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A Plane Symmetric Cosmological Model with Varying Λ - term in Bimetric Theory of Gravitation

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ABSTRACT

We investigated a plane symmetric cosmological model varying Λ term in bimetric theory of gravitation. It is seen that such a model never exist in bimetric theory of gravitation. Further it is realized that for a very large value of T, our model strongly suggest the vacuum universe without viscous fluid and without expansion, rotation and shearing.
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I. INTRODUCTION

In modern cosmological theories, a dynamic cosmological term Λ (t) remains a focal point of interest as it solves the cosmological constant problem in natural way. There are significant observational evidence for the detection of Λ or a component of material content of the universe that varies slowly with time and space to act like Λ .a wide range of observations now compellingly suggest that the universe possesses a non-zero cosmological term. In the content of quantum field theory , a cosmological term corresponds to the energy density of the vacuum, The birth of the universe been attributed to an excited vacuum fluctuation triggering off an inflationary expansion followed by super-cooling. The release of locked up vacuum energy results in subsequent reheating. The cosmological term which is measure of the energy of empty space, provides a repulsive force opposing the gravitational pull between the galaxies. If the cosmological term exists , the energy it represents counts as mass because mass and energy are equivalent. If the cosmological term is large growth, its energy plus the matter in the universe could lead to inflation.

In the early phase of the universe, there is no definite evidence that the present day universe (FRW universe) was of the same type. Therefore it is important to study different Bianchi Type models in the context of the early phase of the universe. In this regards we plan to study various Bianchi Type models in bimetric theory of gravitation. Earlier, Borkar and Charjan (2009, 2010) have developed the models like Bianchi Type I string dust cosmological model with magnetic field in bimetric relativity (2009) , The charged perfect fluid distribution in bimetric theory of relativity (2010) , LRS Bianchi Type I string dust magnetized cosmological models in bimetric theory of relativity (2010), Bianchi Type I bulk viscous fluid string dust cosmological model with magnetic field in bimetric relativity (2010) and Bianchi Type I magnetized cosmological models in bimetric theory of gravitation (2010) . Several new theories of gravitation have been formulated which are considered to be alternatives to Einstein's theory of gravitation. The most important among them is Rosen's (1977) bimetric theory of gravitation. The Rosen's bimetric theory is the theory of gravitation based on two metrics. One is the fundamental metric tensor g_{ij} describes the gravitational potential and the second metric γ_{ij} refers to the flat space–time and describes the inertial forces associated with the acceleration of the frame of reference. The metric tensor g_{ij} determine the Riemannian geometry of the curved space time which plays the same role as given in Einstein's general relativity and it interacts with matter. The background metric γ_{ij} refers to the geometry of the empty universe (no matter but gravitation is there) and describes the inertial forces. The metric tensor γ_{ij} has no direct physical significance but appears in the field equations. Therefore it interacts with g_{ij} but not directly with matter. One can regard γ_{ij} as giving the geometry that would exists if there were no matter. In the absence of matter one would have $g_{ij}=\gamma_{ij}$. Moreover, the bimetric theory also satisfied the covariance and equivalence principles; the formation of general relativity. The theory agrees with the present observational facts pertaining to general relativity[For details one may refer Karade (1980), Katore and Rane (2006) and Rosen (1974, 1977)] . Thus at every point of space–time, there are two metrics

$$ds^2 = g_{ij}dx^i dx^j \tag{1}$$

$$d\eta^2 = \gamma_{ij}dx^i dx^j \tag{2}$$

The field equations of Rosen's (1974) bimetric theory of gravitation are

$$N_i^j - \frac{1}{2}N\delta_i^j + \Lambda\, g_i^j = -8\pi T_i^j \tag{3}$$

where $N_i^j = \frac{1}{2}\gamma^{pr}\left(g^{sj}g_{si|p}\right)_{|r}$, $N = N_i^i$ k = 1 together with $g = \det(g_{ij})$ and $\gamma = \det(\gamma_{ij})$. Here the vertical bar $(\,|\,)$ stands for γ –covariant differentiation and T_i^j is the energy–momentum tensor of matter fields.

Several aspects of bimetric theory of gravitation have been studied by Rosen (1974, 1977), Karade (1980), Israelit (1981), Katore and Rane (2006), Khadekar and Tade (2007) Sahoo (2010). In particular, Reddy and Venkateswara Rao (1998) have obtained some Bianchi Type cosmological models in bimetric theory of gravitation. The purpose of Rosen's bimetric theory is to get rid of the singularities that occur in general relativity that was appearing in the big–bang in cosmological models and therefore, recently, there has been a lot of interest in cosmological models in related to Rosen's bimetric theory of gravitation.

