International Journal of Scientific Research in Computer Science, Engineering and Information Technology

ISSN : 2456-3307 OPEN ACCESS

Available Online at : www.ijsrcseit.com

DOI: https://doi.org//10.32628/IJSRCSEIT

Graph Theory and its Applications in Image Processing: An Overview

Mamatha N[1*], Bhuvaneshwari[2], Bhagyalaxmi B S[3]

[1*]Lecturer, Department of Science, Karnataka (Govt) Polytechnic, Mangalore, Karnataka, India.

[2]Lecturer, Department of Computer Science and Engineering, Government polytechnic for women, Bondel Mangalore, Karnataka, India.

[3]Lecturer, Department of Computer Science and Engineering, Department of Technical education, Government Polytechnic for Women, Ramanagara, 562159, Karnataka, India.

***Corresponding Author :** mamtha123devadiga@gmail.com

ARTICLEINFO

Article History:

Accepted: 01 Jan 2019

Published: 12 Feb 2019

Publication Issue

Volume 5, Issue 1

January-February-2019

Page Number

662-668

ABSTRACT

In a graphical setting, the nodes are referred to as vertices and the connections as edges. Graph theory is a robust area of mathematics that seeks to facilitate the study of relationships between various things. Today, it suffices to say that many fields of sciences and technologies are based on the graph theory which, by the way, was introduced in 1736 by Leonhard Euler in the light of his the famous problem of the Seven Bridges of Königsberg. This strong theory enables modeling and solving of very complex problems, like those related to networked systems. The primary components of the graphs: vertices, edges, adjacency matrices, and graph traversal algorithms allow the representation of data structures, geographical data, and multi-level hierarchical structures. Graph theory is used extensively in other disciplines such as optimization, bioinformatics, computer science, logistics, and AI.

Keywords : Vertices, Edges, Graph Theory, Adjacency Matrices, Graph Traversal Algorithms

Introduction

1. Introduction

Overview of Graph Theory

Nodes (vertices) and edges (links) in a graph structure represent various things, and the investigation of their reliance on each other is the overall theme of graph theory, a strong mathematical discipline. Leonhard Euler established graph theory in 1736 with the now-well-known Seven Bridges of Königsberg problem; decades later, it was the basis of other sciences and engineering fields. Graph theory has a strong basis in graphically modeling and solving complex problems with interacting systems. Graph search algorithms, adjacency matrices, nodes, and edges are the graph elements that well represent data structures, geographical connections, and hierarchical dependencies. Graph theory is applied in most fields, such as optimization methods, biology, logistics, computer science, and artificial intelligence [1].

Importance of Graph Theory in Computational Applications

Graph theory is at the core of data organization and analysis in computational science. Network analysis, algorithm design, and optimization problems are areas where graph models are particularly good due to their ability to model interconnected datasets. Machine learning relies heavily on graphs as data structures in categorization algorithm algorithms, recommendation systems, and knowledge graphs. Graph models also find their way in genome sequencing, protein interaction networks, and brain network connectivity studies in biological computing. The use of graph algorithms to filter and transform complicated signals in structured data, such as social networks and sensor networks, is a significant computational application in digital signal processing (DSP). Furthermore, search engines and AI decision-making systems also benefit from graph-based approaches' ability to facilitate sophisticated grouping, ranking, and data retrieval techniques [2].

Relevance to Image Processing

Graph theory forms a robust foundation for precise data modeling in image processing. When perceiving images as arrays of intensity values, the traditional pixel-based methods may obstruct the visualization of intrinsic global structures and spatial relationships. Images can, however, be modeled as graph models by graph-based methods. In such a model, pixels, superpixels, or regions are represented as vertices, and their interactions are represented as edges. Computational processes, such as classification, segmentation, filtering, and feature extraction, are facilitated by such a graph-based organization. To segment an image into useful sections based on similarity and connection evaluation, graph-based segmentation algorithms employ region adjacency graphs (RAG). Computer vision tasks also utilize graph cuts and spectral clustering, two robust segmentation formulations. Moreover, graph-based denoising approaches such as graph signal processing and graph Laplacian smoothing denoise images while maintaining their inherent structural information. Such approaches significantly improve the precision and reliability of 3D scene reconstruction, object recognition, face recognition, and medical image analysis. Finally, graph-based models are beneficial in modern image processing applications because they outperform pixel-based methods by being relation and structure sensitive [3,4].

