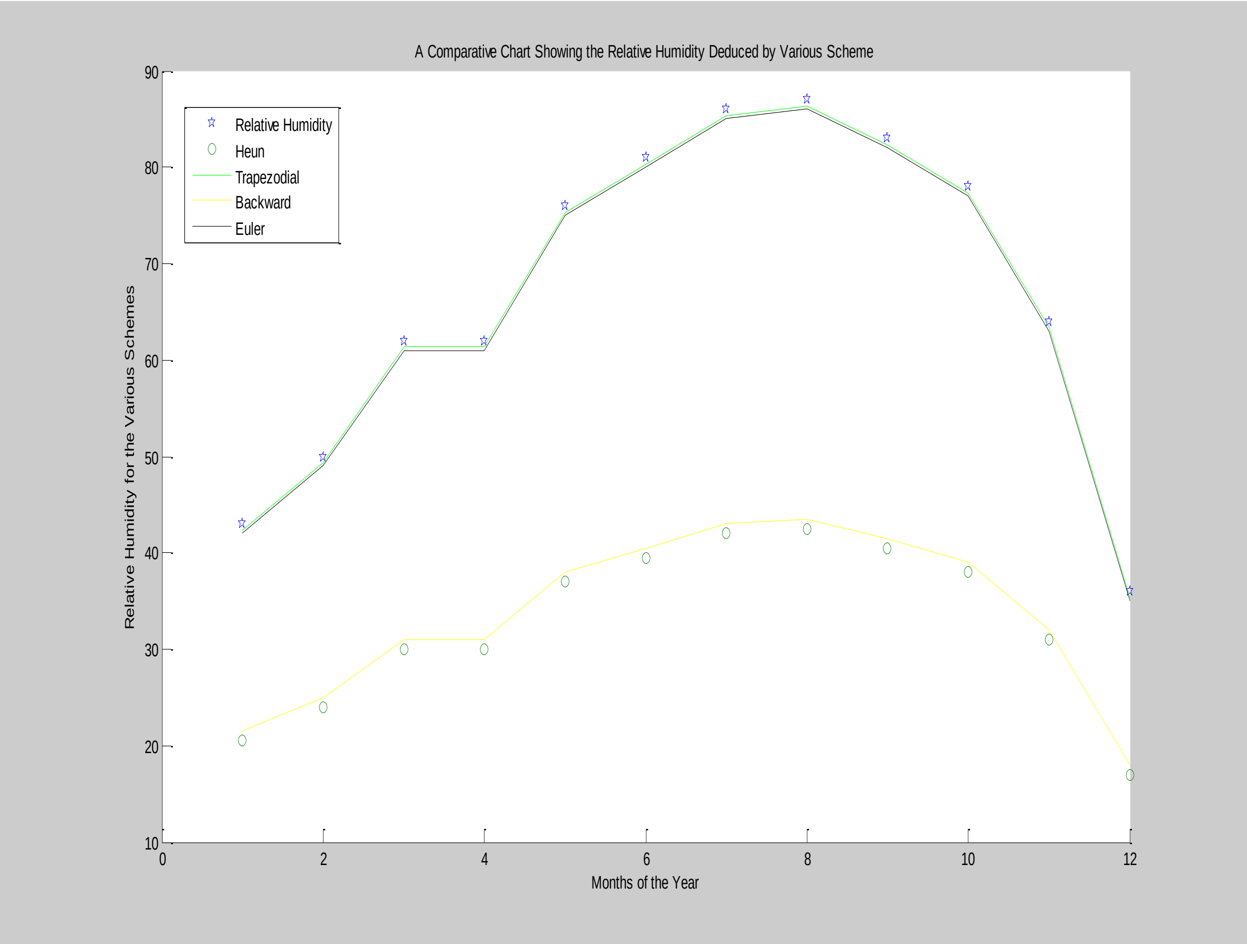

Figure 4 : A Comparative Chart Showing the Relative Humidity Deduced by Various Schemes in one year

As the rain fall increase so does the relative humidity, therefore, observing our choice Trapezoidal Scheme from Figure 4 above it is obviously showing that the relative humidity will start around late February and be very high in June till around October then will begin to reduce and dry season will set in from November.