In bimetric theory, according to Rosen (1974) the background metric tensor γ_{ij} should not be taken as describing an empty universe but it should rather be chosen on the basis of cosmological consideration. Hence Rosen proposed that the metric γ_{ij} be taken as the metric tensor of a universe in which perfect cosmological principle holds. In accordance with this principle, the large scale structure of universe presents the same aspect from everywhere in space and at all times. The fact, however, is that while taking the matter actually present in the universe, this principle is not valid on small scale structure due to irregularities in the matter distribution and also not valid on large scale structure due to the evolution of the matter. Therefore, we adopt the perfect cosmological principle as the guiding principle. It does not apply to g_{ij} and the matter in the universe but to the metric γ_{ij}. Hence γ_{ij} describes a space–time of constant curvature.

we study A plane symmetric cosmological model varying Λ term in bimetric theory of gravitation. It is seen that such a model never exist in bimetric theory of gravitation. Further it is realized that for a very large value of T, our model strongly suggest the vacuum universe without viscous fluid and without expansion, rotation and shearing.

II. Metric and Field Equations

We consider the plane symmetric metrics in the form

$$ds^2 = A^2\left(dx^2 - dt^2\right) + B^2 dy^2 + C^2 dz^2 \tag{4}$$

where A, B and C are functions of t only.

The flat metric corresponding to metric (4) is

$$d\eta^2 = -dt^2 + dx^2 + dy^2 + dz^2 \tag{5}$$

The stress energy-tensor T_i^j in the presence of bulk stress given by

$$T_i^j = (\rho + p)\,\upsilon_i\,\upsilon^j + p\,g_i^j - \eta\,\left(v^j{}_{|i} + v^j{}_{|i} + v^j v^l\,v_{i\,|l} + v_i v^l\,v_{j\,|l}\right) - \left(\xi - \frac{2}{3}\eta\right)\theta\,\left(g_i^j + v_i v^j\right) \tag{6}$$

Here ρ, p, η and ξ are the energy density, isotropic pressure, coefficient of shear viscosity and bulk viscosity coefficient respectively. The quantity θ is the scalar of expansion which is given by

$$\theta = v^i{}_{|\,i} \tag{7}$$

and υ^i is the flow vector satisfying the relations

$$g_{ij}\upsilon^i\,\upsilon^j = -1 \tag{8}$$

We assume that coordinates to be co-moving, so that

$$\upsilon^1 = \upsilon^2 = \upsilon^3 = 0, \qquad \upsilon^4 = \frac{1}{A}. \tag{9}$$

Equation (6) of stress energy tensor yield

$$T_1^1 = \left(p - 2\eta\frac{A_4}{A^2} - \left(\xi - \frac{2}{3}\eta\right)\theta\right), \qquad T_2^2 = \left(p - 2\eta\frac{B_4}{AB} - \left(\xi - \frac{2}{3}\eta\right)\theta\right),$$
$$T_3^3 = \left(p - 2\eta\frac{C_4}{AC} - \left(\xi - \frac{2}{3}\eta\right)\theta\right), \qquad T_4^4 = -\rho \tag{10}$$

The Rosen's field equations (3) for the metric (4) and (5) with the help of (10) gives

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} - \frac{B_4{}^2}{B^2} - \frac{C_4{}^2}{C^2} = -16\pi\left[p - 2\eta\frac{A_4}{A^2} - \left(\xi - \frac{2}{3}\eta\right)\theta\right] - 2\Lambda \tag{11}$$

$$2\frac{A_{44}}{A} - \frac{B_{44}}{B} + \frac{C_{44}}{C} - 2\frac{A_4{}^2}{A^2} + \frac{B_4{}^2}{B^2} - \frac{C_4{}^2}{C^2} = -16\pi\left[p - 2\eta\frac{B_4}{AB} - \left(\xi - \frac{2}{3}\eta\right)\theta\right] - 2\Lambda \tag{12}$$

$$2\frac{A_{44}}{A} + \frac{B_{44}}{B} - \frac{C_{44}}{C} - 2\frac{A_4{}^2}{A^2} - \frac{B_4{}^2}{B^2} + \frac{C_4{}^2}{C^2} = -16\pi\left[p - 2\eta\frac{C_4}{AC} - \left(\xi - \frac{2}{3}\eta\right)\theta\right] - 2\Lambda \tag{13}$$

$\frac{B_{44}}{B}+\frac{C_{44}}{C}-\frac{B_4^{\ 2}}{B^2}-\frac{C_4^{\ 2}}{C^2}=16\pi\rho-2\Lambda$ where the lower suffix 4 at the symbols A, B and C denotes ordinary differentiation with respect to t. The scalar of expansion θ is given by equation (7) and it has the value

$$\theta=\frac{A_4}{A^2}+\frac{B_4}{AB}+\frac{C_4}{AC} \tag{15}$$

3. Solution of the Field Equations

Equations (11) to (14) are four differential equations, in eight unknowns quantities $A,\ B,\ C,\ \rho,\ p,\ \eta,\ \xi$ and Λ, since the scalar expansion θ is in the form of $A,\ B$ and C. To get the solution of the system of differential equations (11) to (14), one has to assume four extra conditions. We first assume that the expansion (θ) in the model is proportional to the Eigen values σ_2^2 of the shear tensor. This condition leads to

$$B=A\ C \tag{16}$$

Secondly we assume that the coefficient of shear viscosity is proportional to the rate of expansion in the model i.e., $\eta\ \alpha\ \theta$ or $\eta=l\,\theta$ and using (15) and (16), we write

$$\eta=2\,l\frac{B_4}{AB}, \tag{17}$$

where l is the constant of proportionality.

