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Two Fluid Cosmological Model in Teleparallel Gravity

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ABSTRACT
In this paper, we have investigated Locally Rotationally Symmetric (LRS) Bianchi type-I space time with two fluid in the $f(T)$ theory of gravitation. The field equations are solved by using some physical assumptions and conservation laws. Some physical and kinematical aspects of cosmological model like expansion scalar θ, shear scalar σ^2, average Hubble parameter H are studied.
Keywords: Bianchi type-I, $f(T)$ Theory, Torsion, Two Fluid.

I. INTRODUCTION

Recent experiments like high redshift supernovae type-Ia [1-4] indicate that the universe is under accelerated expansion. This late time accelerated expansion had been explain by two different approaches. The first approach implies that this acceleration is due to an exotic energy with negative pressure known as dark energy (DE). The second approach deals with modifying the gravitational action. This approach is adopted by modifying the standard theories of gravity. Thus modified theories of gravity have gained a lot of interest in last few years. One of the simplest and most discussed modifications is the $f(T)$ theory of gravity, where T is the torsion scalar.

The $f(T)$ theory of gravity was firstly introduced by Ferraro and Fiorini [5]. Bengochea and Ferraro [6] studied that the late time acceleration of the universe can be explained by modifying Teleparallel equivalent of General Relativity. This $f(T)$ theory is an extension of the Teleparallel theory of gravity. Recently $f(T)$ theory of gravity has attracted many researchers. The $f(T)$ theory of gravity uses the weitzenbök connections which have no curvature but only torsion. Here torsion is responsible for the accelerated expansion of the universe. Wu and Yu [7] have stated that this theory produces field equations of second order and hence easy to solve as compared to the fourth order equations of $f(R)$ theory. Dawande, Adhav and Nerkar [8] have studied the LRS Bianchi type-I universe in $f(T)$ theory of gravity by using a conservation laws.

Sharif and Rani [9] have studied $f(T)$ models with perfect fluid in Bianchi type-I universe. The exact solution of Bianchi type-I are obtained by Nashed [10]. Dawande, Adhav and Khan [11] have studied plane symmetric universe with dark energy in the context of $f(T)$ theory of gravity. Mahanta and Sarma [12] have studied LRS

Bianchi type-I space-time in $f(T)$ gravity. Mete and Raut [13] have derived some exact solutions of Bianchi type-I space-time in the context of $f(T)$ theory of gravitation.

Adhav [14] investigated two fluid cosmological model in Bianchi type-III space-time. Mete *et al.* [15] have discussed two fluid cosmological model using plane symmetric Bianchi type-I metric. Mete, Umarkar and Pund [16] have studied Kasner cosmological model with two fluid. Dagwal and Pawar [17] have studied two fluid FRW cosmological model in $f(T)$ theory of gravity. Coley and Dunn [18] have studied Bianchi type-VI_0 cosmological model with two fluid. Samanta [19] have investigated two fluid anisotropic Bianchi type-III cosmological model with variable gravitational constant G and cosmological constant $\wedge$ in the context of General relativity.

With the motivation from above mentioned work, we have investigated LRS Bianchi type-I cosmological model with two fluid in $f(T)$ theory of gravity.

II. $f(T)$ GRAVITY FORMALISM

The action of the $f(T)$ theory of gravity is defined by generalizing the action of Teleparallel theory of gravity given by

$$S=\int[f(T)+L_{matter}]e\,d^4x \qquad (1)$$

Here, $f(T)$ denotes the differentiable function of the torsion scalar T and L_{matter} is the matter Lagrangian, where $e=\sqrt{-g}$.

The field equations of the $f(T)$ theory of gravity is obtained by varying the action with respect to the tetrads in the following form

$$\left[e^{-1}\partial_\mu\left(eS_i^{\mu\nu}\right)-h_i^\lambda T^\alpha{}_{\mu\lambda}S_\alpha^{\nu\mu}\right]f_T+S_i^{\mu\nu}\partial_\mu T f_{TT}+\frac{1}{4}h_i{}^\nu f=\frac{1}{2}h_i{}^\alpha T_\alpha^\nu \quad (2)$$

where T_α^ν is the energy momentum tensor, $f_T=\dfrac{df}{dT}$ and $f_{TT}=\dfrac{d^2f}{dT^2}$.