Solution of Maximum Temperature Prediction Using Finite Difference Scheme

Using equation (17) and maximum temperature value from Table 1 for the fifth month, we compute the predicted values for the different schemes.

$$U^{(n+1)} = Re\left[\lambda U^{(n)}\right]$$

For Heun Scheme

where $\lambda = 0.5 + i$ and $U^{(n)} = 35.8$ for maximum temperature

then

$$U^{(n+1)} = Re[35.8(0.5 + i)] = Re[17.9 + 35.8i] = 17.9 - 1 = 16.9$$

For Matsuno Scheme

where $\lambda = i$ and $U^{(n)} = 35.8$ for maximum temperature

then

$$U^{(n+1)} = Re[35.8(i)] = Re[35.8i] = 0$$

For Trapezoidal Scheme

where $\lambda = 1 + i/1.25$ and $U^{(n)} = 35.8$ for maximum temperature

then

$$U^{(n+1)} = Re[35.8(1 + i/1.25)] = 35.8 - 0.64 = 35.16$$

For Backward Scheme
where $\lambda = 0.5 + 0.125i$ and $U^{(n)} = 35.8$ for maximum temperature
then

$$U^{(n+1)} = Re[35.8(0.5 + 0.125i)] = 17.9 - 0.015625 = 17.88$$

For Euler Scheme
where $\lambda = 1 + i$ and $U^{(n)} = 35.8$ for maximum temperature
then

$$U^{(n+1)} = Re[35.8(1 + i)] = 35.8 - 1 = 34.8$$

The results of predicted maximum temperature for all the 12 months of the year are shown in Table 6

Table 6: Maximum Temperature 2016 (TMax 2016)

Months	Wavelength λ					Amplitude $U^{(n)}$	Schemes $U^{(n+1)}$(TMax 2016)				
ω	Heun	Matsuno	Trapezoidal	Backward	Euler	TMax'15	Heun	Matsuno	Trapezoidal	Backward	Euler
1	$0.5+i$	i	$1+i/1.25$	$0.5+0.125i$	$1+i$	35.5	16.75	0	34.86	17.7	34.5
2	$0.5+i$	i	$1+i/1.25$	$0.5+0.125i$	$1+i$	37.4	17.7	0	36.76	18.68	36.4
3	$0.5+i$	i	$1+i/1.25$	$0.5+0.125i$	$1+i$	37.7	17.85	0	37.06	18.8	36.7
4	$0.5+i$	i	$1+i/1.25$	$0.5+0.125i$	$1+i$	36.6	17.3	0	35.96	18.3	35.6
5	$0.5+i$	i	$1+i/1.25$	$0.5+0.125i$	$1+i$	35.8	16.9	0	35.16	17.88	34.8
6	$0.5+i$	i	$1+i/1.25$	$0.5+0.125i$	$1+i$	30.2	14.1	0	29.57	15	29.2
7	$0.5+i$	i	$1+i/1.25$	$0.5+0.125i$	$1+i$	28.7	13.35	0	28.06	14.33	27.7
8	$0.5+i$	i	$1+i/1.25$	$0.5+0.125i$	$1+i$	28.7	13.35	0	28.06	14.33	27.7
9	$0.5+i$	i	$1+i/1.25$	$0.5+0.125i$	$1+i$	29.5	13.75	0	28.86	14.7	28.5
10	$0.5+i$	i	$1+i/1.25$	$0.5+0.125i$	$1+i$	30	14	0	29.36	14.98	29
11	$0.5+i$	i	$1+i/1.25$	$0.5+0.125i$	$1+i$	33.7	15.85	0	33.06	16.8	32.7
12	$0.5+i$	i	$1+i/1.25$	$0.5+0.125i$	$1+i$	35	16.5	0	34.36	17.5	34

From the above table, the selection of the scheme to represent the model forecasting for the Maximum Temperature for 2016 is based on the trend of the scheme whose result is closest to the previous year i.e. 2015 and hence among all five schemes in the table it is very obvious that aside Euler's (Forward) Scheme which is the second most closest, the Trapezoidal Scheme is the most closest to the given Maximum Temperature in 2015. Hence we use the Maximum Temperature predicted using the Trapezoidal Scheme.

