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JYOTPATTI

Dr. Sanjay Madhukarrao Deshpande
Head, Department of Mathematics Bhavabhuti Mahavidyalaya, AMGAON, Gondia , Maharashtra, India

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ABSTRACT

Bhaskaracharya ,(Bhaskar II, 1114- 1185 AD), was one of the great mathematicians in India. His text "*Sindhant Shiromani*" (SS) was treated as the base of the further research oriented results by most of all the mathematicians after him.

SS contains of two parts:, *Goladhyaya and Grahaganit*. Jyotpatti is the last chapter in *Goladhyaya. Jyotpatti* consists of 25 *Shloka* (stanzas), all in Sanskrit language. It is a general impression that SS contains *Lilavat*i and *Beejganit* also, but that is not so.

Jyotpatti deals with trigonometry. This was a milestone in developing geometry in India. *Jya* means sine and *Utapatti* means creation. Hence the name *Jyotpatti (jya + upapatti*). *Jya and Kojya(or kotijya)* stand for the Rsine and Rcosine ratios respectively. The trigonometry developed by Bhaskara II is based on a circle of radius R, and not on a right-angled triangle as taught in the schools.

After defining *Jya, Kotijya and Utkrama* (*verse jya)* etc, Bhaskara obtains these ratios for the standards angles of 30,45, 60, 36 and 28,(all in degrees) by inscribing a regular polygon in a circle of radius R. Bhaskara called these angles as *Panchajyaka.* Not only this, Bhaskara developed these results for addition and subtraction of two angles. This result was further developed for the similar results, for the multiple angles. Bhaskara compares *jya and kotijya* with the longitude –latitude of earth and those with lateral threads of a cloth.

Contents in Jyotpatti (Only a few mentioned here)

(1) R jya 45 =R$\sqrt{2}$, and other similar R jya values. (All in degrees)

(2) R jya 36 = 0.5878 approx.

(3) S_n = side of a regular polygon of n sides = D sin (π/n), D is the diameter of circle in which polygon is inscribed.

(4) Derivation of formulae for sin (θ + ϕ) and cosine (θ + ϕ) called as *samas bhavana* and *antar bhavana*.

(5) Concept of derivatives, that is, δ(sin θ) = (cos θ) δθ etc. Which is Rolle's Theorem.

Indian mathematicians developed trigonometry in different way than that of western mathematicians. Though *Jyotpatti* is a small text, it is a landmark in development of ancient and medieval trigonometry.

Keywords : Jyotpatti, Jya, Kotijya, Utkarmajya, Trigonometry

I. INTRODUCTION

Bhaskaracharya, (Bhaskar II, 1114- 1185 AD), was one of the great mathematicians, India has produced. His text "*Sindhant Shiromani*" (SS) was treated as the base of the further research oriented results by most of all the mathematicians after him.

SS contains of two parts:, *Goladhyaya and Grahaganit*. Jyotpatti is the last chapter in *Goladhyaya. Jyotpatti* consists of 25 *Shloka* (stanzas), all in Sanskrit language.

It is a general impression that SS contains *Lilavat*i and *Beejganit* also, but that is not so.

II. THE TEXT *JYOTPATTI*

Jyotpatti deals with trigonometry. This was a milestone in developing geometry in India. *Jya* means sine and *Utapatti* means creation.
Hence the name *Jyotpatti (jya + upapatti*).

Jya and Kojya(or kotijya) stand for the Rsine and Rcosine ratios respectively.

The trigonometry developed by Bhaskara II is based on a circle of radius R, and not on a right-angled triangle as taught in the schools.

After defining *Jya, Kotijya and Utkrama* (*verse jya)* etc, Bhaskara obtains these ratios for the standards angles of 30,45, 60, 36 and 28,(all in degrees) by inscribing a regular polygon in a circle of radius R. Bhaskara called these angles as *Panchajyaka.*

Not only this, Bhaskara developed these results for addition and subtraction of two angles. This was further developed for the similar results for the multiple angles.

Bhaskara compares *jya and kotijya* with the longitude –latitude of earth and those with lateral threads of a cloth.