III. FIELD EQUATIONS

We consider LRS Bianchi type-I space-time of the form

$$ds^2=dt^2-A^2(t)dx^2-B^2(t)\left[dy^2+dz^2\right]. \quad (3)$$

We obtain the tetrad components as follows

$$h^i_\mu=diag(1, A, B, B) \text{ and}$$

$$h_i^\mu=diag(1, A^{-1}, B^{-1}, B^{-1}) \ . \qquad (4) \qquad (4)$$

The energy momentum tensor for the two fluid sources given by

$$T_{ij}=T_{ij}^{(m)}+T_{ij}^{(r)} \ , \qquad (5)$$

where $T_{ij}^{(m)}$ is the energy momentum tensor for matter field with the energy density ρ_m and pressure p_r, $T_{ij}^{(r)}$ is the energy momentum tensor for radiation field having energy density ρ_r and pressure $p_r = (1/3)\rho_r$. The $T_{ij}^{(m)}$ and $T_{ij}^{(r)}$ are respectively given (Dagwal and Pawar, 2020) as

$$T_{ij}^{(m)} = (p_m + \rho_m) u_i^{(m)} u_j^{(m)} + p_m g_{ij} \,, \quad (6) \qquad (6)$$

$$T_{ij}^{(r)} = \frac{4}{3}\rho_r u_i^{(r)} u_j^{(r)} + \frac{1}{3}\rho_r g_{ij} \,. \qquad (7)$$

The four-velocity vectors are given by

$$u_i^{(m)} = (0, 0, 0, 1) \text{ and } u_i^r = (0, 0, 0, 1) \text{ with}$$

$$g^{ij} u_i^{(m)} u_j^{(m)} = 1 \text{ and } g^{ij} u_i^{(r)} u_j^{(r)} = 1. \qquad (8)$$

The components of the torsion tensor are defined by

$$T^{\alpha}{}_{\mu\nu} = \Gamma^{\alpha}_{\nu\mu} - \Gamma^{\alpha}_{\mu\nu} = h_i^{\alpha}(\partial_\mu h^i_\nu - \partial_\nu h^i_\mu) \,, \qquad (9)$$

which gives

$$T^1_{41} = \frac{\dot{A}}{A} \,,\; T^2_{42} = \frac{\dot{B}}{B} \,,\; T^3_{43} = \frac{\dot{B}}{B} \,. \qquad (10)$$

The components of the corresponding contorsion tensor are defined as

$$K^{uv}{}_{\alpha} = -\frac{1}{2}(T^{\mu\nu}{}_{\alpha} - T^{\nu\mu}{}_{\alpha} - T_{\alpha}{}^{\mu\nu}) \,, \qquad (11)$$

which gives

$$K^{41}{}_1 = \frac{\dot{A}}{A} \,, T^{42}{}_2 = \frac{\dot{B}}{B} \,, T^{43}{}_3 = \frac{\dot{B}}{B} \,. \qquad (12)$$

The components of the skew symmetric tensor are

$$S_{\alpha}{}^{\mu\nu} = \frac{1}{2}(K^{\mu\nu}{}_{\alpha} + \delta^{\mu}_{\alpha} T^{\beta\nu}{}_{\beta} - \delta^{\nu}_{\alpha} T^{\beta\mu}{}_{\beta}) \,, \quad (13) \text{ which gives}$$

$$S_1{}^{14} = \frac{\dot{B}}{B} \,,\; S_2{}^{24} = \frac{1}{2}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) , S_3{}^{34} = \frac{1}{2}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right). \quad (14)$$

Using (10) and (14), we get torsion scalar $T = S_{\alpha}{}^{\mu\nu} T^{\alpha}{}_{\mu\nu}$ as

$$T = -2\left(2\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}^2}{B^2}\right). \qquad (15)$$