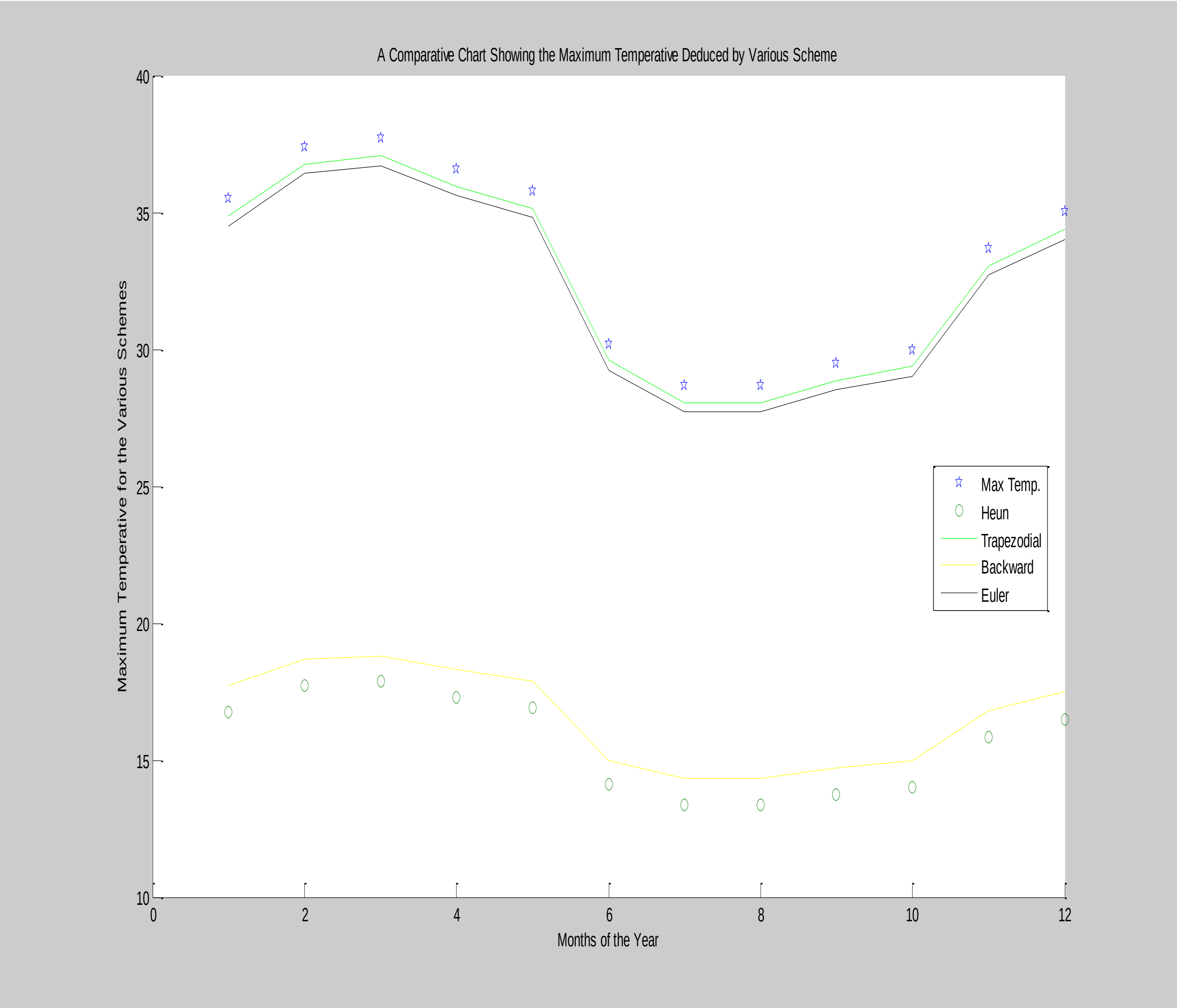

Figure 5 : A Comparative Chart Showing the Maximum Temperature Deduced by Various Schemes in one year

Observing our choice Trapezoidal Scheme from Figure 5 above it is obviously showing that the temperature will be high from January till around May and it begin to decline from June till around October where it will rise slightly in November.