2. Fundamentals of Graph Theory

Basic Definitions and Concepts

Vertices (V) and edges (E) between them in pairs form a graph, a mathematical structure. The structure of the graph is defined as the relationship between edges and vertices. The structure characterizes different types of graphs. If a graph has all its edges directed in one direction, then the graph is referred to as a digraph or directed graph. In an undirected graph, nodes can send messages to one another in both directions because of its undirected edges. Otherwise, weighted graphs are graphs in which edges have a value representing the strength of the relationship between nodes, e.g., intensity, distance, or similarity. Weighted graphs are a class of graph with an abundance of applications in applications such as clustering, shortest paths, and network analysis. Bipartite graphs are another class of graph; in bipartite graphs, vertices are separated into two sets and edges join nodes in different sets. From scheduling to network flow analysis and matching problems, bipartite graphs have an abundance of applications [5].

Efficiency in computation relies on the representation of the graph because different representations yield different levels of performance for specific graph operations. Two of the most widely used techniques utilized to represent a graph include the adjacency matrix and adjacency list. In the adjacency matrix, a two-dimensional array, a non-zero entry means there is an edge between two vertices. If the number of edges is roughly squared times the number of vertices in a dense graph, then such a representation is useful in performing matrix-like operations and quick lookups of edges. Space inefficiency is when adjacent matrices are utilized on sparse graphs, where the number of edges is much smaller than the maximum number of edges. We discover that the adjacency list form is superior in some cases. Each vertex in an adjacency list contains a list of its immediate neighbors; the arrangement is comparable to a linked list or array. Sparse graphs require much less memory with this form and can be traversed and updated efficiently. In any given application, considerations of the computation speed, memory requirements, and requirement for dynamic graph updates dictate whether an adjacency matrix or an adjacency list is more suitable [6].

3. Role of Graph Theory in Image Processing Effectiveness of Graph-Based Approaches Compared to Traditional Methods

Graph-based methods offer a strong and organized platform for the spatial and hierarchical representation of visual information. Graph-based models can support more abstract representations with global and local interdependencies than pixel-based methods which can operate only at the discrete pixel or intensity level. Due to this attribute, they find their best applications in advanced image processing operations like texture analysis, object recognition, and segmentation. The representation of the interdependency of picture regions rather than pixels is a fundamental strength of graph-based methods that enables them to preserve context information. By challenging the noise sensitivity and variation with light of the traditional methods, this structural format improves feature detection and segmentation effectiveness [3,7]. The majority of the traditional image processing methods like thresholding, edge detection, and intensity-clustered grouping tend to work on pixels and therefore succumb to picture noise as well as intensity variation. On objects with featureless boundaries, these lose critical information regarding the structures. Graph-based methods, in contrast, apply spectral grouping, region adjacency graphs (RAG), and graph cuts in a bid to enhance segmentation effectiveness as well as include pixel topology and spatial connections. A few out of a long list of applications overwhelmingly gaining benefit from graph models are medical imaging, biometric identification, and satellite image analysis [7]. Another advantage of graph-based methods is the ability to represent an image at a number of different sizes, accommodating hierarchical processing as well as enhanced computing efficiency. Researchers have proved that graph-based models surpass traditional image processing algorithms in homogeneously dealing with complex visual patterns, texture identification, and object tracking. In order to achieve this, they use state-of-the-art techniques such as graph signal processing, graph convolutional networks (GCNs), and random walk segmentation. Thus, modern image processing systems using graph-based techniques not only provide improved segmentation performance, but also improved robustness and adaptability to a wide range of real-world tasks [7].