III. SINE OF AN ANGLE

Consider the first six stanzas in Sanskrit

आचार्याणां पदवीं ज्योत्पत्त्या ज्ञातया यतो याति ।
विविधां विदग्धगणकप्रीत्यै तां भास्करो वक्ति ।।1।।

इष्टांगुलव्यासदलेन वृत्तं कार्यं दिगङ्कं भलवांकितं च ।
ज्यासंख्ययाप्ता नवतेर्लवाये तदाद्यजीवाधनुरेतदेव ।।2।।

द्वित्र्यादिनिघ्नं तदनन्तराणां चापे तु दत्वोभयतो दिगांत् ।
ज्ञेयं तदग्रद्वयबध्दरज्जोरर्धं ज्यकार्धं निखिलानि चैवम् ।।3।।

अथान्यथा वा गणितेन वच्मि ज्यार्धानि तान्येव परिस्फुटानि ।
त्रिज्याकृतिर्दोर्गुणवर्गहीना मूलं तदीयं खलु कोटिजीवा ।।4।।

दोः कोटिजीवारहिते त्रिभज्येतच्छेषके कोटिभुजोत्क्रमज्ये ।
ज्यचापामध्ये खलु योऽत्र बाणः सैवोत्क्रमज्या सुधियात्र वेद्या ।।5।।

त्रिज्यार्धं राशिज्या तत्कोटिज्या च षष्टिभागानाम् ।
त्रिज्यावर्गार्धपदं शरवेदांशज्यका भवति ।।6।।

The collective meaning of these stanzas is as follows: "One who will acquire the knowledge of Jyotpatti, will be of the rank *acharya* and will be an astronomer".

"The graphical method of obtaining the sine values of various angles can be studied by drawing the desired circle of radius R".

The *jya* of an arc of circle may be defined as the length of half the chord of twice the arc".

"The square of the radius is diminished by the square of Rsineof an arc is equal to the kotijya (R cosine) of the arc".

"Half the radius is equal to the jya(Rsine) if arc is of 30 degrees".
{This means $R\sin\theta = R / 2$. This gives $\sin\theta = 1 / 2$}

"What is really the arrow between the bow and the bowstring is known amongst the scholars as the versed sine".

Thus Bhaskaracharya proves that Jya $45^0 = R / \sqrt{2}$

IV. SIDE OF A REGULAR POLYGON

Bharacharya obtains the side of a regular inscribed in desired circle using the formula given in

त्रिज्याकृतीषुघातात् त्रिज्याकृतिवर्गपःघातस्य ।
मूलोनादष्टहृतान्मूलं षट्त्रिंशदंशज्या ।।7।।

गजहयगजेषु 5878 निघ्नी त्रिभजीवा वायुतेन 10000 संभक्ता ।
षट्त्रिंशदंशजीवा तत्कोटिज्याकृतेषूणाम् ।।8।।

त्रिज्याकृतीषुघातान्मूलं त्रिज्योनितं चतुर्भक्तम् ।
अष्टादशभागानां जीवा स्पष्टा भवत्येवम् ।।9।।

क्रमोत्क्रमज्याकृतियोगमूलाद्दलं तदर्धांशकशिञ्जिनी स्यात् ।
त्रिज्योत्क्रमज्यानिहतेर्दलस्य मूलं तदर्धांशकशिञ्जिनी वा ।।10।।

"Radius multiplied by 5878 and divided by 10000 is Rsine 36^0 and Rcosine of that is Rsine of 54"

This gives $R \sin 36^0 = R \times 5878 / 10000$, or,
$\sin 36^0 = 0.5878$

The formula for side of a regular polygon is
$S_n = (D \times P_n) / 120000$,

where, S_n = the side of a regular polygon of n sides ,
P_n = the coefficients 103923, 84853, 70534, 60000, 52055, 45922 and 41031 and
D = diameter of the circle.

In particular, when D = 120000, $S_n = P_n$.
for n = 3, $S_3 = (D \times 103923)/ 120000 = D \times 0 .8660$,.
similarly, the other values are:
for n = 4, $S_4 = D \times 0.7071$,
for n = 5, $S_5 = D \times 0.5878$,
for n = 6, $S_6 = D \times 0.5000$,
for n = 7, $S_7 = D \times 0.4339$,
for n = 8, $S_8 = D \times 03420$,
for n = 9, $S_9 = D \times 0.3826$,

Using these results, Bhaskara gave a formula for finding sine of an angle. It is $S_n = D \times \sin(\pi / n)$, ,

Bhaskara obtains the values of sine of various angles as:

$\sin(\pi/3) = \sin 60^0$ = 0.866025 approximately ,
$\sin(\pi/4) = \sin 45^0$ = 0.7071083 approximately ,
$\sin(\pi/5) = \sin 36^0$ = 0.587783 approximately ,
$\sin(\pi/7) = \sin (25.71)^0$ = 0.437916 approximately ,
$\sin(\pi/8) = \sin (22.5)^0$ = 0.382683 approximately,
$\sin(\pi/9) = \sin 20^0$ = 0.341925 approximately , etc

The other values of sine of different angles can be used by methods of numerical analysis.
Interestingly, Bhaskara has given six different methods to find the values of sine of an angle these are given in shlok 11 to 20.