Solution of Minimum Temperature Prediction Using Finite Difference Scheme

Using equation (17) and sunshine hours value from Table 1 for the sixth month, we compute the predicted values for the different schemes.

$$U^{(n+1)} = Re\left[\lambda U^{(n)}\right]$$

For Heun Scheme

where $\lambda = 0.5 + i$ and $U^{(n)} = 23.2$ for minimum temperature

then

$$U^{(n+1)} = Re[23.2(0.5 + i)] = Re[11.75 + 23.5i] = 11.6 - 1 = 10.6$$

For Matsuno Scheme

where $\lambda = i$ and $U^{(n)} = 23.2$ for minimum temperature

then

$$U^{(n+1)} = Re[23.2(i)] = Re[23.2i] = 0$$

For Trapezoidal Scheme

where $\lambda = 1 + i/1.25$ and $U^{(n)} = 23.2$ for minimum temperature
then

$$U^{(n+1)} = Re[23.2(1 + i/1.25)] = 23.2 - 0.64 = 22.56$$

For Backward Scheme
where $\lambda = 0.5 + 0.125i$ and $U^{(n)} = 23.2$ for minimum temperature
then

$$U^{(n+1)} = Re[23.2(0.5 + 0.125i)] = 11.75 - 0.015625 = 11.58$$

For Euler Scheme
where $\lambda = 1 + i$ and $U^{(n)} = 23.2$ for minimum temperature
then

$$U^{(n+1)} = Re[23.2(1 + i)] = 23.2 - 1 = 22.2$$

The results of predicted minimum temperature for all the 12 months of the year are shown in Table 7

Table 7: Minimum Temperature 2016 (TMin 2016)

Months	Wavelength λ					Amplitude $U^{(n)}$	Schemes $U^{(n+1)}$(TMin 2016)				
ω	Heun	Matsuno	Trapezoidal	Backward	Euler	TMin '15	Heun	Matsuno	Trapezoidal	Backward	Euler
1	$0.5 + i$	i	$1 + i/1.25$	$0.5 + 0.125i$	$1 + i$	19.3	8.65	0	18.66	9.6	18.3
2	$0.5 + i$	i	$1 + i/1.25$	$0.5 + 0.125i$	$1 + i$	23.2	10.6	0	22.56	11.58	22.2
3	$0.5 + i$	i	$1 + i/1.25$	$0.5 + 0.125i$	$1 + i$	25	11.5	0	24.36	12.48	24
4	$0.5 + i$	i	$1 + i/1.25$	$0.5 + 0.125i$	$1 + i$	25.7	11.8	0	25.06	12.78	24.7
5	$0.5 + i$	i	$1 + i/1.25$	$0.5 + 0.125i$	$1 + i$	24.6	11.3	0	23.96	12.3	23.6
6	$0.5 + i$	i	$1 + i/1.25$	$0.5 + 0.125i$	$1 + i$	23.2	10.6	0	22.56	11.58	22.2
7	$0.5 + i$	i	$1 + i/1.25$	$0.5 + 0.125i$	$1 + i$	22.3	10.2	0	21.66	11.2	21.3
8	$0.5 + i$	i	$1 + i/1.25$	$0.5 + 0.125i$	$1 + i$	22.5	10.3	0	21.86	11.3	21.5
9	$0.5 + i$	i	$1 + i/1.25$	$0.5 + 0.125i$	$1 + i$	22.2	10.1	0	21.56	11.1	21.2
10	$0.5 + i$	i	$1 + i/1.25$	$0.5 + 0.125i$	$1 + i$	21.8	9.9	0	21.16	10.88	20.8
11	$0.5 + i$	i	$1 + i/1.25$	$0.5 + 0.125i$	$1 + i$	21.6	9.8	0	20.96	10.78	20.6
12	$0.5 + i$	i	$1 + i/1.25$	$0.5 + 0.125i$	$1 + i$	17.2	7.6	0	16.56	8.58	16.2