4. Graph-Based Image Representation

Graphs are a strong image representation technique; nodes are pixels or areas, and edges are proximity relationships based on how close they are to each other or how much their features overlap. A pixel graph has one node per pixel and uses edges to represent how close pixels are to each other in terms of intensity or position. Computationally costly for high-quality images, yet offering fine-grained analysis and correct segmentation, pixel-level representations such as these exist. Segmentation of the picture into significant pieces or superpixels and using each as a node, region adjacency graphs (RAG) have efficiency costs in check. To very much simplify things and maintain valuable structural information regarding the picture, RAG models employ edges to represent the relationships between adjacent regions. This is further developed using superpixel graphs, which divide pixels into meaningful regions based on intensity, texture, or color similarities. Reducing the network's overall number of nodes while maintaining picture detail, this assists in enhancing computing efficiency. Due to the fact that they are able to find a compromise between computing frugality and segmentation accuracy, superpixel-based graph representations have extensive use in picture segmentation, object detection, and picture retrieval. Graphs are a strong tool in image analysis and computer vision because different graph-based models enhance feature representation, segmentation accuracy, and stability to noise and changes in illumination [3,8].

5. Graph-Based Image Segmentation Techniques

To segment an image into semantically meaningful segments effectively and accurately, graph-based segmentation algorithms utilize a wide range of mathematical techniques. Minimum Spanning Tree (MST) is one of the most efficient methods in this group, which places pixels in a way that the overall weight of the edges is minimized and thus the connectivity among pictures is maximized. Medical imaging, object recognition, and remote sensing are significantly improved by MST-based segmentation as it can identify regions of low variability within high-variability clusters with high accuracy [1,9]. Normalized Cuts and Spectral Clustering is another widely used method. It uses eigenvector decomposition to uncover general picture structures, and segments the image from its global similarity rather than local interaction. The method works perfectly with complex images with fuzzy object boundaries [9]. Table:1 presents Graph-based Image Segmentation Techniques.

Table:1 Graph-based Image Segmentation Techniques

Category	Method
Discontinuity	
Edge-based	
Gradient-based	Sobel
Gradient-based	Robert
Gradient-based	Prewitt
Gaussian-based	Canny edge detection
Gaussian-based	Laplacian of Gaussian
Similarity	
Clustering-based	K-mean
Clustering-based	Mean-shift
Thresholding	Global
Thresholding	Local
Region-based	Split and Merge
Region-based	Region growing
Region-based	Graph method

Max-Flow/Min-Cut and Graph Cuts One of the most powerful techniques of picture segmentation is to convert the problem into an energy minimization problem. Medical image segmentation and interactive image editing operations are greatly helped by these techniques, which partition an image into two distinct foreground and background regions by minimizing a large cost function [2]. Random Walks for Segmentation's weighted network connectivity estimates pixel class probabilities, producing smooth and consistent segmentations. Texture segmentation and biological image analysis operations rely on this technique's improved border localization and object boundary detection afforded by its probabilistic segmentation formulation [4]. Combined, these graph-based segmentation algorithms greatly improve the accuracy, flexibility, and performance of current image processing software.