The field equations (2) for the metric (3) are obtained as

$$f + 4 f_T\left[\frac{\dot{B}^2}{B^2} + 2\frac{\dot{A}}{A}\frac{\dot{B}}{B}\right] = 2(\rho_m + \rho_r) \,, \qquad (16)$$

$$f + 4 f_T\left[\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{\dot{A}}{A}\frac{\dot{B}}{B}\right] + 4\frac{\dot{B}}{B}\dot{T} f_{TT} = 2\left(p_m + \frac{\rho_r}{3}\right) , \qquad (17)$$

$$f + 2f\left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + 3\frac{\dot{A}}{A}\frac{\dot{B}}{B}\right] + 2\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)\dot{T} f_{TT} = 2\left(p_m + \frac{\rho_r}{3}\right) . (18)$$

Here an overhead dot denotes derivative with respect to cosmic time t.

The physical quantities which are important in cosmological investigation are listed below:

Directional Hubble parameter along x, y and z axes are $H_1 = \frac{\dot{A}}{A}$, $H_2 = H_3 = \frac{\dot{B}}{B}$. (19) The mean Hubble parameter

$$H = \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) \qquad (20)$$

The scalar expansion $\theta = \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)$. (21) Deceleration parameter $q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1$. (22)

Shear scalar $\sigma^2 = \frac{1}{3}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2$. (23)

Anisotropy parameter $A_m = \frac{2}{9H^2}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2$. (24)

IV. COSMOLOGICAL SOLUTION

To solve the field equations (18) - (20), we need some additional constraints. One of the commonly used additional constraints is that the expansion scalar θ is proportional to shear scalar σ which leads to:

$$A = B^m. \qquad (25)$$

Using this, field equations (16) – (18) reduces to

$$f + 4 f_T (1 + 2m)\frac{\dot{B}^2}{B^2} = 2(\rho_m + \rho_r), \qquad (26)$$

$$f + 4 f_T\left[\frac{\ddot{B}}{B} + (1+m)\frac{\dot{B}^2}{B^2}\right] + 4\frac{\dot{B}}{B}\dot{T} f_{TT} = 2\left(p_m + \frac{\rho_r}{3}\right), \qquad (27)$$

$$f + 2 f_T\left[(1+m)\frac{\dot{B}^2}{B^2} + (1+m)\frac{\ddot{B}}{B}\right] + 2(1+m)\frac{\dot{B}}{B}\dot{T} f_{TT} = 2\left(p_m + \frac{\rho_r}{3}\right). \qquad (28)$$

Now, $f_T = \frac{df}{dT} = \frac{\dot{f}}{\dot{T}}$ and

$$f_{TT} = \frac{d^2 f}{dT^2} = \frac{\dot{T}\ddot{f} - \ddot{T}\dot{f}}{\dot{T}^2}\left(\frac{1}{\dot{T}}\right). \qquad (29)$$

Subtracting (28) from (27) and using (29), we get

$$\frac{\ddot{B}}{B} + (1+m)\frac{\dot{B}^2}{B^2} + \frac{\dot{B}}{B}\frac{\dot{F}}{F} = 0, \text{ where } F = f_T. \qquad (30)$$

Now, $F = f_T = \frac{\dot{f}}{\dot{T}}$ and the scale factor B both are functions of the cosmic time t, so without loss of generality we take a relation between F and B (C. Mahanta and N. Sarma, 2018) as

$$F = kB^{1+m}, \text{ where } k \text{ is the constant} \qquad (31)$$

Using this in (30), it follows that

$$\frac{\ddot{B}}{B} + (2 + 2m)\frac{\dot{B}^2}{B^2} = 0. \qquad (32)$$

Integrating above equation, we get

$$B=\{(3+2m)(c_1t+c_2)\}^{\frac{1}{3+2m}}. \qquad (33)$$

Now, (25) gives

$$A=\{(3+2m)(c_1t+c_2)\}^{\frac{m}{3+2m}}. \qquad (34)$$

Thus metric given in (8) reduces to

$$ds^2=dt^2-\{(3+2m)(c_1t+c_2)\}^{\frac{m}{3+2m}}dx^2-\{(3+2m)(c_1t+c_2)\}^{\frac{1}{3+2m}}\left[dy^2+dz^2\right]. \qquad (35)$$