From equations (11-12), we have

$$\left(\frac{A_4}{A}\right)_4-\left(\frac{B_4}{B}\right)_4=16\pi\eta\frac{1}{A}\left(\frac{B_4}{B}-\frac{A_4}{A}\right) \tag{18}$$

Using (16) and (17) in (18), we yield

$$C_4=\beta_1 Ce^{-32\pi l\int\frac{B_4}{B}\frac{1}{A^2}dt} \tag{19}$$

It is difficult to solve the RHS integration in the above equation (19), as both functions $B(t)$ and $A(t)$ are not specified and therefore for simplicity one may assume $A=A(t)=1$, so that the condition $B=AC$ leads to $B=C$. Thus the equation (19) takes the form

$$C_4=\beta_1 CB^{\alpha}$$

where $\alpha=-32\,\pi\,l$ and β_1 is integrating constant.

Using the assuming condition $A=A(t)=1$ and $B=C$, the field equations (11) to (14) reduced in the form

$$2\left(\frac{B_{44}}{B}-\frac{B_4^{\ 2}}{B^2}\right)=-16\pi\left(p-\left(\xi-\frac{2}{3}\eta\right)\theta\right)-2\Lambda \tag{20}$$

$$0=\left(p-2\eta\frac{B_4}{B}-\left(\xi-\frac{2}{3}\eta\right)\theta\right)-2\Lambda \tag{21}$$

$$2\left(\frac{B_{44}}{B}-\frac{B_4^{\ 2}}{B^2}\right)=\ 16\pi\,\rho-2\Lambda \tag{22}$$

which after simplification, we obtain

$$A=A(t)=1 \tag{23}$$

$$B(t) = C(t) = (\alpha t + \beta + \Lambda)^{\frac{1}{\alpha}} \tag{24}$$

Using equations (23) and (24), the metric (4) takes the form,

$$ds^2 = -dt^2 + dx^2 + \left(M \Big/ (\alpha t + \beta + \Lambda)^{\frac{1}{\alpha}} \right)^2 (dy^2 + dz^2) \tag{25}$$

After using the transformation of coordinates

$$(\{\alpha t + \beta + \Lambda)^{\frac{1}{\alpha}} = T, \quad x = X, \quad y = Y, \quad z = Z$$

The above metric (25) can be re-written as

$$ds^2 = -\frac{1}{(32\pi l - \Lambda)^2} dT^2 + dX^2 + M^2 T^{1/16\pi l}(dY^2 + dZ^2) \tag{26}$$

(Using $\alpha = -(32\pi l - \Lambda)$).

This is a plane symmetric viscous fluid cosmological model in bimetric theory of gravitation.

From equation (22), we obtain the energy density ρ as

$8\pi\rho = -1/\,32\pi l T^2 - \Lambda$.

Now we assume the third condition $p = \gamma\rho$, $(0 \le \gamma \le 1)$, as the fluid obeys the equation of state. Then from equation (21), we get the coefficient of bulk viscosity ξ as

$\xi = -(32l - 3\gamma)/1536T - \Lambda$.

From equation (17), we get the coefficient of shear viscosity η as

$\eta = 1/16\pi T - \Lambda$.

The pressure p is given by

$8\pi p = -\gamma\,/\,32\pi l T^2 - \Lambda$.

III. Physical Quantities

The physical quantities like isotropic pressure p, energy density ρ, the coefficient of shear viscosity η, bulk viscosity coefficient ξ, and the scalar expansion θ for the model (26) are given by

$$8\pi p = 8\pi\gamma\rho = -\gamma\,/\,32\pi l T^2 - \Lambda \tag{27}$$

$$\eta = 1/16\pi T - \Lambda \tag{28}$$

$$\xi = -(32l - 3\gamma)/1536T - \Lambda \tag{29}$$

$$\theta = 1/16\pi l T - \Lambda \tag{30}$$

The components of shear tensor (σ_i^j) are given by

$$\frac{1}{2}\sigma_1^1 = -\sigma_2^2 = -\sigma_3^3 = \frac{-1}{96\pi l T} - \Lambda, \qquad \sigma_4^4 = 0 \tag{31}$$

The shear σ and rotation ω in the model are

$$\sigma = 0 \text{ and } \omega = 0 \tag{32}$$

IV. Conclusion

There is no big-bang singularity at $T = 0$, in our model (26). The isotropic pressure p and energy density ρ in the model, both are negative which suggest that the model do not exists. The coefficient of bulk viscosity ξ is negative for $l > \frac{3\gamma}{32} - \Lambda$. The expansion θ in the model decreases and ultimately goes to zero as $T \to \infty$. Also the components of shear tensor σ_j^i are negative. This supported the non-existence of the model.It is note that when $T \to \infty$, then $p \to 0,\ \rho \to 0,\ \xi \to 0,\ \eta \to 0,\ \theta \to 0,\ \omega \to 0,\ \sigma \to 0$. This confirms that our model (26) strongly suggest the vacuum universe without viscous fluid and without expansion, rotation and shearing, for a very large value of T.

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