No other Indian or western mathematician has dealt with these methods.

V. ADDITION-SUBTRACTION THEOREMS

consider the stanzas,

चापयोरिष्टयोर्दोर्ज्ये मिथःकोटिज्यकाहते ।

त्रिज्याभक्ते तयोरैक्यं स्याच्चापैक्यस्य दोर्ज्यका ।।21।।

चापान्तरस्य जीवा स्यात् तयोरन्तरसंमिता ।

अन्यज्यासाधने सम्यगियं ज्याभावनोदिता ।।22।।

Collectively, they mean:

"The Rsines of any two arcs of a circle are reciprocally multiplied by their Rcosines ; and their product is dvided by the radius;

The sum of the quotient is equal the Rsine of the sum of the two arcs and their difference is the Rsine of the difference of the arcs"

Mathematically, it means:

R.jya $(\alpha \pm \beta) = jya(\alpha) \times kojya(\beta) \pm ko\ jya(\alpha) \times jya(\beta)$.

This amounts to

$\sin(\alpha \pm \beta) = \sin(\alpha) \times \cos(\beta) \pm \cos(\alpha) \times \sin(\beta)$.

Bhaskara gave two methods for the proof of this result.

Bhaskara called these addition and subtraction theorem of sine as *Samas Bhavana* and *Antara Bhavana* respectively.

From the above formula, Bhaskara defined the results for $\sin 2\alpha$ and $\sin 3\alpha$ and other values for multiple angles.

VI. *BHUJ-KOTI KARNYA NYAYA*

This means that Bhaskara has given the proof of so called Pythagoras theorem:

if x, y, and z form a right angled triangle, with z as hypotenuse, then $x^2 + y^2 = z^2$,

here, x is `*bhuja*` ,that is, base , `*koti*` means perpendicular and *karna* is hypotenuse.

VII. *TATKALIKA GATI*

Bhaskara introduced an instantaneous method for finding the relative motions of planets.He called this method as *Tatkalika Gati* .

He states: If θ and θ` are mean angles of a moving planet at different instants from a fixed position, then

$$\sin\theta' - \sin\theta = (\theta' - \theta)\cos\theta$$

This result suggests the definition of a differential dθ, if we use

$\theta' = \theta + \delta\theta$. This gives,

$$\sin(\theta + \delta\theta) -- \sin\theta = \delta\theta \cos\theta$$

That is , $\cos\theta = [\sin(\theta + \delta\theta) - \sin\theta] / \delta\theta$

In reference of calculus, if $\delta\theta \rightarrow 0$, this result gives the derivative of sin θ,

This suggests that Bhaskara knew the concept of limiting value of θ and consequently that of a derivative in 12 th century,

Newton (1662—1726) and Leibnitz (1646—1716) introduced the same concept in 17^{th} century,that is, 500 years after Bhaskara..

Not in *Jyotpatti*, but in *Goladhya* (part of Bhaskara`s *Sidhanta Shiromani*) states the mode of derivation of a formula for volume and surface area of a sphere by slicing a sphere into number of parallel circular discs and then their summation.

This suggests the concept of *integration as a limit of a sum.*

VIII. CONCLUSION

Jya and *Kotijya* (now sine and cosine) are defined on a unit circle, and not by using a right-angled triangle, as stated by Pythagoras.

The concept of a bow — bowstring – arrow, used by Bhaskara in defining trigonometric ratios, is from Bodhayan Geometry of period 800years BC. The concept of *Shulb Sutra* is used defining *jya* and *Kotijya*

1. These ratios are defined on the arc of the circle of radius R subtending angle θ at the center of the circle.(and not angles directly.)
2. Initially, graphical methods are applied.
3. *Tatkalika* methods (instantaneous) introduced for motion of planets.
4. Many results have been derived using *Goladhyay*. Without reading *Goladhyaya*, Jyotpatti will be difficult to understand.
5. Though *Jyotpatti* is a small text, it is a landmark in development of ancient and medieval trigonometry.
6. *Bhaskara wrote Lilavati, Beej ganit, Goladhya and Grahaganita*. All text of mathematics in Sanskrit language are written without using any tools as available in modern days.

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