From (15), we get

$$T=-2\left[\frac{(1+2m)}{(3+2m)^2}c_1^{\,2}(c_1t+c_2)^{-2}\right]. \qquad (36)$$

Now, we take Equation of State (EoS) which gives

$$p_m=(\gamma-1)\rho_m,\quad 1\le\gamma\le 2. \qquad (37)$$

Conservation law separated for radiation and matter field are

$$\dot{\rho}_m+[H_1+2H_2](\rho_m+p_m)=0, \qquad (38)$$

$$\dot{\rho}_r+[H_1+2H_2]\left(\frac{4}{3}\rho_r\right)=0. \qquad (39)$$

Using (37) in (38) and using the values of H_1, H_2 we get

$$\rho_m=(c_1t+c_2)^{-\frac{2+m}{3+2m}\gamma}, \qquad (40)$$

$$p_m=(\gamma-1)(c_1t+c_2)^{-\frac{2+m}{3+2m}\gamma}. \qquad (41)$$

Solving (39), we get

$$\rho_r=(c_1t+c_2)^{-\frac{2+m}{3+2m}\left(\frac{4}{3}\right)}. \qquad (42)$$

Since $p_r=\frac{1}{3}\rho_r$, we get

$$p_r=\frac{1}{3}(c_1t+c_2)^{-\frac{2+m}{3+2m}\left(\frac{4}{3}\right)}. \qquad (43)$$

Using (25), (33), (34), (40) and (42) we get

$$f=\frac{1}{T^{1/2}}\int\left[\frac{1}{(c_1t+c_2)^{\frac{2+m}{3+2m}\gamma}}+\frac{1}{(c_1t+c_2)^{\frac{2+m}{3+2m}\left(\frac{4}{3}\right)}}\right]T^{-1/2}\,dT. \qquad (44)$$

Using (20) – (23) we get

The mean Hubble parameter

$$H=\frac{1}{3}\left\{c_1\frac{(2+m)}{(3+2m)}(c_1t+c_2)^{-1}\right\}. \qquad (45)$$

The scalar expansion

$$\theta=\left\{c_1\frac{(2+m)}{(3+2m)}(c_1t+c_2)^{-1}\right\}. \qquad (46)$$

Shear scalar $\sigma^2 = 2\left(\frac{m-1}{2+m}\right)^2$. (47)

Deceleration parameter $q = \left(\frac{6m+9}{2+m}\right) - 1$. (48)

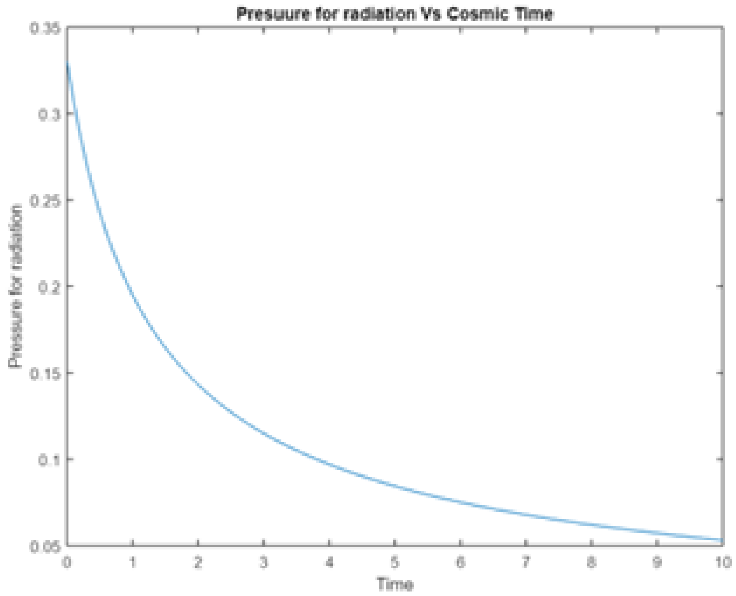

Figure (1): Pressure for radiation verses cosmic time at $c_1 = 1,\ c_2 = 1,\ m = 2$.