From the above table, the selection of the scheme to represent the model forecasting for the Minimum Temperature for 2016 is based on the trend of the scheme whose result is closest to the previous year i.e. 2015 and hence among all five schemes in the table it is very obvious that aside Euler's (Forward) Scheme which is the second most closest, the Trapezoidal Scheme is the most closest to the given Minimum Temperature in 2015. Hence we use the Minimum Temperature predicted using the Trapezoidal Scheme.

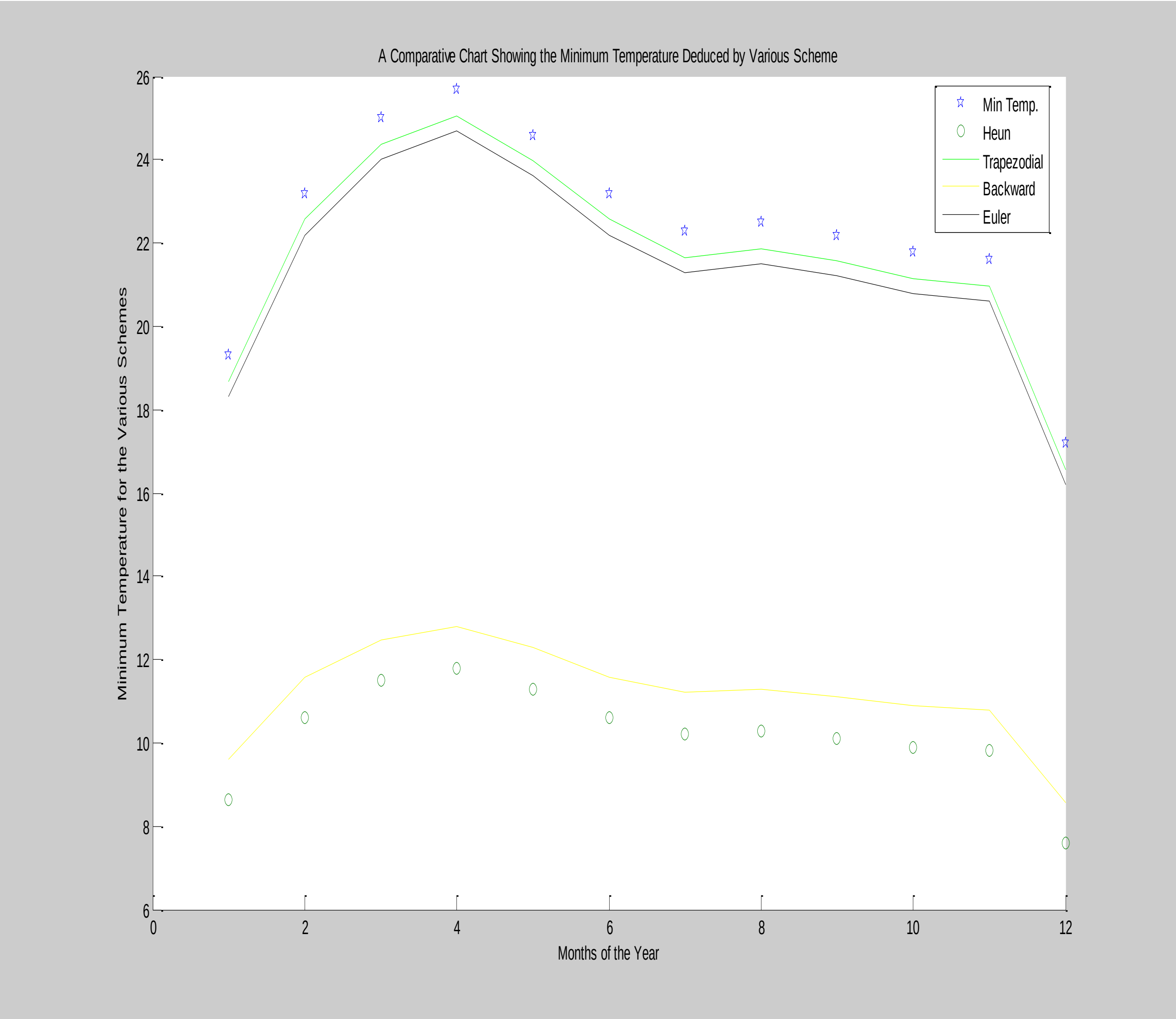

Figure 6 : A Comparative Chart Showing the Minimum Temperature Deduced by Various Schemes in one year

Observing our choice Trapezoidal Scheme from Figure 6 above it is obviously showing that the temperature will be fall from January till around May and it begin to rise from June till around October where it will fall slightly in November.

Summary of Predicted Weather Data Set From Compatible Finite Difference Scheme

Table 8: Compatible FDM Numerical Weather Prediction Year: 2016

Station: ABUJA, NG **Elev: 343.1ft. Lat: 09.15°N Lon: 07.00°E**

STATION NUMBER	STATION NAME	ELEV	LAT	LONG	DATE	RelHum	TMAX	TMIN	RAINFALL	SUNSHINE HRS	WIND SPEED	WIND DIRECTION
65125	Abuja	343.1	09.15' N	09.24' W	201601	42.36	34.86	18.66	-0.64	6.66	2.26	NE
65125	Abuja	343.1	09.15' N	09.24' W	201602	49.36	36.76	22.56	-0.04	6.86	3.06	N
65125	Abuja	343.1	09.15' N	09.24' W	201603	61.36	37.06	24.36	6.86	7.56	2.86	NW
65125	Abuja	343.1	09.15' N	09.24' W	201604	61.36	35.96	25.06	73.56	6.86	4.36	NE

65125	Abuja	343.1	09.15' N	09.24' W	201605	75.36	35.16	23.96	108.56	6.76	4.26	NE
65125	Abuja	343.1	09.15' N	09.24' W	201606	80.36	29.57	22.56	266.56	6.86	4.06	N