6. Graph Theory in Image Filtering and Denoising

A strong method of image smoothing with high preservation of edge features by weighted graph representation is provided by graph-based filtering methods. Graph-based modeling is a representation of the image in terms of a weighted graph, where pixel intensities are nodes and edges represent spatial and feature-based relationships, unlike traditional filtering techniques, which lose edge integrity during noise suppression. This enables structural integrity to be maintained while selective noise reduction is achieved, thus avoiding over-smoothing in important regions [6]. Graph Signal Processing (GSP) is another advanced method that is of extreme significance in image restoration and enhancement since it can extend the domain of traditional signal processing methods for non-Euclidean graph topologies. In images with intricate textures and structures, GSP is a savior because it redirects traditional filtering methods to accommodate graph designs, enabling efficient signal processing of signals distributed in non-Euclidean spaces [5,6]. Further, the Graph Laplacian operator is of extreme significance in image smoothing since effective noise suppression accompanied by preservation of important image characteristics is enabled by it. A quality enhancement in the image accompanied by preservation of important object boundaries is enabled by regulation of intensity variations in the image by application of the Laplacian matrix, derived from the graph structure. Methods in image processing such as medical imaging, remote sensing, and high-resolution photography are significantly aided by the reciprocal effectiveness of these graph-based techniques to image filtering and denoising [6].

7. Graph-Based Image Registration

A core technique in image registration is graph-based feature point matching. The process precisely identifies correspondences among keypoints of individual images. Awaiting structural and relational information in images, graph-based matching algorithms can effectively build these correspondences. The practices guarantee robust feature alignment irrespective of image transformation like scaling, rotation, or noise corruption by representing keypoints as nodes and the interactions among them as edges [7]. By eliminating distortions and optimizing the similarity of matched points, most graph-based correspondence methods like spectral matching and graph edit distance improve the feature alignment process. Object recognition, 3D reconstruction, and medical imaging applications are all improved using these technologies, and registration accuracy is also improved. Compared to more conventional pixel-based matching methods, which tend to suffer from image quality variation and transformation, graph-based matching methods are more appropriate as they can model complex structural relationships [3,7].

Graph-Based Object Recognition, Feature Extraction, Compression, and Retrieval

To properly represent and analyze image information, graph models are essential in object recognition, feature extraction, image compression, and CBIR. Graph models are very efficient in recognizing patterns, shape matching, and texture classification due to their ability to represent object shape, texture, and structural characteristics in an efficient manner. The methods perform efficient pattern and shape detection under changing imaging conditions by modeling objects as graphs with nodes storing segments or keypoints and edges encoding their spatial relationships. Graph-based feature descriptors improve the performance of object detection, biometric identification, and medical imaging accuracy through optimization of keypoint matching performance [8] by considering local and global image features.

To further enhance feature learning and image classification in image processing, Graph Neural Networks (GNNs) have effectively applied deep learning to non-Euclidean data. While, compared to GNNs, which employ graphs to represent complex relationships in non-regular image structures, conventional CNNs handle data structured in a regular grid structure. By applying convolution directly to graph structures, Graph Convolutional Networks (GCNs) enhance the accuracy of picture segmentation, object detection, and classification considerably. When handling pictures predominantly composed of well-structured but non-regular data, which conventional CNNs do not perform well on, such as in autonomous driving, satellite image interpretation, and biomedical picture segmentation, the networks excel [8–12].

Another advantage of graph theory-based methods is that they retain structural relationships rather than raw pixel intensities, preventing redundancy in image data. With respect to high-resolution images and video processing, this is particularly useful because optimal compression maintains small file size without compromising image quality. By saving only the most critical structural content of an image and eliminating redundancy, graph-based methods such as minimal spanning trees (MST), spectral graph wavelets, and graph-cut-based encoding achieve better compression [9, 13-16]. Lastly, picture searches and retrievals are now revolutionized by content-based image retrieval (CBIR) based on graph models. Visual content is favored over metadata using this method. Image search, face detection, and medical image analysis are significantly enhanced by graph-based CBIR methods, in which discriminative picture features may be extracted and matched using similarity measures of graph. Accurate picture retrieval from visual content is of utmost priority in digital forensics, web image search engines, and computer-aided medical diagnosis, among several other applications which benefit significantly by these methods [9]. Intelligent, efficient, and scalable image processing in many applications is addressed by an end-to-end structure by graph-based methods, including object recognition, deep learning, compression, and retrieval methods.