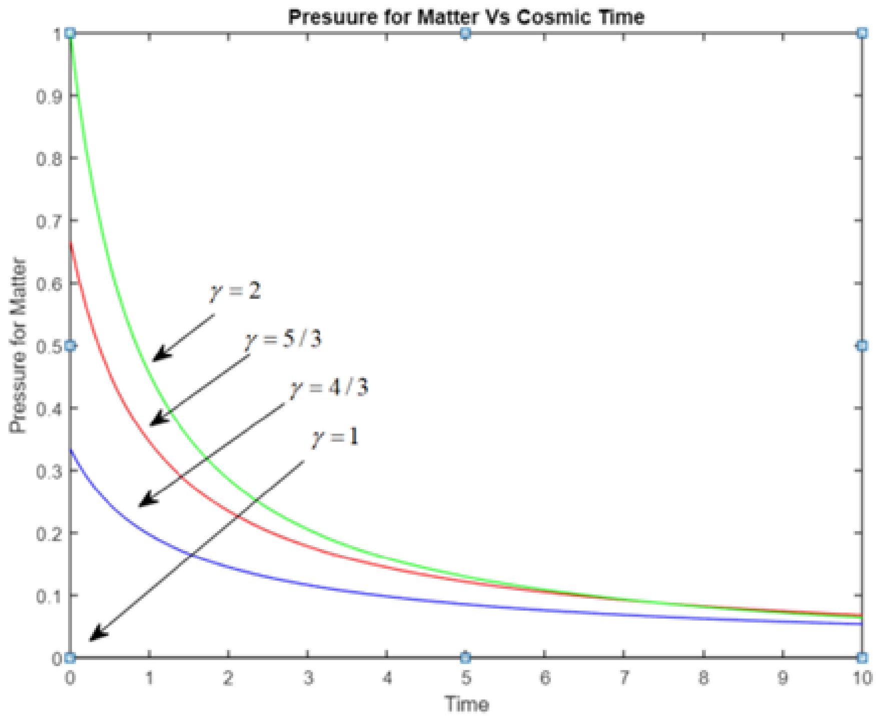

Figure (2): Pressure for matter verses cosmic time at $c_1 = 1,\ c_2 = 1,\ m = 2,\ \gamma = 1, \frac{4}{3}, \frac{5}{3}, 2$.

V. CONCLUSION

In this paper, we have investigated LRS Bianchi type-I space-time with two fluid in $f(T)$ gravity. We have flowing concluding remarks:

- From (45), (46) & (47), we conclude that parameters H and θ are infinite at the initial stage and at $t = -c_2 / c_1$. It means that there is a singularity at $t = -c_2 / c_1$ in the model also Hubble parameter, scalar of expansion and shear scalar are vanishing for large value of cosmic time.
- From (40) and (42), we conclude that energy density for matter and radiation approach toward zero for large value of cosmic time and the energy density for matter and radiation diverge initially and at $t = -c_2 / c_1$.
- From figure (1) and (2), we conclude that pressure for matter approaches toward zero as t approaches to ∞ and the pressure for matter diverge at $t = 0$ and at $t = -c_2 / c_1$.
- Sign of q indicates whether the model expanding or not. The observations of Type la Supernovae and CMBR indicate accelerating model. From (48) we conclude that our model is in agreement with these observation for $m < -1.4$.
- From (35), we conclude that our model has a singularity at $t = -c_2 / c_1$.
- From (46) & (47), we have $\frac{\sigma^2}{\theta^2}$ = constant, thus we conclude that model does not approach isotropy throughout the whole evolution of the universe.

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