65125	Abuja	343.1	09.15' N	09.24' W	201607	85.36	28.06	21.66	314.16	3.86	3.06	NE
65125	Abuja	343.1	09.15' N	09.24' W	201608	86.36	28.06	21.86	277.66	4.56	3.56	NW
65125	Abuja	343.1	09.15' N	09.24' W	201609	82.36	28.86	21.56	257.76	4.56	3.46	W
65125	Abuja	343.1	09.15' N	09.24' W	201610	77.36	29.36	21.16	237.56	6.16	2.66	E
65125	Abuja	343.1	09.15' N	09.24' W	201611	63.36	33.06	20.96	Trace	8.56	2.36	W
65125	Abuja	343.1	09.15' N	09.24' W	201612	35.36	34.36	16.56	-0.64	8.16	2.56	NE

Table 8 shows the values of the predicted weather data values obtained by using the trapezoidal scheme. This compared favourably with the real weather data values collected from Federal Airport Authority of Nigeria (FAAN) Abuja Station shown on Table 4.1.

III.CONCLUSION

Weather prediction for a particular station is mostly accurate in the advent of recursive use of previous predictions or measurement. This research has unveiled that studying the weather trends helps in predicting future weather attenuation using numerical solutions deduced by finite difference method. The finite difference method has been used to deduce compatible models for automated attenuation of various parameters involved in the weather formation with the use of MATLAB in predicting future weather trends. The derivation of the models based on the finite difference method gives a high level of significance. In conclusion, the weather prediction for a station (i.e. Abuja, Nigeria) was flexibly obtained accurately prior to the use of previous determined or forecasted data and a compatible C-grid staggered finite difference method.

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