Conclusion

Graph theory is a strong foundation that is highly compatible with most of the image processing applications. Segmentation, filtering, object detection, and retrieval are only some of the applications where it is superior to the traditional pixel-based approaches because of its higher ability to model complex structures and relationships. To achieve higher efficiency in practical applications, future research should explore even more complete fusion of deep learning techniques with graph models.

References

[1]. Dikholkar, A., Pande, K., Zade, A., Sagne, D., & Paturkar, A. (2015). Image Segmentation Using Iso-perimetric Graph Theory and Its Comparative Analysis. International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering.

[2]. Sanfeliu, A., Alquézar, R., Andrade, J., Climent, J., & Serratosa, F. (2002). Graph-based representations and techniques for image processing and analysis. Pattern Recognition.

[3]. Basavaprasad, B., & Hegadi, R. S. (2016). A Systematic Study on Applications of Graph Theory in Image Processing With a Focus on Image Segmentation. International Journal of Scientific & Engineering Research.

[4]. Lezoray, O., & Grady, L. (2013). Graph theory concepts and definitions used in image processing and analysis. Springer.

[5]. Bershtein L, Bozhenuk A. (2001). A color problem for fuzzy graph. Lecture notes in computer science.

[6]. Parihar, V. R., & Thakur, N. V. (2014). Graph Theory Based Approach for Image Segmentation Using Wavelet Transform. International Journal of Image Processing.

[7]. Tosuni, B. (2015). Graph Theory in Computer Science - An Overview. International Journal of Academic Research and Reflection.

[8]. Sekar, U. (2013). Applications of Graph Theory in Computer Science. International Journal of Electronics Communication and Computer Engineering.

[9]. Brunel, G., Borianne, P., Subsol, G., & Jaeger, M. (2013). Simple-Graphs Fusion in Image Mosaic:

Application to Automated Cell Files Identification in Wood Slices. SCIA Conference Proceedings.

[10]. Broelemann, K., Dutta, A., Jiang, X., & Lladós, J. (2014). Hierarchical plausibility-graphs for symbol spotting in graphical documents. Lecture Notes in Computer Science, 8746, 25-37. https://doi.org/10.1007/978-3-662-44854-0_3

[11]. Du, H., Hu, Q., Zhang, X., & Hou, Y. (2014). Image feature extraction via graph embedding regularized projective non-negative matrix factorization. Communications in Computer and Information Science, 483, 196-209. https://doi.org/10.1007/978-3-662-45646-0_20

[12]. Flasiński, M., & Flasińska, Z. (2014). Characteristics of bottom-up parsable edNLC graph languages for syntactic pattern recognition. Lecture Notes in Computer Science, 8671, 195-202. https://doi.org/10.1007/978-3-319-11331-9_24

[13]. Gaura, J., & Sojka, E. (2014). Resistance-geodesic distance and its use in image processing and segmentation. Lecture Notes in Computer Science, 8887, 238-249. https://doi.org/10.1007/978-3-319-14249-4_23

[14]. Głąbowski, M., Musznicki, B., Nowak, P., & Zwierzykowski, P. (2014). An algorithm for finding shortest path tree using ant colony optimization metaheuristic. Advances in Intelligent Systems and Computing, 233, 317-326. https://doi.org/10.1007/978-3-319-01622-1_36

[15]. Jorge-Hernandez, F., Chimeno, Y. G., Garcia-Zapirain, B., Zubizarreta, A. C., Beldarrain, M. A. G., & Fernandez-Ruanova, B. (2014). Graph theory for feature extraction and classification: A migraine pathology case study. Bio-Medical Materials and Engineering, 24(6), 2979-2986. https://doi.org/10.3233/BME-141118

[16]. Korgaonkar, M. S., Fornito, A., Williams, L. M., & Grieve, S. M. (2014). Abnormal structural networks characterize major depressive disorder: A connectome analysis. Biological Psychiatry, 76(7), 567-574. https://doi.org/10.1016/j.biopsych.2014.02.018